Student book of study visits

Tempus Project "Visuality & Mathematics: Experiential Education of Mathematics through Visual Arts, Sciences and Playful Activities"
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Contents

Preface

This book contains a part of the works that our students did during their one-month study visits to Austria, Hungary, Belgium and Finland. What this book does not contain is something that cannot be presented in the written form: an experience that they will never forget, experience of living and studying in an EU country. Most of these students never traveled abroad. This is one of the most effective results of our VisMath Tempus project.

I want to use this occasion to thank our EU partners for organizing these student study visits and for giving them a first contact to their student-colleagues and professors from our EU partners' universities. Organized vests to historical sites, exhibitions, concerts and other events that they will never forget – these will be the topics of stories that they will be talking to their friend, family members and future children for years!

We also want to thank the Tempus programme for sponsoring these study-visits and our collaborative activities during this two-year project. It was a small effort for the programme and for all of us, but a great achievement with long-lasting positive effects to our students.

Dragan Domazet
Rector of Belgrade Metropolitan University

Study Visits

One of the main challenges and chances of TEMPUS EU project were the study visits of Serbian students and their collaboration with Austrian, Belgium, Finnish and Hungarian students. Intercultural and interdisciplinary exchange took place. They were able to expand their personal and disciplinary potential and co-design innovative solutions for education. Besides their fruitful collaboration, they developed ideas for mathematics education, improved interdisciplinary teamwork and scientific writing skills.

Ruth Mateus-Berr
Chairwoman of the Senate of the University
of Applied Arts Vienna

TEMPUS Study Report Students Visit April-May 2013 Vienna 10.4.–10.5. 2013

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AUSTRIA

dı:'ʌngewʌndtə

Universität für angewandte Kunst Wien
University of Applied Arts Vienna

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PROGRAM

10.4.	10-12am meeting,information, lecture: previous labs	2-4pm Applied Design Thinking Workshop	4-5pm:get to know University of Applied Arts by the students	Evening: occasionally 19:00 Vortrag in den Hofstallungen des Mumok: Rudolf Taschner: Unglaublich, aber mathematisch wahr: Zwei Ziegen und ein Auto
17.4.	1-3pm lecture by Georg Gläser (http://www1.uni-ak.ac.at/geom/staff_gg.php)	3:30-5:30pm Applied Design Thinking Workshop		
18.4.	3pm-5pm "Urban Pilgrimage Vienna": Get to know Vienna with a performative artist: Angela Dorrer. Please bring the following things: good shoes (boots?) and FLAHLIGHT! **MEETING: 2pm Aula Angewandte Kokoschkaplatz**			
24.4.	1-3pm applied design thinking workshop	3:30-5:30pm Applied Design Thinking Workshop		
8.5.	Lecture by Georg Gläser	3:30-5:30pm Applied Design Thinking Workshop	Evening: Get together	

STUDENTS

	Name and Surname	Institution
1	Ivana Supić	ICT
2	Milena Đorđević	ICT
3	Madlena Domazet	Metropoliten
4	Dusan Tatic	MISANU
5	Marina Misanovic	MISANU
6	Ninkov Milovan	UNS
7	Natasa Dzaleta	UNS
8	Jovana Vujicic	UNS
9	Aleksandra Rastik	UNS
10	Andrea Reithofer	UAAK
11	Marie Theres Wakonig	UAAK
12	Christine Gobbi	UAAK
13	Irina Nalis	UAAK
14	Elisabeth Stephan	UAAK
15	Stefan Wykydal	UAAK
16	Klaudia Kozma	UAAK

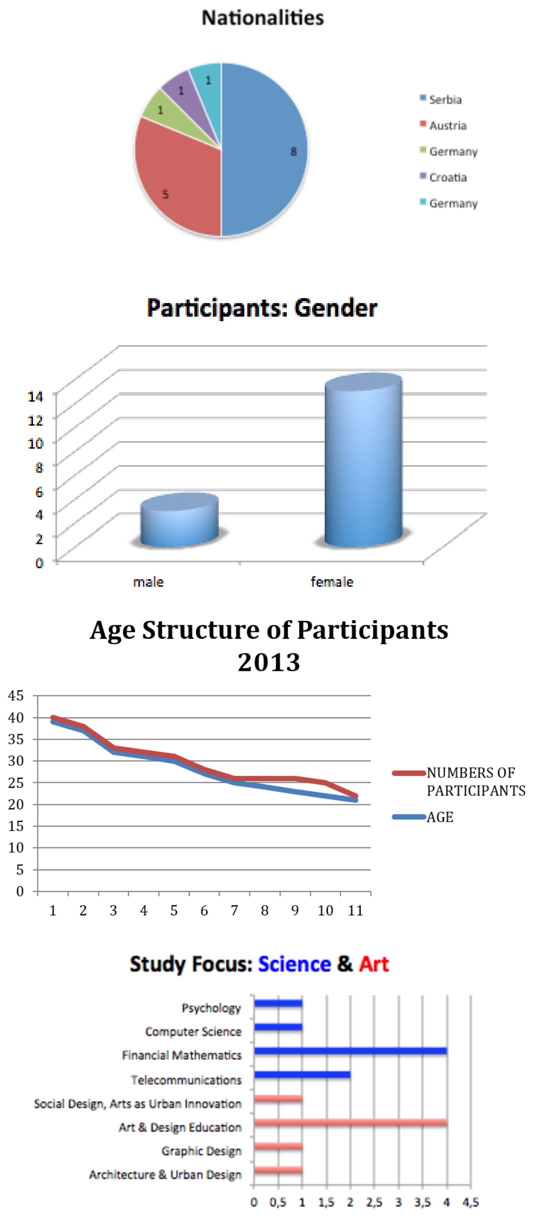

Figure 1, 2, 3: Participation: 1_Nationalities, 2_Gender, 3_Disciplines, Age Structure of Participants
Author: Ruth Mateus-Berr

Applied Design Thinking: Visualizing Mathematics through Art and Design

Ruth MATEUS-BERR

Abstract

Aim of the interdisciplinary workshop at the University of Applied Arts Vienna was to develop educational tools for enhancing playful math education through methods of arts and design. They applied methods of applied design thinking in interdisciplinary teams from Serbia, Hungary, Germany and Austria and developed simple, easily produced manufactures prototypes. Input lectures about math visualizations and design thinking were precondition of work. Results from approaches were to raise understanding of geometrical shapes and their impact on our daily lives through a "play with proportions". By exploring different city floor plans and investigating proportions and geometry one group came up with a task that would give students a chance and freedom to play with and explore geometrical shapes of cities, which have their own sizes and proportions. The second group worked on visualization of finance risk taking. They considered that the topic should be experimented by problem solving through a "risk box" that would allow you to decide the transparency of processes and show the risk percentage. The third group created a tool that would improve the understanding of geometrical shapes by developing a construction, the Augmented Reality Drawing Key, ArtKey. As perspective construction has its roots in the human visual system, added augmented reality widens the activity range. In seeing, touching, moving and listening sustainable learning is guaranteed by implementing all the senses. The fourth group worked on patterns in nature and a possible correlation of patterns in numbers. Mathematical concepts such as Pascal's Triangle, Fibonacci numbers and Golden Ratio were considered in an artistic-mathematical discourse. With the help of an interactive model for exploring ratios and visual relations in human faces, the mathematical concept of Phi shall be addressed. The fifth group had their objectives in connecting arts, mathematics and telecommunication. Explaining how a mobile phone works was considered as an interesting topic for the youth. From physical disassembling mobiles and looking for their antennas until designing individual mobile cases with fractal patterns they developed hands-on tools. The sixth group worked on a new musical notation dictionary, which should offer a new graphical visualization for music, where children have the opportunity to combine geometry with their own visual and acoustic experience.

Keywords: geometry, triangles, squares, proportions, floor plans, urban planning, risk, finances, play, Drawing aid, Augmented reality, perspective, Pascal's Triangle, Fibonacci numbers, Golden Ratio, mathematics, Telecommunication, Fractals, Design education, music, notation, art, geometry, note dictionary, design thinking

1 Introduction: Study Visit Vienna 2013, 2014

Due to the EU TEMPUS Project Visuality & Mathematics: Experiential Education of Mathematics through Visual Arts, Sciences and Playful Activities, Study Visits for Serbian students were planned in Vienna (Austria), Brusseles (Belgium), Jyväskälä (Finland) and Eger (Hungary). In Austria the study visit was planned at the University of Applied Arts Vienna and a schedule of a month was designed for students from Vienna and Serbia to work together.

Expert lectures were planned (Prof. Georg Gläser) as well as descovering Vienna through Urban Pilgrimage Vienna with the performative artist Angela Dorrer in 2013. 2014 Georg Gläser gave two lectures and Prof. Dr. Michael Schreiber another, further the "House of Mathematic" (Dr. Georg Lindbichler) was visited. With methods of Applied Design Thinking they were obliged to develop an innovative educational tool in interdisciplinary teams.

During their visit they were able to visit various recommended English lectures on Architecture and Design as well as a lecture by Prof. Rudolf Taschner or visit various Viennese museums with a special discount. They could participate at lectures for drawing and others which were not part of the entrance examinations clause.

In 2013 sixteen students: nine students from Serbia and Croatia and seven students from Austria, Hungary and Germany deriving from disciplines as psychology; Art & design education; Pedagogics, art history & art education; Graphic design & math, Computer Science & electronic engineering; Painting & graphic design; Applied Mathematics; Financial Mathematics; Theater (film), art history and visual arts, art history & Social Design; Information and Communication Technology, worked one month together with the objective to develop innovative hands-on prototypes as educational tools for the summer school in Eger (Hungary) or in schools in general. In 2014 thirteen: nine students from Serbia, one from Macedonia and four students from Austria, deriving from disciplines as Art & Design Education, Computer Science, Mathematics, Telecommunications, Interactive Media Design, Social Design, Arts as Urban Innovation, Electronic Engineering, Vocational Studies, Information Technology participated.

The Applied Design Thinking Lab (ADTL) Vienna is situated at the University of Applied Arts in Vienna and was founded by the author in 2009. It might be understood as an application model for universities, and institutions of all kind, as it can be considered as a 'hypothesis and action model'. Since 2004 the author followed the vision of inter/transdisciplinary work with diverse institutions, universities cooperation instead of 'single combat' was the strategy.

ADTL approaches inter/transdisciplinary topics with interdisciplinary teams from different departments of universities and institutions. Aim of this Lab is to facilitate innovative solutions for complex problems through interdisciplinary collaboration. Applying tools of Design Thinking strategies, the participants develop their own specific art/design work, inspired by peers, their own broad knowledge, interest of various topics, disciplines and empowered with creativity. The participants derive from different interdisciplinary fields. Experts from various disciplines (mathematicians, fashion designers etc. regarding to each topic) are invited to join the group, reflect on their work and discuss it.

According to Kristensen (2004, 89-96), many design problems arise because there is little integration between the environment, people and technology. He recommends that physical, virtual space and a visual working methodology need to be interconnected in order to enhance a collaborative participation and performance for dispersed teams. The ADTL is such a space and develops innovative and performative transfer of design (and involved disciplines) knowledge. Art and design based research is biased scientifically and applied practically at once. Developed innovative tools may be transferred in the fields of educational subjects (schools and universities), interdisciplinary applied in technologies or unto the fields of the creative industries, change mindsets of companies and enhance creativity of institutions. Design Thinking is a well-established term, a search on Google gives over 340.000.000 hits. But the terminology Thinking rather refers to Aristoteles' episteme (intellectual knowledge) than making (poesis) and doing (praxis). Applied Design Thinking combines both, as Schön described the "reflective practioner" (Schön 1983). Knowledge embodied in art, which has been analyzed as tacit, practical knowledge, is cognitive, though non-conceptual (Borgdorff 2012, 49) and interconnects disciplines. Also basic and applied research is intertwined at the ADTL (Carayannis & Campbell 2012, 25).

In inter/transdisciplinary work, practitioners must be able to cooperate with fellow team members and make referrals and offer educational services (Klein 1990, 150). The National Research Council (NRC) of the USA tracked series of research reports and announced that most significant growth in knowledge production in recent decades was occurring due to Interdisciplinary Design Research (NRC report 1986, 1990). Within curricula of universities in Austria you will hardly find inter/transdisciplinary elements. The University of Applied Arts Vienna has unique academic programmes as Transarts and Social Design. Studying Transarts you may decide your study elements depending your interests, studying Social Design, the teacher staff and the students work in teams.

Previous Projects. Examples of Inter / transdisciplinary projects, ADTL

**(Mateus-Berr 2013 Applied Design Thinking LAB and the Creative Empowering of Interdisciplinary Teams. 2013 In: Springer Encyclopedia on Creativity, Invention, Innovation and Entrepreneurship (CI2E).
Eds: Elias G. Carayannis, Igor N. Dubina, Norbert Seel, David F. J. Campbell, Dimitri Udiszuni. New York: Springer)**

Project "Maths Goes Design, Design Goes Maths" (2007-2010) with Prof. James Skone; in collaboration with the University of Technology Vienna, Discrete Mathematics: Prof. Dr. Reinhard Winkler, Prof. Rudolf Taschner (math.space), University of Vienna: Prof. Ilse Schrittesser, Dr. Eva Sattlberger, Dr. Eveline Christof:

Aim: How can design make complex mathematic phenomena tangible? Developing educational tools with students of two different institutions and disciplines.

Project "The Way Polynomiography Things Go. You real-eyes, what you inhabit" (2009) Prof. Bahman Kalantari developed a computer program Polynomiography© in order to demonstrate the beauty of algebraic equations and reduce the fear of mathematics. The ADTL researched Polynomiography©, its creative potentials and its educational value. 2009 the Applied Design Thinking Lab Vienna presented its outcomes at Rutgers University (USA, DIMACS Workshop on Algorithmic Mathematical Art: Special Cases and Their Application, May 2009) Aim: The central question was, if, and how Polynomiography© stimulates creativity and where it lead to. Interdisciplinary approaches took place within the knowledge and associations of the participants (The polynomial pattern of a butterfly, the symmetry of a baroque garden etc.). The program itself is a beautiful metaphor for Applied Design Thinking: To achieve a zero of a complex polynomial function, you start from any arbitrary point and approach the researched points in iterative steps, similar a design process. Design Thinking strategies involve parts of the philosophy of the radical constructivism. There are no objective conditions and viewpoints of the world and there is not one right solution, but various.

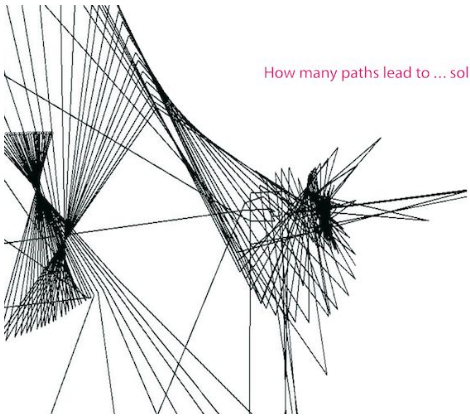

Figure 4: How many paths lead to ... solution? convergence points, Polynomiography© 2009
Author: Walter Lunzer

Project "Math goes Design - Post It!" (2010) Aim: The main questions were: How and what could those two disciplines, maths and design, learn from each other? Therefore students and teachers had to develop various ways of communication and learn a new language. Dialogues through Applied Design Thinking methods might develop a new approach to mathematic-education. The result of the project was unfolded throughout the method of the presentation: art performance with Post-its (Peran 2008). The aim was to create and provoke a space for reflection through questions and statements about mathematics. Writing, talking, discussing and postulating were expected in a determined chaos. By giving simple instructions how to act (Algorithm, Equation), the results were put into the initial values of the equation (Iteration) and developed chaos due to the determined initial conditions. An important role was played by bifurcations, situations, where decisions had to be made. These situations might end with an overshoot-and-collapse or order through bifurcation. In this situation systems break locally through the structure of the system and temporarily through the periodical dynamic of the decision itself (Briggs & Peat 1999). This mathematical metaphor reassembles decision-making in mind-mappings and design processes. For the presentation 3M-post-its were used and all participants became part of the performance. Post-its appeared and disappeared, could be removed without traces. Post-its have become an item for temporary improvisational design in urban landscapes, where urbanites recover space for their needs (e.g. free running/parcours, barbecuing, playing golf with tin holes, ...). They are a symbol and reaction in form of civil disobedience. Post-its were used as a metaphor for disobedience in traditional mathematic- education. There is no space there for pupils needs, either.

Figure 5: Post-its 2010
Author: Walter Lunzer

Figure 6: Math goes design – post-it! Performance am Stephansplatz
Author: Franz Morgenbesser

Project "Math Goes Fashion" (2010-2011). Since 2010 the Applied Design Thinking Lab Vienna concentrated on mathematics and fashion. In Western Europe, the making of patterns in garments mainly comes from one tradition. So far, no one has yet thought about an approach based on platonic solids or reformulated the traditional S, M, L, and XL sizes with a new mathematical interpretation, Body-Index-Cloth. Rotational forms of conic sections enables for example to find forms for a hyperbolic ball gown or Moebius-fashion. The Lab covered a broad range of problem domains from pattern making to fashion for buildings with inflatable membranes. Recent experiments revealed new perspectives for fashion and, additionally, brought up educationally fruitful methods for working with mathematical topics using a creative base.

Figure 7: Body-Index-Cloth
Author: Jasmin Schait


Figure 8, 9: Platonic Solids
Author: Walter Lunzer


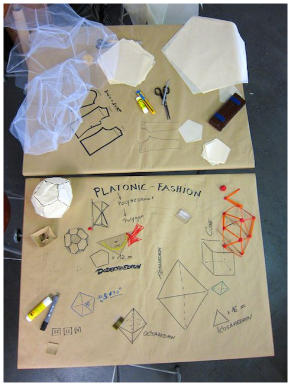

Figure 10, 11, 12: Platonic Solids
Author: Walter Lunzer

Figure 13: The Giant Cloud, experimenteller Workshop für Kinder und Jugendliche im Rahmen des Carnevale dei Ragazzi Biennale Venedig 2011. Authors: air-shaped cloud group – dieAngewandte (Maria Walcher, Elena Waclawiczek, Isis Várkonyi, Peter Michael Schultes, Stefanie Pichler, Niki Passath, Kerstin Nowotny, Klaudia Lässer, Dora Kuty, Carmen Fetz, Konrad Černohous, Clelia Baumgartner, Daniel Aschwanden, Agnes Achola) The Giant Cloud by air-shaped cloud group © dieAngewandte is licensed under a Creative Commons Attribution-Share Alike 3.0 Austria License. This project was realized in collaboration with the department for Textiles - Free, Applied and Experimental Artistic Design/University of Applied Arts Vienna (Prof. Barbara Putz-Plecko).

Figure 14, 15: Inflatable membrane wear
Authors: Konrad Cernohous, P. Michael Schultes

2 Material & Methods: Applied Design Thinking Methods combined with Arts, Design & Maths

For the kick-off meeting, the Stanford d.school material (d.school) of a Design Thinking lecture was used and adapted to the topic. The author considers this approach as very useful to get-to-know (empathize) each other and jump into the topic right away. Further on a full design process can be experienced in just 90 Minutes.


Figure 16, 17: Empathize/Applied Design Thinking Lab
Author: Ruth Mateus-Berr


Figure 18, 19: Prototype & Test/Applied Design Thinking Lab
Author: Ruth Mateus-Berr

The students had to work in groups of two and start with interviews. The workshop usually starts with the brief to design a "perfect, ideal" – in this case Vismath education tool. After three minutes the groups are asked if this was possible. Of course no ideal solution can be found because perception of "perfect and ideal" are very subjective and relies on some ones experiences. But this small introduction let people think about constructivist methods in education and "making beliefs" (Reich 2008). Then the workshop continues with interviews about the topic, digging deeper, defining and redefining the problem and ideating various solutions and iterating feedback. They get to know each other and develop a first project in a team. The process itself has an iterative character, consists of social skills like increasing the ability to listen, further on encourage in decision- making and risk-taking. The final workshop part lets them build up a prototype of a solution, which makes it possible for people to "experience an individual solution". Very valuable is the reflection phase at the end, where participants present their design and discuss their solutions. Unfortunately five of the Serbian students could not participate because they arrived due to organizational circumstances two days later. They had to hand in a scientific essay at the end, adjusted to a template and scientific context. Papers were written about: Life of Geometry; Financial Mathematics: Risk and risk free financing; Construction of an Augmented Reality Drawing Key; Perception of Phi: Patterns in Numbers; How does a Mobile Phone work? Fractals; Note Dictionary for the Visual Experience, decimal numbers: Operating with decimal numbers; The principle of decimal numbers. How can 10-12 year-olds imagine a newly learned numbering system? Infinitop –or how to explain infinity to children; Sinniluminate – or how to use GeoGebra for functions and how to paint them with light; Circle: How to experience Circles by hands-on material; Crocheting on computers/with IT; Mathician: or how to combine geometric formulas with music.

With this paper they should learn how to put their work in a scientific context and improve writing for degrees, journals and conferences.

3 Results 2013

The students built teams of two or three persons. Most of them were interdisciplinary concerning arts/design & math.

Group 1:

Irina Nalis, Austrian, is 32 years old, studied psychology, specialized in economic and social psychology. Since she graduated from University, she was working in different companies in Austria and Germany as a strategic consultant for communication, mainly brand communication and advertising. Since march 2013 she is a Ph.D. student, where she aims at an interdisciplinary approach which combines the knowledge and techniques of the University of Applied Arts Vienna and at the University of Vienna at the department of psychology to find out more about the future of work and the drivers and barriers of career change in general and in the field of the education of teachers.

Andrea Reithofer, Austrian, is 24 years old, studied between 2007- 2012 at University of Applied Arts Vienna, Institute of Art Science and Education, Department: Design, Architecture and Environment and Art and Communicative Practice for Teacher Training in Fine Arts. Her thesis for her diploma was about insulin pump carrying systems for young female users, which she passed with excellence. Since 2013 she is participant of the research project INTERACCT, funded by the

Austrian Research Promotion Agency. (FFG) is the national funding institution for applied research and development in Austria. In this interdisciplinary project INTERACCT (Integrating Entertainment and Reaction Assessment into Child Cancer Therapy), which is a project where two universities in Vienna (University of Applied Arts Vienna: Institute of Art Sciences and Art Education, Department of Design, Architecture and Environment for Education; University of Vienna: Faculty of Informatics, Computer Science Didactics and Learning Research, and Research Group Entertainment Computing), the CCRI (Children's Cancer Research Institute), children of an Austrian high school (Schulschiff Bertha von Suttner) and T-systems (a division of Deutsche Telekom, systems integration, computing and network services and e-business), University of Vienna (Department of Applied Psychology: Health, Development, Enhancement and Intervention; Faculty of Psychology, Traumata Research) have been involved within the Applied Design Thinking LAB Vienna from 2009 until today. Aim of this project is to enhance multidisciplinary and participatory approaches and methods of Design Thinking in design and technology education. Case study is the design of an interactive web based communication platform for improving quality of life for patients of the St Anna Children's Hospital Stem Cell Transplantation (SCT) Outpatient Department and to improve medical communication and education in outpatient care after pediatric hematopoietic SCT. Spin-off is design of a serious game where healthy children and children with malignant diseases are participatory involved as well as the students of different teaching subjects (informatics, art and design education), the caring medical staff and industry partners.

Since 2012 Andrea teaches art education in school, too and in spare time her main interests are listening to music: Blues, Blues-rock, Guitar music, Postpunk, Indi pop ... and in Art. She likes to observe the behaviour of people, does a little in Sport (Rugby) and loves painting.

Figure 20: Group 1 (l-r): Irina, Andrea, Aleksandra
Author: Ruth Mateus-Berr

Aleksandra Rastik, Serbian, is 22 years old and lives in Novi Sad, Serbia. Currently she is at her Bachelor studies of Architecture and Urban Design at University of Novi Sad. When it comes to architecture itself, she really enjoys designing residential spaces and interiors, which is why she would like to enroll to a Master course connected to interior design or even continue her studies at the architecture departments of the University of Applied Arts Vienna (Zaha Hadid, Greg Lynn, Hani Rashid). Apart from architecture, she loves sports, music, reading books and spending large amount of time outside.

This group did an impressing research and followed up a well-done documentation. The following work was documented in their research documentation:

In their meetings they discussed interesting topics as Pythagorean theorem, the use of triangle shape in todays spaces ant it's use in the past, further on they were interested in 3D Models and modeling and researched the Israeli Neri Oxman, named to ICON's list of the top 20 most influential architects to shape our future (2009), and Richard Sweeney with his extraordinary paper models. They exchanged knowledge about contemporary application and possibilities of 3D printers and visited a 3D studio (Metalab). Andrea considered triangles as used in the past for layouts of floor plans, and could find them today in concepts for complex surfaces of buildings. Irina remarked that she found triangles in the past as the pyramids and today as rooftop application also in countries with high rainfalls and cold winters. They researched on the use of triangles and organic shapes, the application of the triangle as static element in construction (Norman Forster, Buckminster Fuller). Then they focused on the emotional concept of space, discussed how it feels and did a research about symmetry in humans. The posed questions as: What is a good floor plan and why is it a good floor plan? Is it a good floor plan because of its: rhythm, it's scale or the proportion? They did research on subjective likes (like the palace of Knossos and Nowgorod) and dislikes of floor plans and discussed the conditions, about what children and youth would think about well designed floor plans and arranged some excursions in the field. They came to their conclusion that spaces must be well designed, where you feel welcomed and discussed at public spaces, shopping halls and new architectural signs like the Zaha Hadid building at the Viennese Prater. They figured out that "some buildings seem to be stuck in a limbo – between a past they don't relate to and a future they still cannot create" and that some spaces don't work with the social environment.

They discussed urban plans and proportions and which might serve most. The following discussions led them to playing with proportions (Erwin Wurm's distorted parent's house). So they were inspired by architecture, art and psychological perception, which was reflecting their disciplinary origin. The shape they were interested most was the triangle and the interest in "shaping well-being of places". The decided working title was "play with proportions" and their approach: "from Pythagoras to 3D modeling". For the educational approach at the Summer School Eger they wanted to let students explore and define what they wish to find in a place, consider its measurement (size and proportion), let them construct such a space with a paper model, according to layouts of the country sides via “Google earth”. At their interactive presentation they let the auditory fill out mini-questionnaires and let us draw a desired place.

Figure 21: Designs for ideal places
Authors: Participants of the Workshop

They examined the drawing workshop and came to the result that nobody created an indoor place. During the workshop no floor plan or architecture painting was made. They discussed that for their group it was more important what was happening at this place as the place itself. It was important for them that they can meet nice people there. “That kind of places are, where we can think about something, enjoy and where also we can find some good friends.” (places = meet and find some friends). Three participants drew a park (garden) because they considered it great place to be. They drew a classical garden with trees, a bench and a river (Hypothesis: a river is calming). A place where one other person feels comfortable was at the sea (Hypothesis = It is more the feeling of holiday). And there were only two persons, which designed a balcony (Balcony = a surface between inside and outside).

Also important was good light, colour and the smell. They came to the conclusion that their topic could be “The surface between inside and outside and garden design”.

For their final presentation they prepared “a puzzle of floor grounds”. Their goal was to raise understanding of geometrical shapes and their impact on our daily lives and the social function of environment.

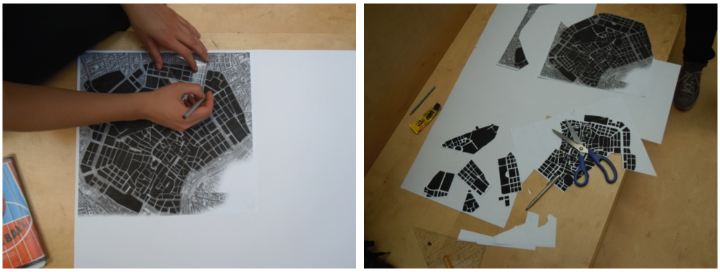

Figure 22, 23: Designing Floor Plans
Authors: Irina Nalis, Andrea Reithofer, Aleksandra Rastik

Group 2:

Christine Gobbi, Austrian, 39 years old, studied pedagogics, art history & art education and studies art and design education at the University of applied Arts Vienna. She teaches in schools and workshops of the Reggio Emilia pedagogics, illustrates books for children and is engaged in education at museums (ZOOM, Austrian Theater Museum).

Madlena Domazet, Serbian. She studies graphic design in Belgrad. She is interested in fashion, new media, brand building and visual mathematics. She lived in LA, then lived in Singapore for 10 years, and came back to live in Belgrade. There she studies at Metropolitan University and is assistant for drawing. She considers herself as a total art freak, anything she can make more creative or interesting, she will do! She is addicted to sports and music. She loves tattoos, graffiti's, yoga, and animals.

Nataša Džaleta, was born in Split in 1990, Croatia, but lives in Serbia since she was one year old. She always loved drawing and math. She decided to study math. Apart from that she loves languages and speaks English and Spanish. She has two cats, and hopes that she will have an animal shelter one day.

Figure 24: (l-r): Christine Gobbi, Madalena Domazet, Nataša Džaleta
Authors: Ruth Mateus-Berr

This team worked about the relation between financial risk and award. As many people do not have a chance to feel that kind of risk, they developed a simple game that enables players to determine a risk level at which he wants to play was designed. A puzzle, which stands for the financial resources, can be composed in different visible conditions in a bordered period of time with one hand. The visible conditions, which embody risk-taking can be calculated by the player. For a possible maximal profit the player works completely blind Minimal profit is therefore gained by having the best visual condition. The psychological aspect and the corresponding dynamics can be experienced in this project.

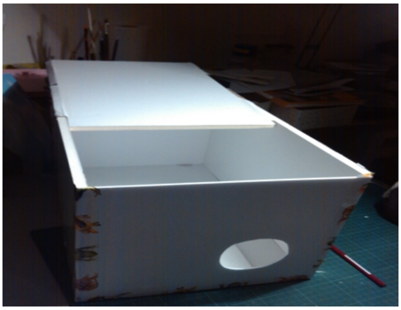

Figure 25, 26: Risk Box
Authors: Christine Gobbi, Madalena Domazet, Nataša Džaleta

Figure 27: Risk Box Puzzle
Authors: Christine Gobbi, Madalena Domazet, Nataša Džaleta

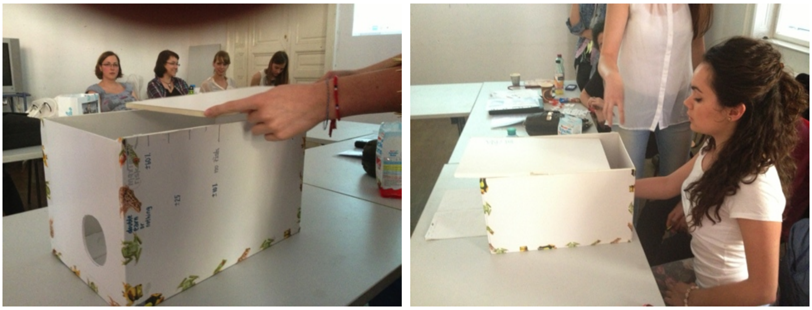

Figure 28, 29: Risk Box Final Presentation
Author: Ruth Mateus-Berr

Group 3:

Dušan Tatić, 31 years old Serbian was born in the city of Niš, where he lives and studies. Currently he is a student of the third year of postgraduate studies at the Faculty of Electronic Engineering, University of Niš, Serbia, where he also got Master of Science degree in the field of Computer Science. Since his master thesis was about IT and multimedia, he continued to work in that area. Interdisciplinary projects are something that inspires him most. He loves to work in a group and learn from different scientific fields. He has been working on the research projects that combine IT technologies with medicine or archeology. He is highly motivated, professional and engaged. Currently, he also works as research assistant on the project "Development of new information and communication technologies, based on advanced mathematical methods, with applications in medicine, telecommunications, power systems, protection of national heritage and education", supported by Ministry of Science, Republic of Serbia. Realization of his project activities are at CIITLab - Computation Intelligence and Information Technologies Laboratory – at Faculty of Electronic Engineering, University of Niš, while the coordinator of the project is Mathemetical Institute of Academy of Sciences, Republic of Serbia. His research is focused on technology of Augmented Reality (AR). This technology combines in real time computer generated and real-world images. Taking into account that this technology is a new type of user interface, it can be implemented in variety of areas as interesting visualization tool. He is involved in of research concerned the application of this technology in the area of national heritage digitalization. Recently, in CIITlab he has formed an interdisciplinary group ARhiMedia which consist of individuals who have and various computer skills and come from different faculties. The goal of the group is to represent Niš city history using new information technologies such as augmented reality, projection mapping, and 3D reconstructions. One of recent project that they realized was ARchiMed – Mobile Archeological Tourist Guide based on the Augmented Reality. These systems used to augment and improve the exhibition of archaeological site of Medijana in

Niš during the tourist visit. He also published several papers and participated in conferences related to these issues.

Due to multidisciplinary nature of research related to visualization, he is eager to learn new systems and working technologies and upgrade his skills and knowledge. He enjoys doing investigative and innovative work, especially when it leads to practical applications.

He likes to spend his free time in nature and often plays basketball with his friends. Especially, he enjoys travelling and exploring foreign countries and different cultures.

Stefan Wykydal is 37 years old, Austrian. He studied painting and graphic design, since then working as an artist, concentrating on painting for some years now. He won the Bauholding Strabag award in 2005 and is an awesome painter. At the moment he studies art & design education at the University of Applied Arts Vienna. Mathematics is for him like a foreign country that he likes, considers charming and interesting. He is visiting it often, but still not feeling perfectly at home there. He is convinced that, though talents vary, individual abilities and usage concerning maths, can be improved through better and more diverse educational practice. Mathematics is for him above all, also a cultural phenomenon. The western attitude towards maths is specific, as other regions have their own approach towards numbers. Of course there is a lot in common also, but he is especially interested in the field of ethno-mathematics. The way we deal with maths also has influence on the way we think. In schools mathematics is mostly dealt with as something totally sure, nothing is more secure than mathematics and its right solutions. He wanted to focus on the differences between different cultural traditions, Western – Asian for example, in order to make clear the special qualities of different systems and to create an awareness that our western mathematics and also its aesthetic is very specific and not universal. His last Design object was linked to stargazing, what is considered to be one of the first mathematical approaches.

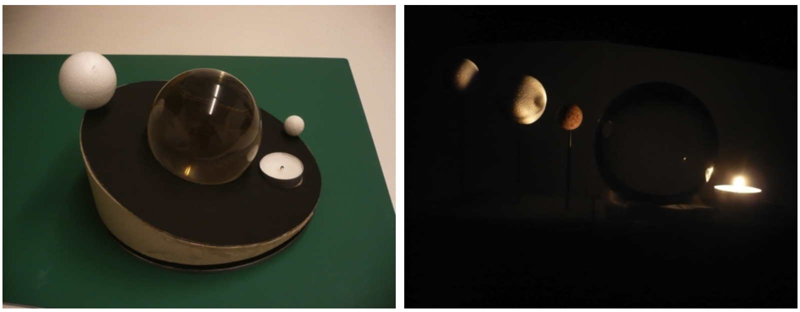

Figure 30: Tellurium
Author: Stefan Wykydal

Figure 31: Team 3: (l-r) Dušan Tatić, Stefan Wykydal
Author: Ruth Mateus-Berr

This interdisciplinary team developed a project to improve instruction of mathematics. The objective was to create a tool that could improve the understanding of geometrical shapes. A tent-like construction, called an Augmented Reality Drawing Key, is developed. The ARtKey can be placed on a table. Geometrical shapes are placed underneath this design in order to be drawn or captured through an augmented reality application. Using ARtKey, students can get a deeper and more personal understanding of geometry, as they get involved into several different activities. They experience how perspective construction has its roots in the human visual system. Further, the added augmented reality feature widens the activity range. Through using multiple senses, i.e., seeing, touching, moving, and listening, the user learns much better than in the case where only one sense is involved.

Geometry is, for most students, something abstract and theoretical. ARtKey not only links theory with practice, but it also enhances deeper understanding of the methods of representation. In linking the world of hand-drawing and the technology of augmented reality, this interdisciplinary solution brings geometry closer to the personal world of a student.

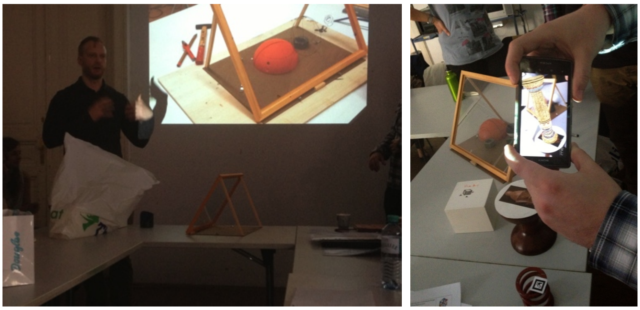

Figure 32, 33: ARtKey
Author: Ruth Mateus-Berr

The next step for the ARtKey project will be to test it with students. Their reactions will lead to further questions and developments. Through analysis of data gained in such an interactive use of the system, further improvement of the features of ARtKey is planned.

Group 4:

Elisabeth Stephan, originally from Germany, worked in photography and studied theater (film), art history and visual arts in the United States. At the moment she is studying at the University of Applied Arts, Vienna, Social Design – Arts as Urban Innovation. Her interests in math concepts gather around patterns in numbers such as Pascal's Triangle, Fibonacci Numbers and the controversy surrounding the Golden Ratio. Other interests of her are cycling, horseback-riding, diving, learning new, unexpected things and playing with language.

Milovan Ninkov, 23 years old, has a bachelor degree in Applied Mathematics Module of Financial Mathematics. Currently he is student of Master studies of Applied Mathematics at the University of Novi Sad. He is reliable, well-organized, and used to work on his own initiative. He is comfortable working on his own and also as a part of a team. He is Interested in foreign cultures and loves to travel, loves music and was part of a musical orchestra. He is interested in the field of Mathematics called Probability and Stochastic analysis. Further on he is interested to find some kind of relation between Mathematics and music and is highly interested in financial mathematics.

Jovana Vujicic, 22 years old and lives in Novi Sad. Currently she is student of Master studies of Applied Mathematics at the Universitiy of Novi Sad. She has always been interested in science and math, since her father is Professor of Physics and she adored all of his stories when she was little. However, science is not all she is interested in. She loves dancing, especially classical ballet, and was practicing it for 12 years. Besides that, she enjoys reading a good book while drinking a cup of coffee.

Figure 34: Team 4 (l-r): Jovana Vujcic, Milovan Ninkov, Elisabeth Stephan
Author: Ruth Mateus-Berr

This team derived from Graphic design & math, Computer Science & electronic engineering; Painting & graphic design; Applied Mathematics; Financial Mathematics; Theater (film), art history and visual arts, art history & Social Design; Information and Communication Technology. This group worked on Perception of Phi: Patterns in Numbers. Of great interest thus became the expectation of this ratio in their daily lives. Do we expect to find relations of phi in everything we do or see? A game of finding proportional lines in faces, will, without making any commentary or judgement on what is considered beautiful, rather through the exploration of prevailing lines and ratios of these lines, introduce the concept of phi playfully and interactively. The user should gain the ability to see relations and derive a cognitive understanding of this abstract mathematical concept.

Figure 35, 36: Perception of Phi: Patterns in Numbers
Author: Group 4

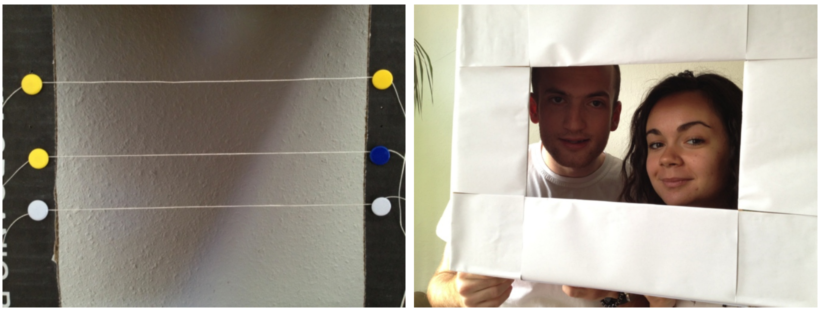

Figure 37, 38: Perception of Phi: Patterns in Numbers
Author: Group 4

The fourth group worked on patterns in nature and a possible correlation of patterns in numbers. Mathematical concepts such as Pascal's Triangle, Fibonacci numbers and Golden Ratio were considered in an artistic-mathematical discourse. With the help of an interactive model for exploring ratios and visual relations in human faces, the mathematical concept of Phi shall be addressed. The playful exploration, a Perception Prototype, of visible relations should increase awareness of human predilection for seeing patterns.

Group 5:

Marie-Theres Wakonig was born 1984 in Klagenfurt, in the south of Austria, she came to Vienna to study International Realtions and foreign languages and finish now at the University of Applied Art „art and design education“. Her main interest is communication. She is very interested in learning more about people and their languages, to improve her knowledge. Further interests are painting, photography, literature and cinema. For her it is important to travel a lot, to get new impressions and make different and intense experiences.

Marina Mišanović comes from Novi Sad, Serbia and is twenty three years old. She is a second - year student of master studies of Applied Mathematics (Mathematics of Finance) at the University of Novi Sad. She likes dancing, music (especially pop and R&B) and volleyball, likes to travel, visit sights, meet new people and different cultures. She is interested in fashion design, so I like to cut out again old clothes.

Milena Đorđević lives in Pančevo (Serbia) and is 21 years old (the youngest in this program ☺). She is finishing her second year of studies at the College of ICT (College of Information and Communication Technology), loves to travel and is looking forward to meeting new people.

But most of all, when she goes to another country, she likes to try their specialities and meet their customs. She likes all kinds of music, from classical to folk music, likes to draw, and when she was little, she made different models houses out of wood. What interests her is how to connect her profession, mathematics, electronics, electrical engineering etc. with the design.

Figure 39: Team 5 (l-r): Marie-Theres Wakonig Marina Mišanović, Milena Đorđević
Author: Ruth Mateus-Berr

The fifth group had their objectives in connecting arts, mathematics and telecommunication. Explaining how a mobile phone works was considered as an interesting topic for the youth. From physical disassembling mobiles, looking for their antennas until designing individual mobile cases with fractal patterns they developed hands-on tools.

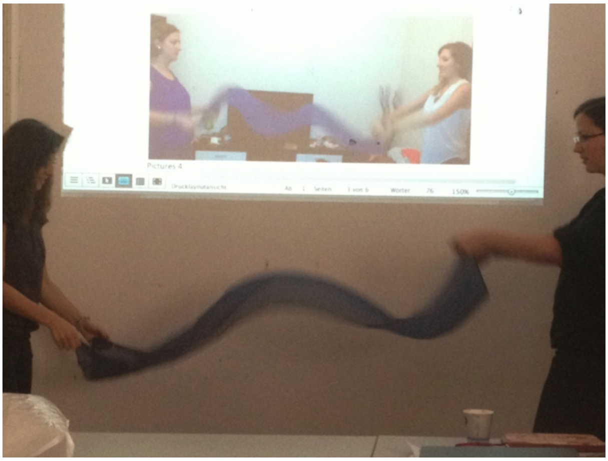

Figure 40: How does a mobile work? Fractals
Author: Ruth Mateus-Berr

Figure 41, 42: How does a mobile work? Fractals
Author: Group 5

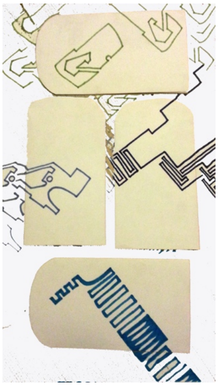

Figure 43: Design for Mobile Cases
Author: Group 5

Group 6:

Ivana Supić is 22 years old and comes from Serbia. She is studying ICT at the College in Belgrade, study programme of Telecommunications. She is interesting in technology and mathematics. In her spare time she enjoys reading books and listening to music.

Klaudia Kozma was born on 03.08.1987 in Székesfehervar in Hungary. Since 2009 she studies art and textile pedagogics at the University of Applied Arts Vienna. In her spare time she engages with art, drawings, paintings, music and dancing. In her artwork she engages with math, especially with the topic of Moebius and Kleinbottle.

Figure 44: Moebius © Klaudia Kozma;
Performer: Daniel Aschwanden
Author: Thomas F. Berr

The sixth group, both passionately interested in music, worked on a new musical notation dictionary, which should offer a new graphical visualization for music, where children have the opportunity to combine geometry with their own visual and acoustic experience. These educational tools can be applied in math and art class, regarding intercultural and interdisciplinary interest. This group was especially challenged because Klaudia only speaks Hungarian and German and Ivana a little German but could exchange their interests regarding their visible communication.

Figure 45: Team 6 (l-r): Klaudia Kozma, Ivana Supić
Author: Ruth Mateus- Berr

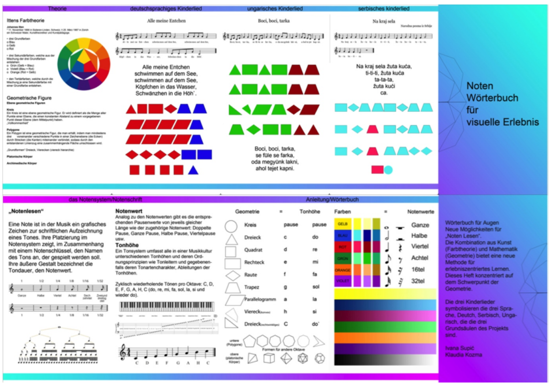

Figure 46: Musical Notation Dictionary
Author: Group 6

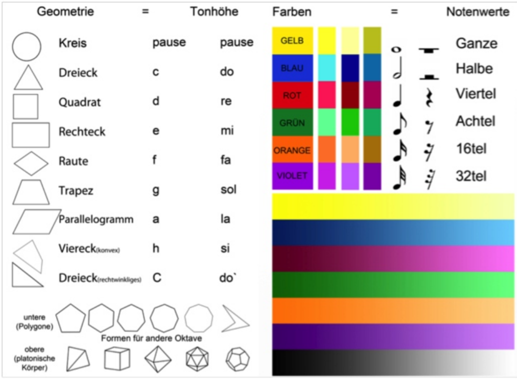

Figure 47: Musical Notation Dictionary
Author: Group 6

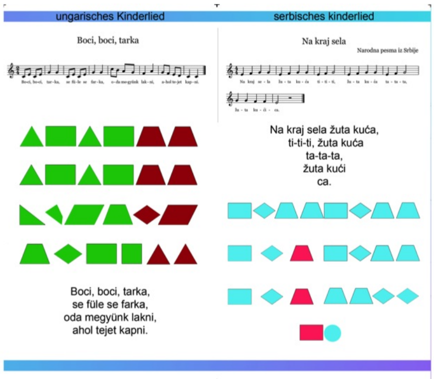

Figure 48: Musical Notation Dictionary
Author: Group 6

TEMPUS Study Report Students Visit April-May 2014 Vienna 29.4.–29.5. 2014

Ruth MATEUS-BERR[1,a,b]
Dejan Todosijević[d], Djordje Manoilov[d], Jovana Radenović[d], Vuk Vasić[e], Filip Popović[e], Milan Ličina[e], Jelena Kričković[h], Ivan Marić[h], Lilian Wieser[a], Kleijlia Zivkovic[a], Clemens Göller[a], Julian Gris[a], Stefan Wykydal[a];

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[e]Metropolitan Belgrade University (BMU)
[f]Faculty of Electronic Engineering, University of Niš, Serbia (MISANU)
[g]College of Professional Studies in Information and Communication Technology, Serbia
[h]ICT College, University of Belgrade

STUDENTS

	Name and Surname	Institution
1	Dejan Todosijević	MISANU
2	Djordje Manoilov	MISANU
3	Jovana Jezdimirović	MISANU
4	Jovana Radenović	UNS
5	Vuk Vasić	BMU
6	Filip Popović	BMU
7	Milan Ličina	BMU
8	Jelena Kričković	ICT
9	Ivan Marić	ICT
10	Lilian Wieser	UAAK
11	Kleijlia Zivkovic	UAAK
12	Julian Gris	UAAK
13	Clemens Göller	UAAK
14	Stefan Wykydal	UAAK

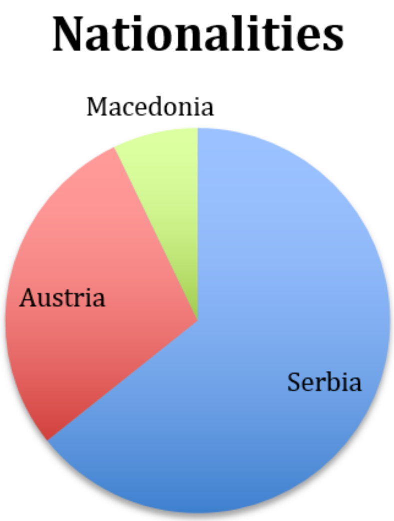

Figure 1: Nationalities
Source: Ruth Mateus-Berr

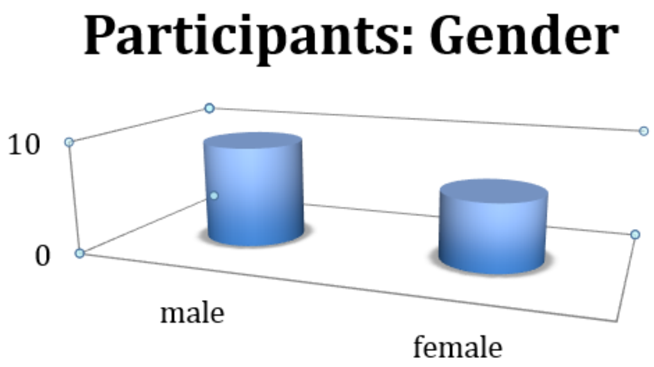

Figure 2: Participants: Gender
Source: Ruth Mateus-Berr

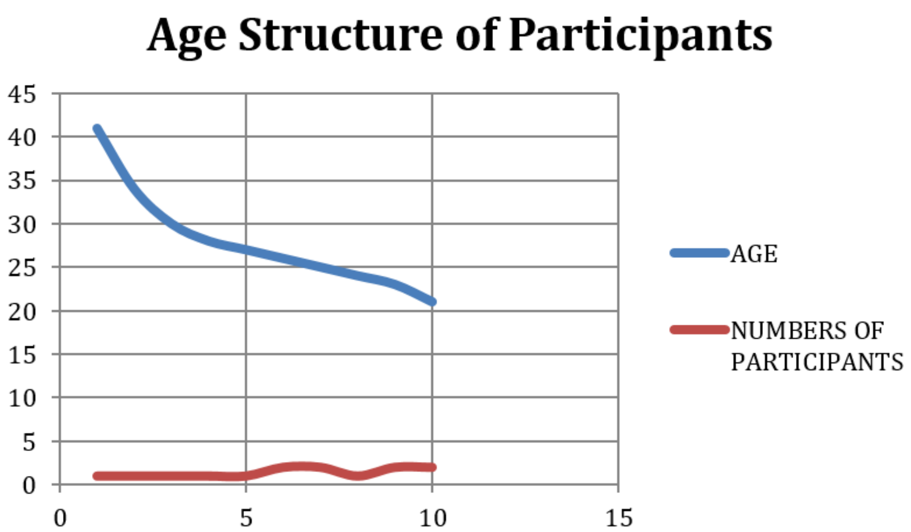

Figure 3: Age Structure of Participants
Source: Ruth Mateus-Berr

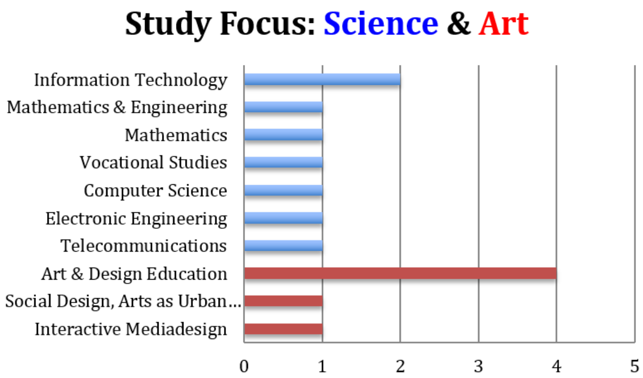

Figure 4: Study Focus: Science & Art
Source: Ruth Mateus-Berr

The students built teams of two or four personas. All of them were interdisciplinary concerning arts/design & math and international concerning teams (just one group were only Serbians).

Group 1:

Dejan Todosijević, is 26 years old, studies Computer Science. He got MSc in Software Engineering from the Faculty of Electronic Engineering Nis. From an early age he was interested in computers, and wanted to know how they work so his career choice did not surprise anyone. During his studies, he participated in a lot of interesting projects, both individual and in team. What impressed him the most was a project in which we had to use the technologies of artificial intelligence. He considered it very interesting to learn how it works and how he can make something that uses artificial intelligence and succeeded in that. With his team he developed a game that could make own conclusions, which he found very interesting. He enrolled in a doctoral program at the same University where he received a master's degree (MISANU, NIS). As a topic for research he combined his biggest interest's, which is the Semantic web usage in E-learning, which makes extensive use of artificial intelligence technology which he found really interesting to explore. He loves physical activities that include running. It is his belief that having physical activity often is very important for health. In his spare time he fills with activities such as football, running, basketball or workout at the gym, likes to listen to all genres of music, such as pop music and others, depending on the mood, and of course the situation and company.

Djordje Manoilov, 26 years old, enrolled at the Faculty of Electronic Engineering in Niš where he got a MSc in Information Technologies. Now, he is a PhD student 2nd year at the same faculty and looking for the topic of his doctoral dissertation. He loves music of 90s, and his favorite bands are EKV from Serbia and Pearl Jam. He is also a real a real football fanatic and big fan of FC Red Star from Belgrade.

Jelena Kričković, 23 years old, studies telecommunications at the ICT college in Belgrade. She is a second year student and participated in Tempus project 2013, e.g. Jyväskylä, Finland and Eger, Hungary. She hoped that she could contribute to this project with her experience and that her knowledge of visual mathematics would expand further during the course in Vienna.

Stefan Wykydal, 41 years old, studied visual arts at the University of Applied Arts Vienna and studies Arts & Design Education since 2011. Since 2013 he is teaching besides his studies. Initially he is a visual artist , but as a second obligation he also works as an Art/Design educator at a Viennese High School. For his arts work he concentrated on painting the last years and started more in 3 dimensional work recently. His teacher identity is very new, as he has just started last autumn, he considers it fun and interesting and can imagine to continue with it. He is interested in the field of inter-disciplinary work in order to have new experiences in the academic field of the teacher education department. Maths education is, from his point of view, a problematic subject in the Austrian school system, as it causes too much fear in students lives. Thererfore he considers it necessary to intervene in the daily practice of maths education, in order to address all different kinds of personalities among students. Last year he participated not only at the study visit, but also at the Summer University in Eger.

Figure 5: Group 1 (l-r): Djorde Manoilov, Stefan Wykydal below, Dejan Todosijević, Jelena Kričković
Author: Lilian Wieser

Since the beginning this group decided to work with the topic of decimal numbers. Their first ideas and discussions were about the whole and the divided, emotional aspects of dividing something, because organisms are often killed when divided, which evokes often feelings of aggression (fear and/or pleasure of it), the design of a puzzle out of a divided circle, to create a ceramics project for a target group of 10-11 year old children in technology (design education in schools), to use the classic cake principle, not original but effective (the eye of Horus, 4 thousand years old) or to design augmented reality applications that show children how to solve some math problems with decimal numbers.

Jelena Krickovic and Stefan Wykydal then decided to create a tool, that explains the decimal numbers by haptic experience. Stefan analyzed the problem by discussing it with a mathematic teacher in his school. They localized the transitional period from primary school to secondary school causing difficulties for children, because the lessons are short, schoolbooks divers conceptualized. The main topic fort hem was identified by the chapter of decimal numbers: on the one hand the principle of decimal numbers itself, on the other hand the calculations done with decimal numbers. They imagined to design a “cake” as a metaphor of a divided cylinder or sphere and used this to develop an interdisciplinary workshop or lesson period of the subjects of design & technology and mathematics, where the children should first capture images of spheres by photography and further work hands-on to design “cakes of clay”. They believed that in producing such design objects, scholars have a very strong personal relation to the newly introduced decimals and that proportions that are felt with all senses (Steininger 2008) come more easily into ones mind, than just numbers and that they co-created user- oriented design tools. They experienced the collaboration of art, design and mathematics especially fruitful.

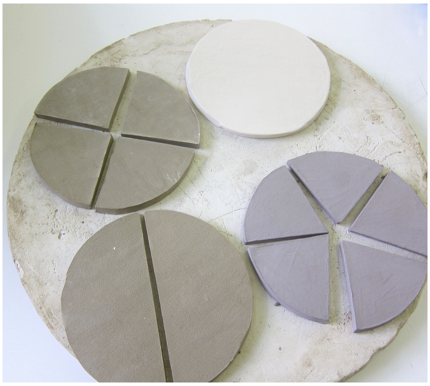

Figure 6: Clay Spheres
Author: Stefan Wykydal

Further on this group developed an application, which uses the capabilities of augmented reality technologies. This was designed to show scholars step by step solutions to specific mathematical problems with decimal numbers. To enter this toolkit a simple QR marker could be placed in schoolbooks, linking an augmented reality application. Augmented reality is a technology, which combines information from the real world with computer-generated content and such integrated, it displays them on the screen of user mobile phone or computer in real-time. This technology is very new and its usage is quite widespread. For now, the most commonly it is used for promotional purposes, and for fun. First they designed a lecture about decimal numbers. By starting this part of the app users get the ability to run a demo animation of solving a particular problem with decimals. Concrete problem with operating with decimals is linked to the specific marker, and after scanning appropriate marker app launches animation (video) with detailed step-by-step solution to the problem. This approach was adapted because this group came to the conclusion that children of that age just need a little more visual representation of problem solving processes. The idea is that the child scans markers and watches the video that shows the solution to the problem, learn how problem can be solved step by step, and then do the assignment independently on paper. This approach is enhancing individual learning with new technologies in combined methods, allows the child to gain knowledge through divers approaches, information by real and "virtual" teachers and allows the parents to save money in private additional teaching.

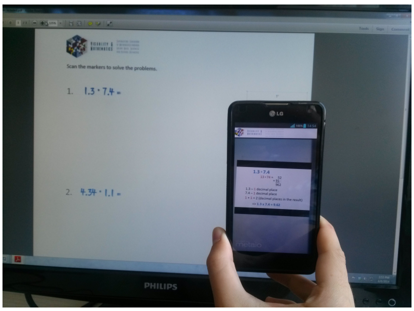

Figure 7: AR Animations part of the app, this part shows a demo animation of solving a particular problem with decimals
Source: Đorđe Manoilov, Dejan Todosijević

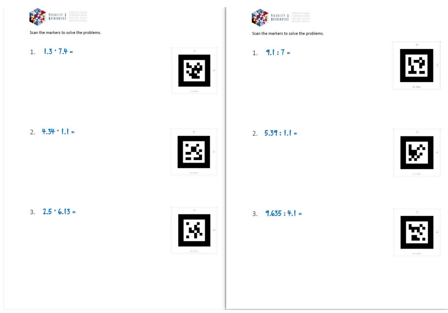

Figure 8: Prepared test sheets as an example of usage in math classes
Source: Đorđe Manoilov, Dejan Todosijević

Group 2:

Clemens-G. Göller was born in Vienna and is 30 years old. He graduated from secondary school in 2002 with excellent results, writing on a psychology-specific piece of work concerning children's drawings. In 2003, he started studying Design at the University of Applied Arts Vienna, specialising in graphics and advertising. His diploma about earned him an award from the Creative Club Austria 2011. Göller and Philip Reitsperger established the cooperation project 'das kleine büro' in 2010 – aimed at concept and branding. They offered graphic design, sound, installation and video.

2013 Göller started his second Master's degree at University of Applied Arts Vienna: "Art and Communication Practices"/"Design, Architecture and Environment". Additionally he works as a teacher and graphic designer.

Ivan Marić was born in eastern Serbia and is 21 years old. He finished PTT (Post Telegraph Telephone) high school in Belgrade, section telecomunications and is student at ICT College of Vocational Studies, University of Belgrade. He loves math and calculating in general, to travel, meet new people and experience new stuff.

Figure 9: (l-r): Ivan Marić, Clemens G. Göller
Author: Lilian Wieser

This group had their objectives in explaining infinity to children. Infinity is one of the topics very hard to explain and further more cannot be prooved by practical examples but explained by imagination. They approached this idea by researching the youthculture and related their work to examples as A popular example for a kid's movie is Pixar's Toy Story (1995) in which a character named Buzz Lightyear says "To infinity and beyond." Also video games use the concept: for example in the fun racer Mario Kart 8 (2014) for Wii U by Nintendo there is a track that has the physical appearance of a **Moebius Strip** (Hideki Konno, Kosuke Yabuki), M. C. Escher's Penrose stairs or the latest movie "Inception" (2010) by director Christopher Nolan (Warner Bros., Legendary Pictures). Further on they did a participatory research in school and asked children to draw infinity, then evaluated their definitions. With all generated knowledge they developed a card came, which includes historical input of mathematics as well as playful approaches.

Figure 10: Packaging Design
Author: Clemens-G. Göller (2014)

Group 3:

Milan Ličina is a new media designer/artist from Belgrade, Serbia and 23 years old. Currently, he is working on his final project for Master degree in New media design at Metropolitan University in Belgrade. After graduating from high school, he earned Bachelor degrees in Graphic design and Industrial design at Belgrade Polytechnic College and also Bachelor diploma in Interactive media design at Metropolitan university. For him, these study-visit-workshops present innovative way for developing new principles and tools for better understanding of mathematics. It is kind of fulfilling a personal wish because he was not very good at math due old-fashioned approach to lecturing. link: `http://vimeo.com/86750777`

Jovana Jezdimirović was born in Cacak, Serbia. She studied at the Faculty of Mathematics, University of Belgrade. She followed up studies for the Master Degree of Mathematics. Master work title: "E-lessons of the Differential Calculus created using the software package GeoGebra". Currently she is at the Faculty of Mathematics, University of Belgrade, Department of Mathematics, engaged in PhD studies. She participated at international conferences and published in conference proceedings as e.g. "Interactive E-course of the Differential Calculus Created within GeoGebra" London International Conference on Education-2013 Proceedings.

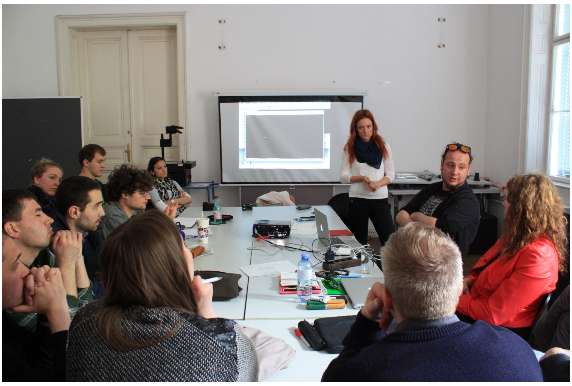

Figure 11: (l-r): Jovana Jezdimirović, Milan Ličina (sitting, talking)
Author: Klelija Zhivkovikj

Aim of their project was to present a visual approach to the theory of functions, within the theoretical content, detail graphic examples created in GeoGebra package software and meaningful use of new media tools. They perceived difficulties of scholars in drawing a function graph even they have had become familiar with its concept at preschooler's level. In order to foster scholars motivation and attention, new technologies and media were involved in way of creating mathematically significant images by light painting. They suggest this technique which they called "sinilluminate". Created educational materials are publicly available at `http://alas.matf.bg.ac.rs/~ml08060/`, also in the form of QR codes and text step-by-step instructions. Using mobile or tablet device, this "sinilluminate" project is easily reachable for teacher in order to create and design their own workshop, seminar or any other kind of mathematical educational activity on this topic.

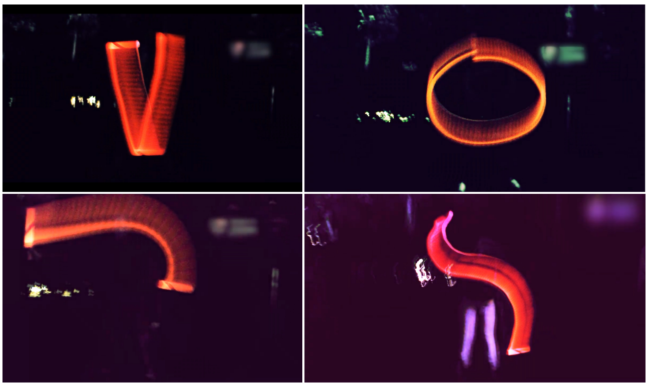

Figure 12: Light painting function example
Source: Milan Ličina, Jovana Jezdimirović

Group 4:

Klelija Zhivkovikj is 34 years old and was born and raised in Skopje, Macedonia. She got a BSc in Industrial Design from the Mechanical Engineering Faculty at St. Cyril and Methodius University. While she was studying she was involved in many art, design & public space projects and workshops with the Dutch Embassy in Macedonia, EU Culture program etc. During her studies she started working at the Balkan Investigative Reporting Network (BIRN) in Macedonia, where she handled financial and administrative tasks. This was a critical point where she has chosen to steer her professional endeavors in a new direction – one that combines art, architecture and social sciences. Now she lives in Vienna where she studies Social design at the University of Applied Art. Since March 2014 she started working at the Tempus VisMath Project at the University of Applied Arts Vienna. She loves nature, music, literature and people. She is particularly fond of activities that include all four. She loves post-rock, punk, indie, but also IDM, ambient etc. One of her favourite foreign authors is Haruki Murakami, her favourite colours are the cold colours (blue, green, violet) and black, and favourite means of transport – bicycle.

Jovana Radenovic is 27 years old and was born in Serbia. She studied mathematics at the University of Novi Sad and is PhD. student at University of Novi Sad, Mathematics in Engineering, since October 2012. She has been teaching assistant at the Faculty of Technical Sciences, University of Novi Sad for the following courses: Mathematical Statistics for Traffic Engineering, Mathematical Analysis 1 for Power, Electronic and Telecommunication Engineering, Mathematics for Graphic Engineering and Design, Statistical Methods for Environmental Engineering. She published Mathematical Models of University Funding, master's thesis in 2011 and Secant Methods for Unconstrained Optimization, bachelor thesis in 2010.

Figure 13: Klelija Zhivkovikj, Jovana Radenovic
Author: Lilian Wieser

Following one month of interdisciplinary work, the students conceptualized and designed a hands-on-tool that will embody the principles of playful learning and personal growth. They set out to design a tool about the geometry of a circle that will enable the pupils to learn math while developing a sense for visual expression and understanding. The process explores math as a visual language, and learning as an interdisciplinary process that begins at an early age. Through explaining the creation process, the hands-on method and the elements that are employed in the tool, this paper will give a clear and coherent overview of the process and expected results. By tactile movement and visual experience scholars may experience the geometry of a circle. This tool can be prototyped easily by scholars and applied. Similar as Maria Montessori used hands-on materials for teaching they created an inspiring tool.

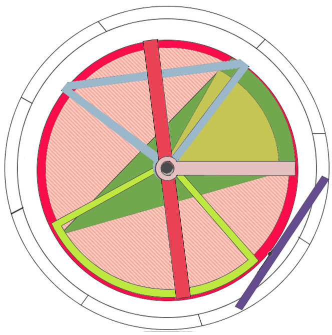

Figure 14: Source: Author Klelija Zhivkovikj, Jovana Radenovic

Group 5:

Lilian Wieser was born in in Vienna/Austria and is 29 years old. She finished the fashion school Hetzendorf at the department of fashion knitting and warp-knitting. Now she is studying art education for textile arts and teaching mathematics to combine both disciplines. Her work is concerning mainly crocheting mathematical thoughts and the facilitation of its use for science and art. She is working on several workshops and projects as for example the international EU project Visuality and Mathematics and seminars for teachers of textile arts in Vienna.

Vuk Vasić was born in Belgrade and is 21 years old. Since he was a little boy I was always interested in computers and computer science. His parents were shocked when neighbuors called him to install the operating system on his new computer when he was 6 years old. They didn't believe he could actually do that, but that day he came back home with a hands full of candies. When he was 8 he started programming in program language C++ and nowadays he knows over 20 different programming languages. He finished Technical High School for Computer management and is currently studying Information Technology at Metropolitan University in Belgrade.

This team worked on the topic chrocheting. As well as a mathematical two-dimensional space, a simple crocheted plane consists of elements – called single meshes – which are situated in a perpendicular net of lines, where the meshes are situated on the crossing points. This fact makes it possible to express mathematical thoughts in crocheting patterns and also to experience basic geometry as an educational tool. The described task of crocheting a circle is only one possible application of this connection of mathematics and textile arts. The developed webpage can be used as an introduction into this precious activity to build a relationship to one of the participating disciplines. It is held in a informal appearance to interrupt habitual perception of mathematical appearance in society and to underline the creativity of doing mathematics on one hand. On the other hand it can open a hand crafter or an artist a different access to creating their projects. Either way, it opens borders and tries to open a self-motivated learning experience.

Since Daina Taimina wrote the book "Crocheting Adventures" which describes how to crochet a model of a hyperbolic plane (Taimina 2009), crocheting settled as an adequate medium to make mathematical thoughts not only visual but also tactile. The hyperbolic space is a mathematical idea that had its problems with getting understood by mathematic students and other interested people before the publication of this book.

Group 6:

Julian Gris, 25 years old, grew up in Vorarlberg, which is the most western part of Austria, very close tot he boarders of Switzerland and Germany, at the Constance Lake. After finishing school at the age of 18, he spent his "social year" in a refugee camp run by a NGO in Vorarlberg, and afterwards he worked and travelled in South America (Ecuador and Colombia) for three months, where he fell in love with the Spanish Language. Already some time before his travels, he decided to become a teacher. So, after coming back to Austria, he moved to Vienna to study Design Pedagogics and Spanish, which are both fascinating subjects to him until today. He love music in all it's different ways, visits a lot of concerts and plays the guitar since more than ten years, besides that my interests are art, design, travelling, languages, cultures, ...He is very excited to find different ways to express and teach mathematics in funny and interesting ways!

Filip Popovic, 23 years old, is a second year student at Metropolitan University from Belgrade, department for Information Technology. He was born in Loznica and after graduating from high school, he moved to Belgrade to start studying at University. From his point of view, this Vishmath project represents great opportunity to do something innovative for children, and maybe bring math to them in some interesting way. It's also a chance for him to meet new people and learn a lot from them.

Figure 15: (l-r): Filip Popovic, Julian Gris
Author: Lilian Wieser

Aim of the project was to overcome the difficulty of teaching mathematics to schoolchildren in an understandable and true – to – life way using interdisciplinary methods and developing knowledge. The goal was to find a simple and user oriented way to teach mathematics in combination with music and music theory principles. Following the guidelines and principles of an ordinary C major scale, a computer application was developed, where mathematical formulas can be tipped in on a keyboard simulation. Every key on the keyboard has its mathematical equivalent, therefore if the Pythagorean theorem $a^2+b^2=c^2$ is played correctly on an ordinary computer keyboard, a triangle appears on the screen.

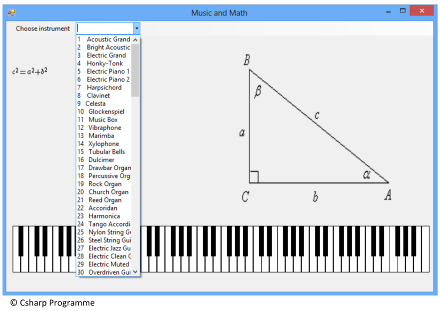

Figure 16: Source: Filip Popovic, Julian Gris, Cacharp Program

Lessons Learnt

By defining a problem as a task in the student groups they were challenged in their disciplinary thinking. This appears quite difficult for experts in a discipline because they cannot understand that there are any problems because they are easy for them to understand. With the method of Applied Design Thinking they needed to empathize with each other, trying to understand deeply what the other partners need. Speaking different languages (German, Hungarian and Serbian) doubled the possibilities of misunderstandings, therefore repeating interviews, visualizations of problems, iterative feedback strengthened the collaboration and communication.

Interdisciplinary, Intercultural communicative competence

"I have learned to listen to others ideas and thoughts and combine them with my own. What's more, I've got an insight into completely different way of thinking, one of an artist and successfully implemented it with my, mathematical, way" (J.V.)

"I improved my cooperation skills. List of friends improved also" (I.M.).

"During a first idea generation session, where the D-School Design Thinking Lab took place, I realized that knowing personal interests would still bring you only half way to the finish line. Key is to really understand the final design should accomplish, i.e. what purpose (or what need for someone) the thing I'm designing should fulfill. This first Design Thinking Lab was particularly difficult for me, as my partner was a math expert (she studies financial mathematics and the like) and I had a very hard time to identify her need. Finally I was able to pin it down to her need to apply math concepts to real world applications" (E.S.)

"Design thinking was interesting topic for me. I have learned how to notice the problem and how to find the solution. Problem can have many solutions but you must decide what way is the right for you or for your group. For me the most interesting topic was my project with Klaudia. We tried to find a connection between art and mathematics, precisely between music and mathematics. (...) I will think more about art and visualisation the mathematics problems. Design thinking will help me in that. If you want to find a solution, you must to see over the border" (I.S.)

"During VisMath project I feel I have improved my communicational skills concerning working in a interdisciplinary team as well as more creative way of thinking or to say how to think outside the box" (I.V.)

"My design thinking skills and skills in working in an interdisciplinary team as well as more understanding it" (A.R.).

"I learned that interdisciplinary work is possible, if you keep on asking the other to explain his point of view, so that you can follow and develop together the project. So the need to come to a common language is essential. As I am interested in communication I tried to observe this during the project. The results were differing from the personal motivation of each member of the group. This is another point of view, I realized as an important one: the personal commitment and motivation to keep on working on the project. Everybody of us has other motives why she or he decides to start an interdisciplinary process. So even the level of interest, knowledge, curiosity, personal commitment, time and energy are different. All these factors you have to have in your mind, during the working process. And all

of them are in change. Another important point I learned is, that I will ask in the beginning of a project very concrete questions about the goals, because in this case, the goal (scientific essay) was defined too late to have the possibility to come to an satisfying output.

As a designer I am used to develop things in an open process, so the prototype we created can be seen as interesting starting point to go deeper into the materia, but to reflect in a scientific way our prototype as well as putting it into an international context, time was not sufficient enough. Also concerning organization I learned, that often simple things turn out to be more difficult as thought. I often realized that things to which I am used to, are totally different for the other. And to explain why and how my perspective is another, takes a lot of time and needs a high level of empathy" (MTW).

"The interdisciplinary work and design thinking methods, done at the workshop, helped me to improve a creative process and team building skills. New approaches at the seminar were a great asset to my knowledge and they inspired me for my further research. Also lessons that I learned, gave me a wider perspective how mathematic education and other disciplines can be improved using design thinking methods. (...) The fast approach about design thinking, at one of the workshops, was something that fascinated me the most. In a short period of time we have gone through all important design thinking stages and used most important methods for practical work. (...) As my research interests are connected with team work, I will use new methods for new innovative projects. Design thinking will be the tool which I will use for further development." (D.T.)

"Team skills, knowledge in mathematics. Combining music and mathematics, and also with art. Using design thinking in most of my new projects, working more in groups, combining subjects that even seem unmatchable" (M.D.).

"I improved my creativity.I improved design thinking.I improved team work.I learned how to use art to make learning mathematics easy and more interesting. Since I was doing a project that combines art and mathematics, in the future if I have a chance to do something similar, I have the experience and then it will not be difficult to me (M.M.).

"I have learned how to more meaningfully use visualization and design in mathematics education!" (J.J.)

"I improved my team and presentation skills" (M.L.)

"I've improved my way of thinking. Group work and the view from the design point has really helped me see things from a different perspective and to first imagine in my head what I need to do and then find a solution. (...) The knowledge I have gained here at the university will help me in understanding the problem when I should be working on a project in the framework of a team. I have the experience and it will not be difficult to me to work in a group (M.Do)".

"I learned a lot on how to create new ideas,how to function as part of a team and of course gained a lot of experience. (...) We will use knowledge in the way

that I will work on ideas to apply this knowledge to the team spirit and I will look to further training to learn in a more creative way (M.N.)".

"I think I improved interdisciplinary skills, since we worked in teams formed of people from opposite fields. (...) As I am now on Master studies of Applied Mathematics where we work a lot in teams it is useful to have good team skills and brainstorming abilities" (N.D.).

"First of all I had to reflect my experiences with mathematics. My experiences at school had not been well so I had been curious what happens. One of my prejudices had been that the mathematic way of thinking is completely different from the creative way of thinking. Of course I knew that there had been connections between maths and art or design (middle age: building churches, golden section, tables with colors from Goethe and others...) but we never touched such topics in school.

I realised that mathematics in school had been something else than the students of mathematics from Serbia were talking about and that you can really be fascinated about this subject. I learned that if I have to work with an "unknown" and "disliked subject" it is very important for me to be in contact and communication with somebody who is an expert. In the group it had been fast to develop interesting questions. The possibilities between maths and design take shape" (C.G.).

"I discovered new and interesting perspectives from inspiring pupils. I learned how to communicate besides language – visual. We had the possibility to get to know new methods (ADTL), intensive workload and how to cope with time limits" (K.K.).

"This was in every sense a very valuable life experience, both on professional and on a personal level. I'm very glad I had the opportunity to participate in such project and had a chance to do the things that interest me in a new and creative way" (D.T.)

Although we have had language for the last 200.000 years, 95 per cent of the evolution of our brain took place in a language called "mentalese" which works more like a computer language and especially non-linguistically and we still think mainly "mentalese" today (Petty 2009, 8). Therefore visualization of problems and solutions help us to understand each other in a more appropriate manner, according to the LOTH: Language of Thought Theory.

Future work needs teamwork and has to be practiced already at University level. Involving students with their personal interest at this level should also inspire them to use this in teamwork situations and classrooms. Interdisciplinary and intercultural work facilitates innovative solutions

Passionate Expert Input: Art & Mathematics – competence of knowledge

The most inspiring part for allmost all of them were lectures by an expert in mathematics: Georg Gläser.

"The most interesting topic to me was when Professor Georg Gläser explained and compared the geometric shapes in nature. For example, a parabola, or an ellipse is compared with bridges, rhombus is compared with the fishing net etc..

It was a lecture of professor Georg Gläser who was talking how he connects mathematics with art. He finds forms in nature and identifies them with geometric shapes. I found out that when you combine two candles you get a form of Cassini ovals, trace of a car has a form of fern, horns of antelopes have a spiral shape, the bridge structures can be parabolic ..."

"The most inspiring part for almost all of them were lectures by an expert in mathematics: Georg Gläser" (D.J.)

"The most fascinating topic during my stay was Applied Design Thinking Workshop!" (J.J.)

"Art in general. We were teamed in a way that in every group has to be at least one artist" (I.M.).

Competence in a discipline seemed to dispel fear of subjects.

Scientific Writing Competence

The information about the scientific essay was given too late and students had difficulties in finishing their project and meeting to get their work done. Although the information for the scientific essay was given too late, the students from different study levels and experiences, all of them with "English as a second language" handed in excellent papers.

They should encourage them to study in foreign countries, apply to international conferences and become part of the scientific community.

Self determination, Empathize

Letting students define a task or topic out of their interest encourages them to "learn for sense and not just remembering, reproducing". Reasoning had an impact on their conversations and they had to show, to visualize it. Through interactive dialogues with the peers they had to empathize and learn from feedbacks, at presentations in front of the groups, the individuals got another feedback, outside the peers and had to learn to think outside the box.

Feedback, Test

Developing innovative presentations, where pupils could participate and practice their suggestions of educational tools they had the possibilities for immediate testing. Feedback methods are designed to check, improve and explore learning and should take place often and constructive in educational context. John Hattie (1999) and Black und Wiliam (1998) discovered, that constructive feedback has one oft he highest impact in education. Learning disabilities refer to „Not understanding a goal" (Petty 2009, 89), a good feedback should discover this evidence and show possibilities of directions. Especially in school subjects of arts and mathematics the definition of talent hast o be cleaned up.

With the method Applied Design Thinking the visual part of the brain is challenged as well as the linguistic part (Petty 2009, 133). It is important to regard the feedback. "Activity-based programs" often show less success, subsumes Hattie (Petty 2009, 67) but it becomes extremely sufficient by giving feedbacks. Visualization und and haptic prototypes were designed. Referring to the "Mentalese" part of our brain, these methods help to increase learning efficiency in a high degree, regarding to qualitative research studies of Marzano (Petty 2009, 77).

Teamwork, social consciousness

Working in a team can be very challenging when expectations and goals are different and need a lot of patience and developing team skills. Finding the same language needs time for translation, empathy, relying and trusting each other which means social consciousness.

Missing lectures and workshops disable comparing evaluation.

Timemanagement

Coping with time limits encourage excellent outcomes.

Further studies

The workshops will have to be continued at the Summeruniversity in Eger and in schools and further on evaluated. Some of the topics can be worked out. In Autumn/Spring a workshop will be held with Parsons University New York (Carol Overby, Aaron Fry) on visualizing-finance.

Competence in a discipline seemed to dispel fear of subjects.

Presentation Skills

"During my stay in Vienna I think I improved my creative as well as my presentation skills. As an engineer I'm used to thinking primarily in a structured way so this kind of team work with creative people was very interesting to me, it was new to me but also very useful for future professional development" (D.J.).

Scientific Writing Competence

The information about the scientific essay was given too late and students had difficulties in finishing their project and meeting to get their work done. Although the information for the scientific essay was given too late in 2013, the students from different study levels and experiences, all of them with "English as a second language" handed in excellent papers, the improved in 2014.

They should encourage them to study in foreign countries, apply to international conferences and become part of the scientific community.

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Conference Presentations

The students presented in a very terrific way their interim results at a conference on "Applied Praxis" at the University of Applied Arts Vienna.

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Life of Geometry

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Abstract

In this paper art and mathematics are combined as a result of an interdisciplinary work. The goal is to raise understanding of geometrical shapes and their impact on our daily lives through an approach that was named "play with proportions". By exploring different city floor plans and investigating proportions and geometry, students should get a chance and freedom to play with and explore geometrical shapes, which have their own sizes and proportions. The idea is also to show the amount of presence of shapes in our daily lifes. Therefore not only geometrical items should be used, because we live in geometrically designed houses and towns, but the social function and personal meanings of places played part in the field research.

Keywords: geometry, triangles, squares, proportions, floorplans, urban planning, golden section

Introduction

Proportions

"Proportion is a correspondence among the measures of the members of an entire work, and of the whole to a certain part selected as standard. From this result the principles of symmetry. Without symmetry and proportion there can be no principles in the design of any temple; that is, if there is no precise relation between its members as in the case of those of a well shaped man". —Vitruvius, The Ten Books of Architecture (III, Ch. 1).

Every item, object, house, plant, human being has its own proportions. Playing with proportions can evoke different feelings as it was demonstrated before in art works (eg. Pipiloti Rist, Das Zimmer (The Room), video installation, 1994/2000/2007). Proportion in architecture is the relation between elements and a whole. In architecture the whole is not just a building but the set and setting of the site. Criteria that are supposed to make a building and its site "well shaped" include how a person can find orientation of the site and the buildings on it to the features of the grounds on which it is situated. From psychology, especially evolutionary psychology but also recent studies in media psychology (e.g. Christian Korunka, The Forbidden Place, 1995) it is known that the better one person's overview and orientation inside a building the better the general feeling of wellbeing and safety is. This also takes into consideration that there are many different aesthetic perceptions, which vary not only from person to person but also in situations and moods. Light, shade, wind, elevation, choice of materials, all should relate to a standard and say what is it that makes it what it is, and what is it that makes it not something else.

Vitruvius thought of proportion in terms of unit fractions, such as those used in the Greek Orders of Architecture although not all Greek temples used the golden section and for example, the frieze and architrave vary from 3/4:1/2 in the Doric style to 5/8:5/8 in the Ionic and Corinthian styles. Capitals are 1/2 in all styles except Corinthian, which is 3/4. The shaft width is always 5/6 at the top. Column shaft heights are Tuscan 7, Doric 8, Ionic 9 and Corinthian 10. Column bases are always 1/2. In the Pedestal, caps are always 1/4, dies are 8/6 and bases are 3/4. In the quarter of the column entasis, Tuscan styles are 9/4, Doric are 10/4, Ionic are 11/4 and Corinthian columns are 12/4.

Scribes had been using unit fractions for their calculations at least since the time of the Egyptian Mathematical Leather Roll and Rhind, Mathematical Papyrus in Egypt and the Epic of Gilgamesh in Mesopotamia.

One example of symmetry might be found in the inscription grids of the Egyptians, which were based on parts of the body and their symmetrical relation to each other, fingers, palms, hands, feet, cubits, etc. Multiples of body proportions would be found in the arrangements of fields and in the buildings people lived in. A cubit could be divided into fingers, palms, hands and so could a foot, or a multiple of a foot. Special units related to feet as the hypotenuse of a 3/4/5 triangle with one side a foot were named remen and introduced into the proportional system very early on. Curves were also defined in a similar manner and used by architects in their design of arches and other building elements.

These proportional elements were used by the Persians, Greeks, Phoenicians and Romans, in laying out cities, stadiums, roads, processional ways, public buildings, ports, various areas for crops and grazing beasts of burden, so as to arrange the city as well as the building to be well proportioned. Architectural practice has often used proportional systems to generate or constrain the forms considered suitable for inclusion in a building. In almost every building tradition there is a system of mathematical relations, which governs the relationships between aspects of the design. These systems of proportion are often quite simple; whole number ratios or incommensurable ratios (such as the vesica piscis or the golden ratio, which were determined using geometrical methods.

Generally the goal of a proportional system is to produce a sense of coherence and harmony among the elements of a building. Going back to the Pythagoreans there is an idea that proportions should be related to standards and that the more general and formulaic the standards the better. This idea that there should be beauty and elegance evidenced by a skillful composition of well understood elements underlies mathematics in general and in a sense all the architectural modules of design as well. So the idea is that buildings should scale down to dimensions humans can relate to and scale up through distances humans can travel as a procession of revelations which may sometimes invoke closure, or glimpses of views that go beyond any encompassing framework and thus suggest to the observer that there is something more besides, invoking wonder and awe.

Human Scale and the impact on design and well-being

As a study conducted by the Robert Wood Johnson Foundation 2005 about the influence of urban design on walkability, a term which is used to describe the ability to use the city and its infrastructure without the help of a car and therefore supposed to have a positive impact of the wellbeing of people in places, found out there are up to at least eight different factors that have a very big and positive impact on the walkability. Human Scale being one and is described according to the authors of that study:

"Human scale refers to a size, texture, and articulation of physical elements that match the size and proportions of humans and, equally important, correspond to the speed at which humans walk. Building details, pavement texture, street trees, and street furniture are all physical elements contributing to human scale" (Reid et al 2006).

Buckminster Fuller

Buckminster Fuller's "operational mathematics" stems from his observation of the lack of connection between conventional mathematics and reality. He writes in "Synergetics: the Geometry of thinking," The prime barrier to humanity's discovery and comprehension of nature is the obscurity of the mathematical language of science. Fortunately, however, nature is not using the strictly imaginary, awkward, and unrealistic coordinate system adopted and taught by present-day academic science." (Fuller 1979, 1). Fuller proposes that mathematical principles should be derived from experience of the natural world. If we start with real things, and make deductions based on them, then our resulting generalizations will reflect and apply to the world in which we live (Child1999, 1).

The research

To be able to elaborate idea the series of research and surveys were conducted. The kick- off was a mini questionnaire that was framed very open and that was given to a group of 11 students who were participating at an interdisciplinary study program where students of visual arts worked together with students of mathematics. The students were asked to share insight of their thoughts of the ideal places and spaces. There were no set boundaries or presentation requirements since the idea was to get what first come to their minds in order to get assuredly results.

The results of the survey are given bellow.

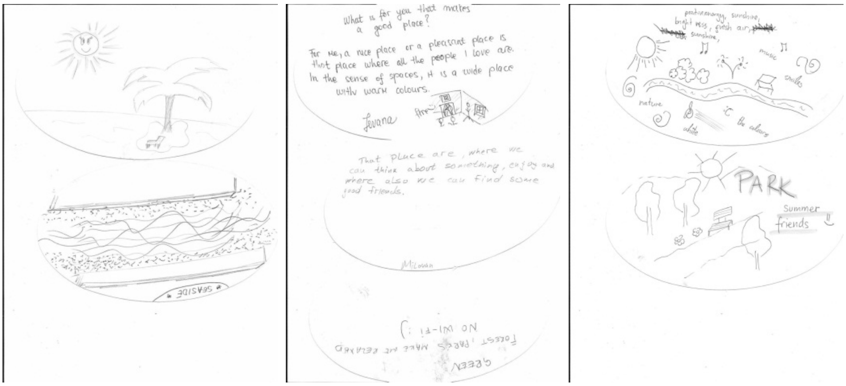

Figure 1–3: Results of the survey
Authors: Irina Nalis, Andrea Reithofer and
Aleksandra Rastik & Participants of the workshop

One of the most striking result was that nobody created a place indoor. During the workshop no floorplan or architecture painting was made, neither were there any other descriptions of rooms. The importance of the space is measured for them by the presence of the people they know or “nice” people. Nice place equals meet and find some friends.

Park (garden) is a great place to be. They draw a classical garden with trees, a bench and a river. Hypothesis: A river is calming.

Place where the other feels comfortable is next to the sea. Hypothesis: It is more the feeling of a holiday than the idea of a place itself that attracts people’s imagination. So to say more benefit oriented than concentrated of the features of the place itself.

The excursion to Prater,WU and the OMV building in Vienna

In order to understand better spaces and shapes that surround us an excursion to some parts of Vienna was organized among the group. The result of the excursion and observation was that a great place depends not only on good buildings. Good planning at a local scale involves spotting ways to balance economic viability, public safety, accessibility, and amenity (Hague 2000, 6). Space is more than the sum of individual buildings. It is the relation of buildings and activities to each other, to spaces, to the scale and grain of the area that gives us the sense of place, a feeling of identity, an appreciation of the character of an area (Hague 2000, 6).

Figure 4–7: Documentation of the excursion
Authors: Irina Nalis, Andrea Reithofer and Aleksandra Rastik

The task

After the research was done the main goals were to raise awareness of the omnipresent existence of geometrical shapes in our daily lives. In order to do so students of a summer school for visualization of math in Hungary will be given a map of Vienna. In fact, not the entire city map but a series of geometrical shapes that form parts of Vienna's downtown in a certain scale. Those geometrical shapes are movable parts and could be rearranged in a different manner. So the task for the students would be to take those shapes and rearrange them and come up with a different relation between them. When combining the shapes they can use they own criteria on how a town should look like to make them like it. They can re-arrange those shapes according to the golden section or make them symmetric or asymmetric – there is no right or wrong it all depends on their beliefs and feeling of a beautiful place where they would feel good. The process is shown in the pictures underneath where a prototype consisting of a map of Vienna (Google Map) that was coloured via the architectural graphics program Auto CAD that afterwards had been cut into pieces enables to rearrange parts of the city into new shapes.

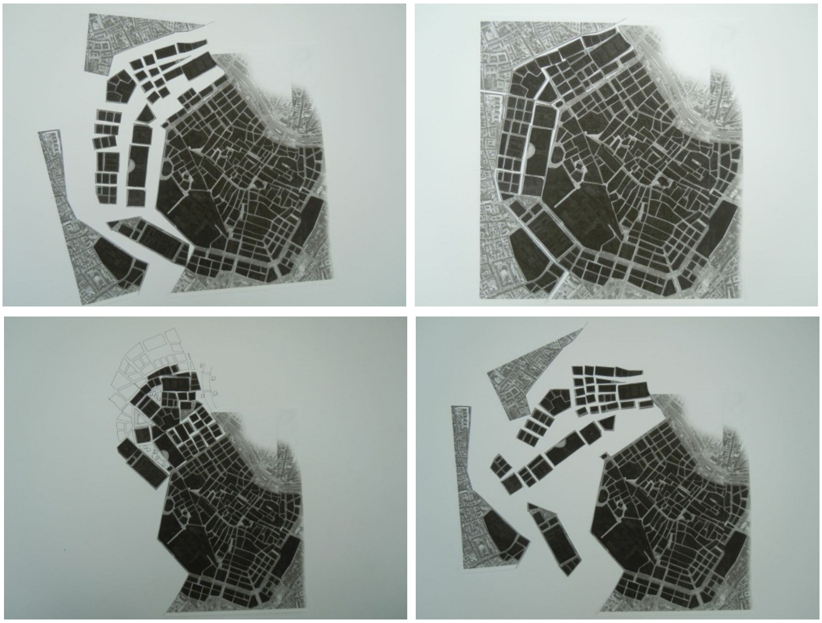

Figure 8–11: Documentation of the excursion
Authors: Irina Nalis, Andrea Reithofer and Aleksandra Rastik

The whole idea is based on the concept of "play with proportions". By giving students floorplans we are raising awareness about how much geometrical shapes are infiltrated into our lifes. We are living and moving according to geometry. By adding functions (e.g. social functions such as parks, meeting points or commercial functions) to the block there is a possibility to see how much space they think is needed for a certain function and if it fit their needs.

Play with proportions

The task is also visually expressed in a manner of graphic and visual presentation so that it can be easier to understand. The logo for the math task that was designed to appeal to younger target group shows the main items in a pictographic style: A map on which you can apply the laws of proportions has infinite solutions.

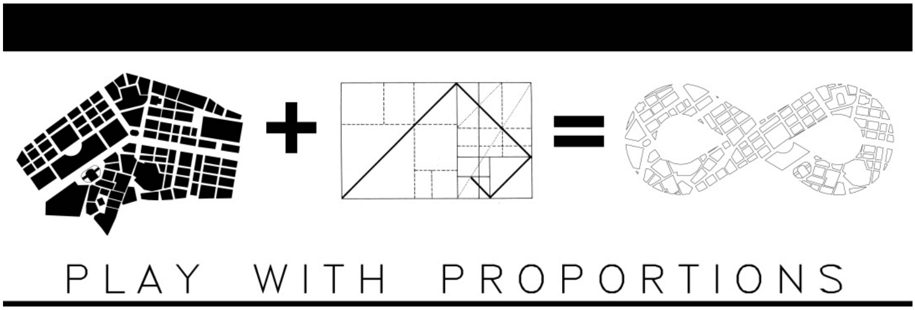

Figure 12: Play with proportions
Author: Aleksandra Rastik

The aim of the task is to show that this way of rearranging shapes would mathematically give an infinitive number of possible solutions. Playing with shapes of towns and applying the laws of proportions and sizes would therefore give infinitive number of new solutions and thus show interesting approaches to urban design. Also the idea is to give a sense of freedom in playing with shapes and letting participants free their minds in experience the endless array of solutions and truly experience geometric shapes that rule our lives.

Future work

This type of approach can later be used in further research of town proportions and social context of living.

Conclusion

We are surrounded by geometry thus mathematics without noticing most of the times. In combining the social notions of spaces, living together and wellbeing with the underlying mathematical structures this education tool should add an extra dimension to geometry.

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Financial mathematics: Risk and risk free financing

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Abstract

This work shows the relation between financial risk and award. As many people do not have a chance to feel that kind of risk simple game that enables player to determine a risk level at which he wants to play was designed. A puzzle, which stands for the financial resources, can be composed in different visible conditions in a bordered period of time with one hand. The visible conditions, which embody risk-taking can be calculated by the player. For a possible maximal profit the player works completely blind Minimal profit is therefore gained by having the best visual condition. The psychological aspect and the corresponding dynamics can be experienced in this project.

Keywords: risk, finances, game, feel

Introduction

Risk is the possibility you will lose money if an investment you make provides a disappointing return. All investments carry a certain level of risk, since investment return is not guaranteed. According to modern investment theory, the greater risk you take while making an investment, the greater your return has the potential to be if the investment succeeds.

For example, investing in a startup company carries substantial risk, since there is no guarantee that it will be profitable. But if it is, you are in a position to realize a greater gain than if you had invested a similar amount in an already established company.

After many sessions of brainstorming, trying out a lot of different ideas, discussing with people and researching their relationship to finances and mathematics it could eventually be concluded that the average young person needs to use his own senses to really understand, especially when it comes to a topic this complicated. Moreover in Figure 1. you can see children hands dipped in paint. This was an inspiration to make a game that would enable a player to use his/her own senses and therefore comprehend the risk better. The point was to explain it in a most simple way, and not to implement a certain opinion on risk, whether it is positive or not.

Figure 1: Feeling the world around us

Materials and Methods

The initial idea was to form a game where two players could interact inside of a box representing interaction on a financial market (Figure 2). The box should be filled with some kind of reward, in this case balls of different sizes representing rewards of different value. They would both work trying to collect as much as they can. As profit of these two parties on such a market does not depend only on them, the possibility of introducing more participants and making the game more realistic was the next step. However, after a lot of researching, a way to represent risk in multiplayer type of game could not be found.

Figure 2: Initial Idea
Authors: N. Džaleta, C. Gobbi, M. Domazet 2013

Decision to keep the idea of a box game was made, this time with just one player (Figure 3). Simple box with a hole on one side where participant puts his hand and tries to solve the puzzle inside the box was constructed using easy-to-find materials.

In practice financial risk is connected with not having enough information of a financial market, which makes it harder to perform efficiently. Therefore a move-able lid on top of the box was designed, which player could adjust to their own preferences for risk. Putting the lid in a position, which reveals the inside of a box corresponds to risk free situation, as you can see and control everything in the game (Figure 4). As you move the lid towards you, hiding the inside of a box, the risk rises and eventually it reaches its highest level when the lid is completely covering the top of the box (Figure 5).

Figure 3: Continuing with the idea of a box
Authors: N. Džaleta, C. Gobbi, M. Domazet 2013

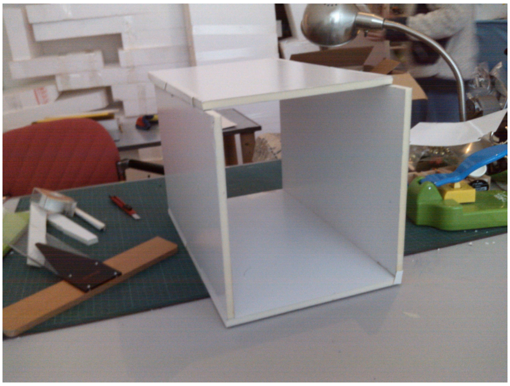

Figure 4: Risk free
Authors: N. Džaleta, C. Gobbi, M. Domazet 2013

Figure 5: The highest level of risk
Authors: N. Džaleta, C. Gobbi, M. Domazet 2013

Having designed the box now, it was necessary to make a task for the participant to complete in order to gain a reward. At first an easy puzzle was constructed (Figure 6) but as soon as its simplicity was examined with the lid completely covering the box, it turned out to be impossible to complete. Then simplification was to be done (Figure 7) by cutting out shapes that can be felt more easily using just one hand.

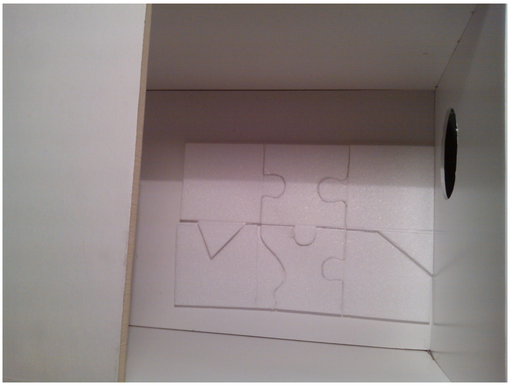

Figure 6: Initial idea of a puzzle
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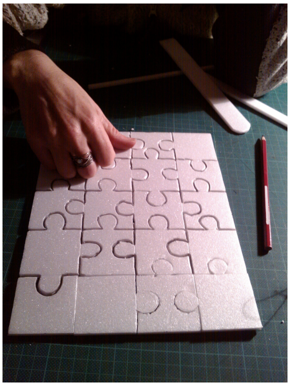

Figure 7: Simplified puzzle
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Conclusion

In the end a game was designed, where a player invests a certain amount of "money" which for the purpose of a game would be candies, decides what amount of risk he/she wants to experience and finally tries to complete a puzzle under those circumstances. If he/she chooses to go with a higher risk his/her award will be proportionally higher as he/she puts more effort into completing the task.

Oppositely, if he chooses lower risk level the award is going to be smaller.

Construction of an Augmented Reality Drawing Key

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Abstract

This paper presents an interdisciplinary project developed to improve instruction of mathematics. The objective was to create a tool that could improve the understanding of geometrical shapes. A tent-like construction, called an Augmented Reality Drawing Key, is developed. The ARtKey can be placed on a table. Geometrical shapes are placed underneath this design in order to be drawn or captured through an augmented reality application. Using ARtKey, students can get a deeper and more personal understanding of geometry, as they get involved into several different activities. They experience how perspective construction has its roots in the human visual system. Further, the added augmented reality feature widens the activity range. Through using multiple senses, i.e., seeing, touching, moving, and listening, the user learns much better than in the case where only one sense is involved.

Keywords: Drawing aid, Geometry, Mathematics instruction, Augmented reality, Perspective

Introduction

Drawing tools, used to help depict reality or geometry, have a long tradition - from the wooden Proportion Mannequins of figures and animals, to the more space-oriented constructions, like the camera obscura or the famous drawing constructions of Albrecht Dürer. The British painter David Hockney has collected scientific and visual evidence in masterpieces of western art for frequently used machineries and inventions. Often, they consisted of glass lenses (Hockney 2001, pp. 202-225). As J.V. Field points out in his work about the renaissance artist Piero della Francesca, some artists even contributed significantly to the development of geometry, as their will to depict led them to unique solutions.

Students are confronted with geometrical forms when they learn mathematics at school. In order to think geometrically it is necessary to depict those forms. Often it is not easy to translate the object that is seen into a drawn representation on paper. Learning is much more intensive if the student uses more than one sense. Seeing, writing, drawing, and thinking are the common activities in math classes. What is missing often are the touchable forms themselves. But just touching mathematical forms alone is also limiting. Therefore, we wanted to create a tool that also includes the representational (drawing) part and analogue thinking using the technology of augmented reality.

Augmented Reality

Augmented reality (AR) is a technology that combines a live view from real world with computer-generated images (Carmigniani et al. 2011, pp. 341-377). Thus, reality is augmented by additional multimedia information presented on the screen of a particular device, such as smartphone or a tablet PC. The user's perception of the world is, in this way, augmented by additional multimedia information (audio, video, and 3D objects). According to Milgram the possible combinations of real space and virtual elements are shown in Figure 1. In this project augmented reality was created through implementation of virtual objects into a real environment (Milgram et al. 1994, pp. 1321–1329).

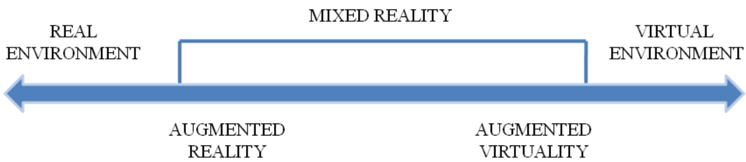

Figure 1: Milgram's reality-virtuality continuum (Milgram et al. 1994)

An augmented reality application must have the following characteristics: it combines real and virtual objects in a real environment; it runs interactively and in real time; and it aligns real and virtual objects with each other (Azuma 1997, pp. 355–385.). Advancements in mobile hardware, which can satisfy these characteristics, lead to a variety of innovative applications. The applications realized with augmented reality are becoming increasingly more numerous, especially in the field of medicine, entertainment, military or cultural heritage. A particular area of application is the field of education.

This project should encourage students to learn and overcome difficulties when drawing 3D mathematical shapes. The technology of augmented reality is applied to provide more information about the drawn objects, in the form of videos and 3D objects. The remainder of the paper is organized as follows. In Section 2, the experimental setup will be described. Construction of ARtKey and recorded results are discussed in Section 3. Section 4 closes the paper with main conclusions and some directions for future work.

The Experimental Setup

The experimental setup consists of two parts:

- the drawing aid, and
- the mobile application, using the technology of augmented reality.

The Drawing Aid

The drawing aid is defined as a tool that opens possibilities of experience for drawing. The construction has a tent-like form. One side is a glass plain and the other side is open, so that geometric forms can be put underneath (Figure 2). The used geometric shapes (spheres, cubes, spirals, etc.), which are everyday objects, should have clear forms. This solution was created using a common picture frame that was transformed by adding two wooden sticks, forming the tent situation. The goal was to make an object that was easiest to build and affordable for students. The used picture frame is a cheap and easy to find product. The geometric shapes put underneath should be collected from the students themselves. Consequently, students also learn to focus on geometry outside the classroom and find the practical usage of geometry in various designed objects or natural materials.

Figure 2: Augmented Reality Drawing Key
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The Augmented Reality Application

The application is created for mobile phones, which have Android operating system. In this application, we use the technology of augmented reality. In this work the software metaio Mobile SDK (Metaio Inc 2013) was used, because it applies fast, modular libraries and algorithms since it was designed to be used for mobile devices. It supports different types of tracking. Further on marker-based (Kato et al. 1999, pp. 85-94) and marker-less tracking (Herling et al. 2011, pp. 255-272) was used.

For multimedia content, videos in 3GP file format, and, for 3D models, OBJ file format were used.

Construction and Results

The ARtKey is a link between spatial perception and the hand that tries to render forms. The specific areas in the brain that mostly concentrate on shapes and distances are activated, whereas other brain activities are reduced. The hand simply follows lines.

At the same time, the tent-like drawing tool and its self-made geometric objects are filled with information through augmented reality programming. The ARtKey also activates abilities different or even contrary to drawing, such as following an instruction or associating analogies in forms.

In order to get a convincing perspective picture on the glass plain, the drawer has to close one eye and fix the position of his head. Otherwise, it is not possible to fix a line. This experience shows that our cultural convention to use perspective is always based on a fixed, one-eyed position. We are used to this experience because of our widespread use of photography and film. Martin Kemp has focused on the beginnings of western central perspective in Italian renaissance (Kemp 1997, pp. 121-171). As far as the relativity of western traditions is concerned, Sigrid Schade and Silke Wenk have pointed out in their work about visual culture that ways of representation vary a lot in different cultures and can lead to great misunderstanding (Schade et al. 2011, pp.13-28). Therefore, it is of importance to be aware of one's traditions of depicting reality. Otherwise, one tends to believe that one's own way of representation is the only one, which does not belong to a constructivist's world-view.

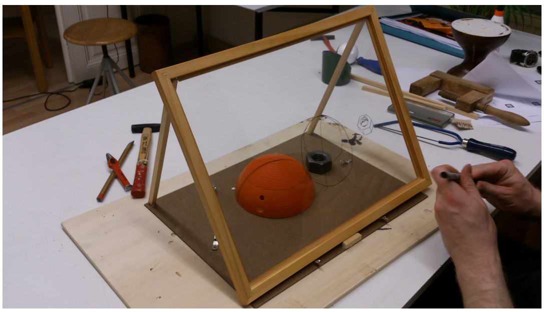

Figure 3: Drawing Aid
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The other main characteristic of multidisciplinary approach is a software tool that uses the technology of mobile augmented reality. An application using mobile AR is designed to give more information about the mathematical objects used to investigate. Mathematical objects have markers or images on their surface. The application is designed so that the user, while using the camera on the mobile device, can detect markers or images on the geometrical forms. Afterwards, the screen of the device shows multimedia information about mathematical objects at the exact location of the recognized marker. Using this new type of interface there exists the ability to present further information. Multimedia content in form of videos, 3D models, images, and sounds (Figure 4) can be added.

The typical feature of AR is that the user is motivated to move around an object. Through moving around, he investigates and explores a shape much more intensively than through merely looking at it. A shape seen from different angles offers larger amount of information. In this way, a moving person can widen his view of reality simply by shifting his position in space.

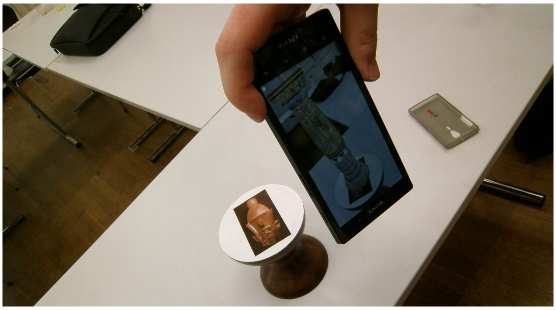

Figure 4: Augmented Reality Mobile Application
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The added information has several objectives. On the one hand, it furthers mathematical knowledge, such as formulas. On the other hand, it provides analogue ideas (nature, architecture, design, mechanics, etc.) connected to the observed shape.

Conclusions and Future Work

Geometry is, for most students, something abstract and theoretical. ARtKey not only links theory with practice, but it also enhances deeper understanding of the methods of representation. In linking the world of hand-drawing and the technology of augmented reality, this interdisciplinary solution brings geometry closer to the personal world of a student.

The next step for the ARtKey project will be to test it with students. Their reactions will lead to further questions and developments. Through analysis of data gained in such an interactive use of the system, further improvement of the features of ARtKey is planned.

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Perception of Phi: Patterns in Numbers

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Abstract

This paper investigates the present of patterns in nature and a possible correlation of patterns in numbers. Mathematical concepts such as Pascal's Triangle, Fibonacci numbers and Golden Ratio will be considered in an artistic-mathematical discourse. With the help of an interactive model for exploring ratios and visual relations in human faces, the mathematical concept of Phi will be addressed.

Keywords: Pascal's Triangle, Fibonacci numbers, Golden Ratio, visual perception, patterns in numbers.

How fundamental is that Nature's design is aesthetic. If the aesthetic response to pleasing pattern designs we sentient beings experience extends all the way down to and through Nature's most fundamental levels of existence...and by that we mean way beyond leptons and quarks...to the abstract space-time of numbers...to those very same numbers we count apples and oranges. If Nature is number and number is Nature...in whole or part (and try to find the difference, pattern and design are the essence defining us all. The spiritual implication implied here, and by others, is not lost nor limited by journeying deeper and deeper into Nature's mystery...if anything it is greatly enhanced! What, then, could be the relation between the existence of Phi's concept in nature and the human perception?

Before going on to link up Phi (Φ) with Fibonacci it will be useful to recapitulate where Phi came from: it was in fact from Euclid, the Greek mathematician of the Alexandrian centre of learning of 2300 years ago. He first recorded what he did not claim to have originated, namely a ratio, writing "A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so the greater is to the lesser." For centuries the Extreme and Mean Ratio was given the Greek letter Tau (T) as its symbol, from the initial letter of the Greek word meaning to cut. Only in the twentieth century did Phi become more common. Apparently deriving from an association with the Greek sculptor Phidias, who lived much earlier than Euclid, in Athens, and who was credited with using this ratio as a proportion for buildings like the Parthenon, and for his sculptures. He believed that there was a particularly fitting proportion in this ratio, something beautiful, something golden about it. The now very common name of the Golden Ratio has its roots in this sculptor's visual aesthetic preferences. The ancient Greeks had discovered Pi (Pi) as well, and the whole principle of irrational numbers, which are neither integers nor fractions based on integers, and can never be stated completely. Phi too is irrational.

A simple construction of Pascal's Triangle proceeds in the following manner. The number on the top is always 1. As can be seen in Figure 1, to construct the elements of following rows, add the number above and to the left with the number above and to the right to find the new value. If either the number to the right or left is not present, substitute a zero in its place.

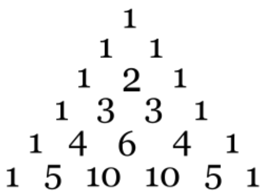

Figure 1: Pascal's Triangle
Author: Conrad Irwin

One property of the triangle is the diagonal bands sum to successive Fibonacci numbers. This relation is illustrated in Figure 2.

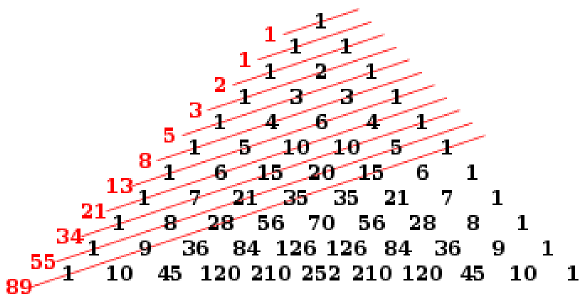

Figure 2: Fibonacci Numbers
Author: Arpit Shah

In mathematics and the arts, two quantities are in the golden ratio if the ratio of the sum of the quantities to the larger quantity is equal to the ratio of the larger quantity to the smaller one, visually represented in Figure 3.

Expressed algebraically:

$$\frac{a+b}{a} = \frac{a}{b} := \varphi,$$

where the Greek letter phi (φ) represents the golden ratio. Its value is:

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.6180339887\ldots$$

Expressed visually :

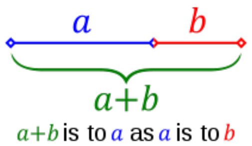

Figure 3: The Ratio
Author: Xiong Chiamiov

The golden ratio can be found in many objects in nature. Scientist found the golden ratio expressed in the arrangement of branches along the stems of plants and of veins in leaves, also in the skeletons of animals and the branching of their veins and nerves, the proportions of chemical compounds and the geometry of crystals . . .

A short introduction of a few famous, yet simple number patterns thus presented should lay the foundation for an understanding of the remarkable patterns found in nature. While ancient reference to many of these patterns can be found in Indian, Greek, Hebrew, Chinese and Arab cultures, our references will be those found in Western sources. It should also be noted that the examination of these very simple, but profound number patterns in no way detracts from the marvellous patterning found in fractal, virtual, imaginary, irrational or complex numbering systems (for the most part). Simple, whole integer counting numbers, but irrational numbers - as the before mentioned Golden Mean and the Fibonacci - form the bridge from the patterns also found in the simple whole numbers to that found in the double helix of DNA. These patterns have both a numerical and a visual presentation (Rose 1998, p. 967).

Rose and Blake argue in Motion Perception: From Phi to Omega:

> Human visual perception continuously seeks structure and meaning in the dynamic patterns of light imaged on the retina. Even when the spatio-temporal information in that pattern of light is impoverished or underspecified, constructive, synthetic processes fill in the gaps (Gregory 1970; Rock 1983, p.97), whether those gaps exist in space (as

> exemplified by subjective contours: Kanizsa 1955, p. 29) or in time ('phi motion': Wertheimer 1912, p. 176. The visual system assumes, in other words, that the world is orderly and structured, and, when confronted with unstructured or partly structured input, the brain literally completes the picture. These constructive propensities of human vision are so powerful that they even operate when the retinal input is completely random. For instance, people report seeing regular and repetitive patterns after a few seconds of viewing a dot pattern that is genuinely random (MacKay 1965, p. 192) (Rose 1998, p. 967)

One may argue that we have been conditioned to recognize such patterns; that laypersons, as well as artists and scientists, commonly both see and are attracted to subjective pattern-imposition in almost everything scrutinized by the senses. We may even over-interpret our 'pattern-izing', imposing our expectant visions upon other people, other cultures and other sensibilities. Because pattern-recognition - even if not scientifically-based - gives us an plausibly rational approach to subjects such as art, poetry or history, it has become an ingrained aspect of our educational culture; and as many teachers and writers have used such concepts, those questing for knowledge have happily leaned upon mathematically shaky structures. For while the golden ratio is inherent in nature's programming, we, in our love and derivation of art from nature, have falsely chosen to exaggerate its importance in a subjective, impositive manner.

Of great interest thus becomes the expectation of this ratio in our daily lives. Do we expect to find relations of phi in everything we do or see? A game of finding proportional lines in faces, will, without making any commentary or judgment on what is considered beautiful, rather through the exploration of prevailing lines and ratios of these lines, introduce the concept of phi playfully and interactively. The user should gain the ability to see relations and derive a cognitive understanding of this abstract mathematical concept.

Figure 4: Use of the ratio-finder-box
Source: Photo, Jovana Vujicic

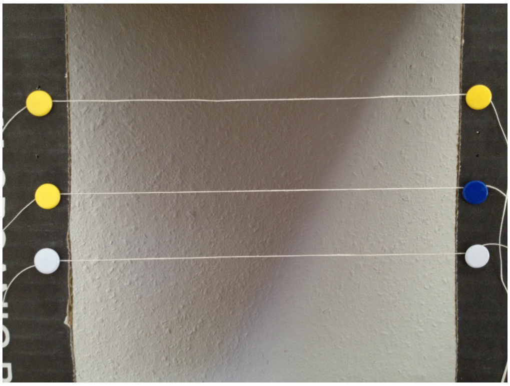

Figure 5: Finding the ratio
Source: Photo, Elisabeth Stephan

The Perception Prototype is a means to explore the multitude of visual relations of lines in the face that can be related back to patterns in numbers in mathematics. The aim is to find many lines in the face that have a relative distance relation, often very close to phi. With a partner, the user will to discover the relations of facial features by pinning imaginary lines across three different levels they see in their partner's face. After measuring the distance between the two upper lines and then the difference between the two lower lines, the ratio is calculated.

This playful exploration of visible relations is a way to increase awareness of human predilection for seeing patterns, often influenced by previously recognized visuals. If humans tend to see what they know, may one even suggest it's in our DNA to see these patterns?

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Shah, Arpid: Available from:
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Chiamiov, Xiong Available from:
`http://upload.wikimedia.org/wikipedia/commons/a/a6/Golden_ratio_line.png`

How does a Mobile Phone work?: Fractals

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Abstract

In this paper it will be described how a topic for the project was chosen, what was studied and what was considered as the most import to know in this area.

Since the working group consisted of three students, who are studying at different faculties, we tried to find a topic that will connect arts, mathematics and telecommunication. It was discussed about the areas that we like and dislike concerning mathematics. The final decision was made to investigate in the topic of fractals.

During the discussions a very interesting topic was found, which was associated with fractals. We decided the topic "How does a mobile phone work?" because it connects telecommunication and mathematics.

Questions like: "Does mobile communication use fractal geometry?", "How can we explain the connection between fractal geometry and communication to children? " arose, and educational tools for the topic developed.

Keywords: Mathematics, Telecommunication, Fractals, Design education.

Introduction

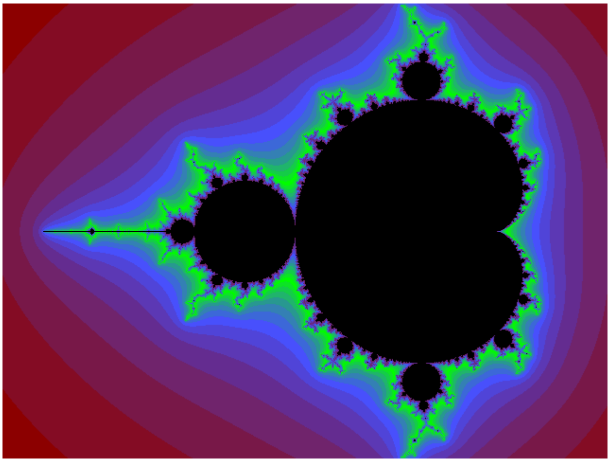

Figure 1: Mandelbrot set
Author: Wolfgang Beyer
Available from:
http://fi.wikipedia.org/wiki/Tiedosto:Mandelbrot-Menge_farbig.png

Figure 2: Romanesku cauliflower
Author: Juha Salin
Available from:
http://www.cartinafinland.fi/fi/imagebank/image/46/46337/Romanesku+kukkakaali+46337.jpg

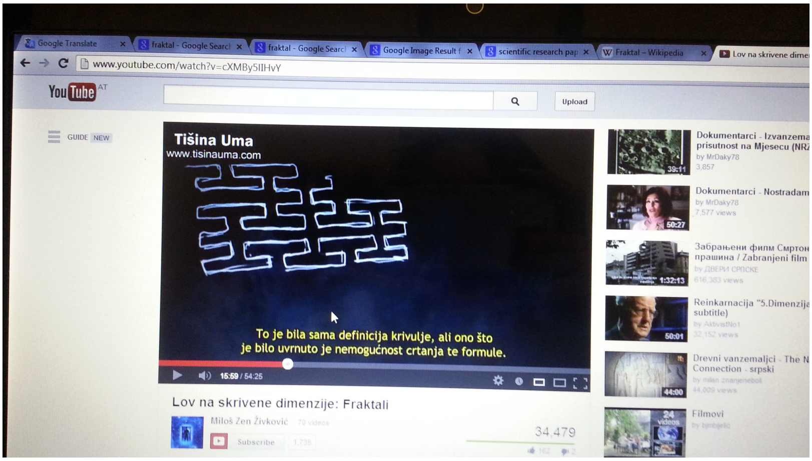

Figure 3: Fractals; Hunting the Hidden Dimension
Author: John Haptas
Available from:
`http://www.youtube.com/watch?v=cXMBy5lIHvY`

The study of fractals started in the 1980s by Benoit Mandelbrot, a French scientist, (Mandelbrot, Benoît 1983), who was playing by drawing complex numbers on a computer. A fractal is a mathematical object that displays self-similarity on all scales.

The object does not need to perform exactly the same structure at all scales, but the same "type" of structures must appear on all scales. Fractals can be found in plants, flowers, forests, lungs, weather systems, rhythm of heart...

The most important area of application of fractals is by microwave devices where size has a fundamental importance, for example by satellite systems (GPS), mobile systems(GSM), Bluetooth applications, and others.

90's years a Boston radio astronomer, Nathan Cohen (Cohen 2002, pp.6, 7, 256, 452, 553, 751) used fractal geometry for technological discovery in electronic communication. He wondered if it was possible to make an antenna using the same fractals. He has not only managed to make much smaller antenna using fractal design, he has already managed to expand their range of frequencies that can catch.

Cohen developed his findings at the time the telephone company had problems with mobile phones.

Today fractal antennas are used among tens of millions of mobile antennas and other devices of world wireless communication.

Since the 19th century our famous Serbian scientist Nikola Tesla, said:

“You might think that I'm a dreamer and I'm going too far to say that if you pay my hope for in the future. I can tell you with absolute confidence that I believe I will send messages from one country to another, without wires! Also, I hope that I will on the same way, without wires, transmit electricity without losses. In success wireless control and signaling distance will no longer be any doubt" (Tesla in conversation with Artur B. Jovanović: 2013).

Methods

The objective of this project was to explain how antennas work by showing it in a performative way. Antennas in the mobile phones work by collecting electromagnetic waves, which are invisible to the human eye.

To make electromagnetic waves visible, it is possible to demonstrate them by using a scarf. A scarf imitates electromagnetic waves.

What can be seen are the oscillations they are making. The number of these oscillations, which can be increased, is called frequency.

Looking at a period of time, oscillations can be slower and may be faster.

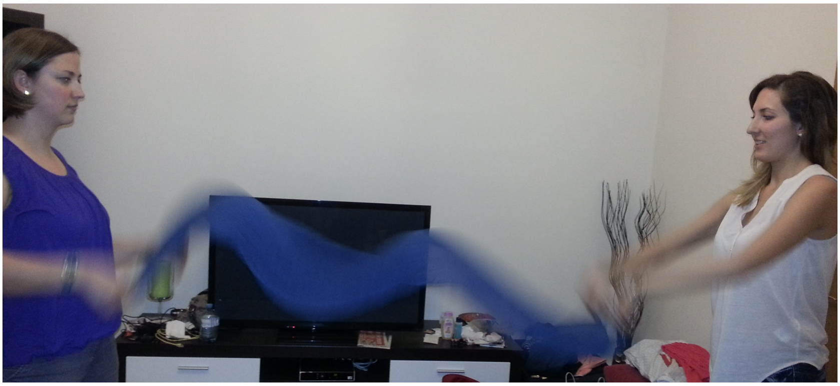

Figure 4: Demonstration of electromagnetic waves
Authors: Marina Mišanović, Milena Đorđević, Marie-Theres Wakonig, 2013

Results

Looking for old mobiles, destroying them, looking for the antenna can be a first step for understanding circumstances.

Figure 5–8: Mobile antennas
Authors: Marina Mišanović, Milena Đorđević,
Marie-Theres Wakonig, 2013

Students in workshops could design mobile cases in order to engage with fractal form and technological impact:

Figure 9, 10: Mobile phone cases
Authors: Marina Mišanović, Milena Đorđević,
Marie-Theres Wakonig, 2013

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Note Dictionary for the Visual Experience

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Abstract

The "Note Dictionary" offers a new graphical visualization for music. With this method, children have the opportunity to combine the basics of other areas and subjects. Thus, the learning of notes may offer a visual experience.

The two elements that are defining it are the value of the note and pitch. In the "Dictionary" the coloured geometric shapes substitute the conventional written record of the notes.

The different note durations are indicated by different colours. The arrangement of the colour refers to the colour theory of Johannes Itten. The classification is designed to use primary colours, secondary colours, and tertiary colours. After this order, we found the following relationships: the whole note: yellow, the half note: blue, the quarter note: red, the eigth note: green, the sixteenth note: orange, the thirty-second: purple.

The naturals (C major) based on the rules of geometric shapes (regular polygons hierarchy of squares). The upper-lower octaves are assigned more complicated shapes or body.

The finished catalog / Leporello (Illustration 1. Leporello to the "Dictionary") is the basic explanation of the application and three children's songs as examples. Using this method, children, according to individual drawings, are able to "compose" music in a playful way and get to know the elementary concepts of music, geometry and art. For this adventure centered learning only crayons, paper and body resonance are needed.

Keywords: music, color theory, art, geometry, experience-centered learning, new graphical visualization, note dictonary

Introduction

The goal of the project is a visually interesting reformulation of the notes. The logic structure, or the connection between geometry and art, will be presented. This simple visualization offers newcomers the note reading reference points for general education.

The academic knowledge in mathematics (Figure 3) define the pitch of the notes and the learned colour mixing basics in art class (Figure 2) determine the note values. In this way, the written representation of music needs no "abstract" knowledge.

The focus of this kind of working specializes in basic geometric shapes (Euclidean geometry, hierarchy of squares, platonic solids). The increase of musical difficulty (complexity level) offers sophisticated mathematical challenges. The more difficult the musical composition is, the more complicated / multilayer geometric shapes / bodies are applied.

After Kueppers[1] "In mathematics there are increasing levels of difficulty. From stage to stage, the calculations get more complicated ... This view options are not dissimilar in colour theory. In this project two colour mixing laws were used, which are explained by this primitive stage name, subtractive mixture 'and additives mixture.' "

The structure of music (pitch, note value) is necessary for the understanding of musical notation, and for children, music students and music lovers useful and profitable. The users of the "Dictionary" put the colours and geometry in a creative order to enable the cognitive thinking.

Experimental Setup, Results

This work is based on the linkages of several scientific areas.

The circle in its completeness, its end and its beginning is one and the same, called the break. The octave C major is the regular triangle, square, rectangle, rhombus, trapezoid, parallelogram, defined as convex quadrilateral, rectangular triangle. At higher and lower octaves, the shapes and bodies are more complicated.

The colours identify the six most frequently collocated note values. Children have the option of individual treatment because they can select and use the shade according to their own taste. Blue is blue, whether light or dark, king or prussian blue, it depends only on the imagination of the users.

Work

The finished Leporello (Figure 1) consists of 3 parts:

- the manual (Figure 5.)
- a short scientific explanation (in terms of color theory of Johannes Itten and polygons)

[1] (p.75). Kueppers, Harald: The Logic of Color: theoretical basics of color theory / Harald Kueppers. - 2. thoroughly revised and supplemented edition. - Munich: Callwey, 1981.

- Examples (three children's songs that present the possibility of the application and consistency of the three countries (Austria, Hungary, and Serbia) representing the project) (Figure 3)

Conclusion

In this method, the focus is always in contrast to the traditional touch sensitive to the child and their individual needs and wishes. The experience and enthusiasm for notes arouses interest by using colorful shapes.

Within the workshop, the children learn, or even dyslexic inside, know the application of the method and have the ability to produce according to your own drawings, music and / or graphically represent new classical music pieces. Another future possibility is to use a computer program that provides playful and musical media skills in using computers.

The benefit of the "dictionary" is enjoyable and accessible for all.

Illustrations

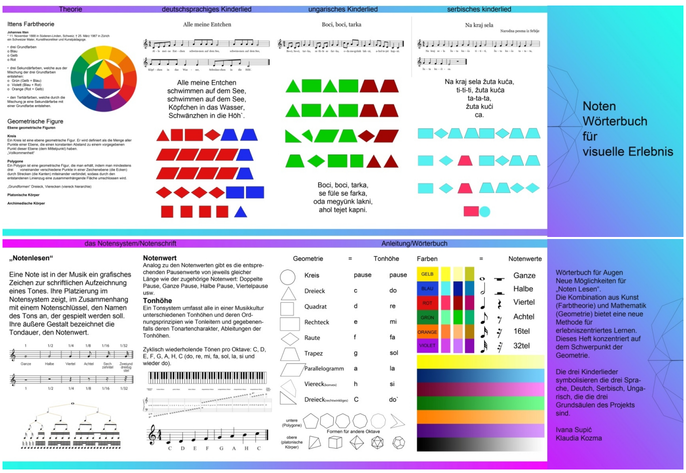

Figure 1: Leporello to the "dictionary" of graphic design Klaudia Kozma
Author: Klaudia Kozma

Figure 2: Color Circle by Johannes Itten 1961
Author: Malte Ahrens

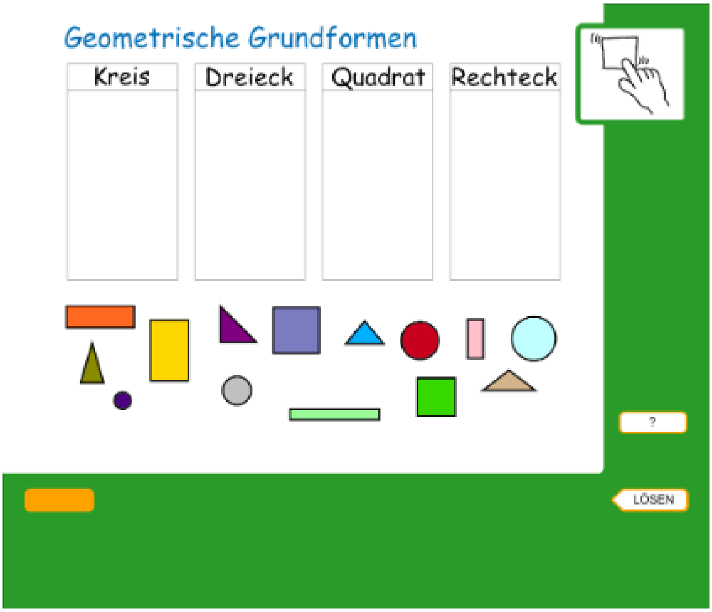

Figure 3: Interactive panel paintings: Sort Basic geometric shapes, SMONS: Geometry
`http://exchange.smarttech.com/details.html?id=98d8f54d-80b9-47f3-8a96-bfc8ddb39e9c`

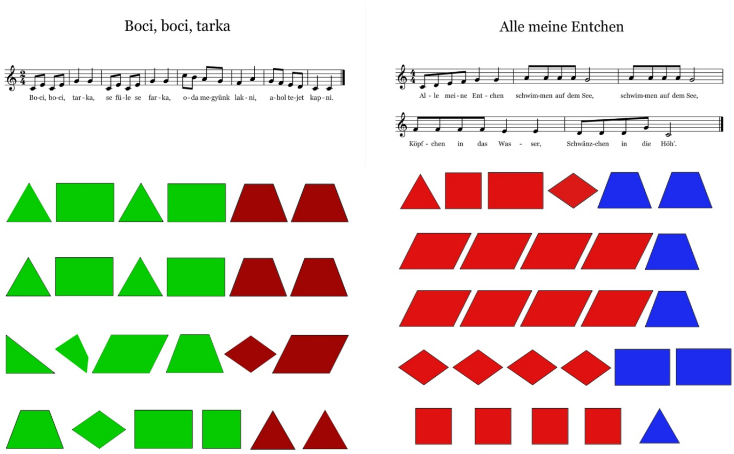

Figure 4: Children's Songs: "Alle meine Entchen", "Boci, boci tarka", Graphic Design by Klaudia Kozma Copyright © Klaudia Kozma, Ivana Supić

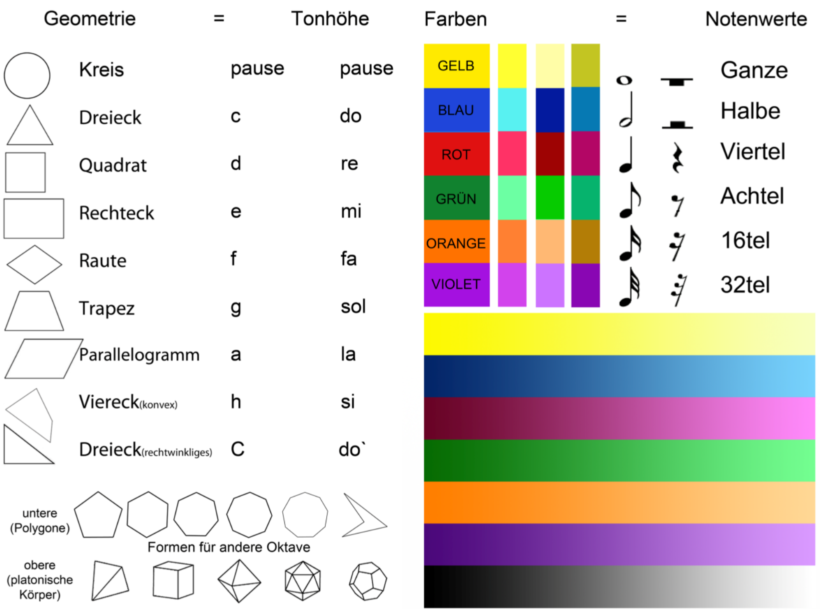

Figure 5: Instructions for "dictionary" Graphic Design by Klaudia Kozma Copyright © Klaudia Kozma

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Decimal numbers: Operating with decimal numbers

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Abstract

The aim of the project was to develop a tool for 10-12 year old children for better understanding of operations with decimal numbers, to be more precise in multiplication and division. The idea was that through interdisciplinary and intercultural collaboration we get to the final product that can be used as a toolkit in math classes for better understanding of decimal numbers. The result of the project is an application that uses the capabilities of augmented reality technology to the children of this age, clarify the process of problem solving multiplication and division of decimal numbers.

Keywords: decimal numbers, augmented reality, math, multiplication of decimal numbers, division of decimal numbers, interdisciplinary, art, solving math problems.

Introduction

During this student exchange program at the University of Applied Arts in Vienna we had the opportunity to learn how primary schools in Austria teach. We collected a lot of useful information about the way in which mathematics is taught in primary and secondary schools. Prior to participating in this project, we were clear from our own experience that teaching "science" subjects is not an easy task. Topics are not easily prepared for presentation and consequently for students it is hard to understand them.

We came to the conclusion that teaching mathematics and related subjects can be improved by using some kind of visuality, for example using new technologies such as computer and mobile phone applications in the classroom.

The link between primary and secondary school is often causing difficulties for 10-12 year old students. The difference in teaching methods is considerable, especially in math education, and the time schedule of primary is open, whereas the secondary school has 50 minutes units for each subject. Also schoolbooks for math differ a lot compared to primary – secondary schools. As a consequence a large number of secondary students have difficulties in math at that age.

The target group focused on these studies are therefore the 10-12 year olds. Furthermore, a maths teacher at a Viennese secondary school investigated his tests of the last several years in order to find out which field caused the major problems for students. It was the chapter of decimal numbers. On the one hand the principle of decimal numbers itself, on the other hand the calculations with decimal numbers.

The main problem for children in this age is understanding multiplication and division of decimal numbers. As a solution to this problem, an application which uses the capabilities of augmented reality technologies was designed to show students step by step solutions to specific mathematical problems with decimal numbers.

Children nowadays are surrounded by technology, use of computers, tablets and smartphones in classrooms is something that looks natural to them and they can very easily be adapted to use these technologies for learning.

Research

Augmented reality is a technology which in real-time combine information from the real world with computer-generated content and such integrated, it displays them on the screen of user mobile phone or computer. This technology is very new and its usage is quite widespread. For now, the most commonly is used for promotional purposes, and for fun. However, Augmented Reality is used in various fields of science such as medicine, archeology, economics, etc.

The basic idea of using augmented reality technologies in education is to give students a possibility to decide themselves what they will learn. Interaction with real environment and virtual object can give students a chance to manipulate virtual objects in real life situations and on that way to acquire new skills and learn some tasks. The good side of all this is that there can't be any "real" mistakes. If we use augmented reality in medical education, for example when surgeon learns how to perform some kind of operation in augmented reality situation and make some mistake during the training there will not be any real consequences. Learning using these technologies give possibilities for more authentic learning. Using new technologies in learning is something that is to be expected nowadays. Augmented reality can enhance textbook materials in a way that earlier has not been possible, and can give students possibilities to acquire some new skills on a new way. There are a lot of ways to incorporate AR in education, for example augmented reality app can provide very unique experience for students during the visit to the museum or some historical landmark. Examples of using AR in education can be found here [1].

The largest number of applications of this type is created for mobile devices with Android and iOS operating system.

There are two primary types of AR implementations: Marker Based and Markerless.

- Implementation which is based on markers, uses some type of images such as a QR codes. When a marker is sensed by a reader, typically a camera on a smart phone, result is shown on the screen of the phone.

- Implementation without markers depends on the characteristics of the device being used such as the GPS location, velocity meter, etc. It may also be referred to as Location-based or Position-based AR.

Both implementations require AR specific software or browsers to function. Marker-based AR is currently the most prevalent and easiest to accomplish. While Markerless AR is emerging, it is currently rather limited due to sensor accuracy (i.e. GPS accuracy anywhere between 10 – 50 meters), service limits (i.e. indoors vs outdoors), bandwidth requirements (4G is not a reality in all places nor can the devices currently in existence actually handle it), and power pulls on the devices.

Material & Methods

As already mentioned the focus group were 10-12 years old children. The problem that had to be solved, was the understanding of operations with decimals, to be more precise in multiplication and division of decimal numbers. The programmed application that results from an interdisciplinary work is an augmented reality application which has the main purpose to help students to solve practical problems in operations with decimals. The main idea was to create an application that will allow students to understand the entire process of solving specific problems.

Application is made for android operating devices (smartphones and tablets) and it is based on augmented reality technology. For augmented reality part of the application Metaio technology was used. Metaio represents SDK (Software Development Kit) that is used to connect computer made data with object in real world and all those data connected are shown on display of android device. This application can also be made for phones that use iOS operating system.

When the application starts, a welcome screen is shown to the user. On that screen users can see three controls. As a cover of the application pictures were used that show the cake principle of decimal numbers, and can be found in everyday life. On Figure 1 the welcome screen of the application is presented.

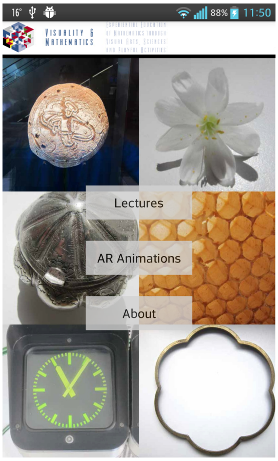

Figure 1: Welcome screen of the application, cover of the app shows cake principle that is used to explain principal of decimal numbers to children
Source for the cover: Stefan Wykydal

The first control starts the part of the application that is called Lectures. In this part of the application, users are able to read lessons related to decimals. On Figure 2 is shown the part of the application with lectures about decimal numbers.

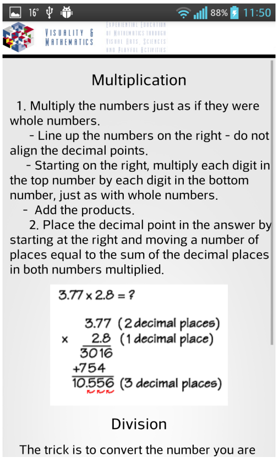

Figure 2: Lecture part of the app, this part shows the lessons about multiplication and division of decimal numbers
Source: Đorđe Manoilov, Dejan Todosijević
Source for lesson:
`http://www.math.com/school/subject1/lessons/S1U1L5GL.html`

The second control launches augmented reality part of the application. By starting this part of the app users get the ability to run a demo animation of solving a particular problem with decimals. Concrete problem with operating with decimals is linked to the specific marker, and after scanning appropriate marker app launches animation (video) with detailed step-by-step solution to the problem. This approach was adapted because we came to the conclusion that children of that age just need a little more visual representation of problem solving processes. Augmented reality is very interesting to children and because of that we decided to use this technology as a tool for making fun but also educative application. On Figure 3 is shown the augmented reality part of the application.

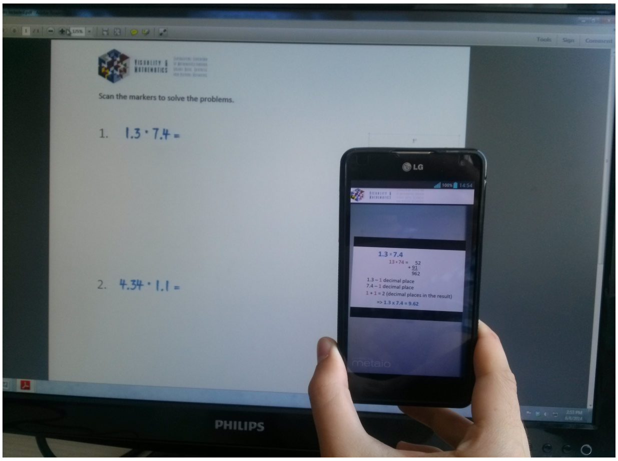

Figure 3: AR Animations part of the app, this part shows a demo animation of solving a particular problem with decimals
Source: Đorđe Manoilov, Dejan Todosijević

The third control launches part of the application in which users can be informed about how the cover of the app was made and what it means.

Results

What would it look like using this application in math classes? As one possible example of using this application, we have prepared the tests on which there are given tasks (assignments) with decimal numbers. After each assignments, there is blank space for children to solve the assignments by themselves using pencil, on the right side is presented augmented reality marker related to the problem. The idea is that the child scans markers and watches the video that shows the solution to the problem, learn how problem can be solved step by step, and then do the assignment independently on paper.

In this test there are several assignments so that children have enough examples to practice a particular procedure of solving the problem. On Figure 4 are shown prepared tests with some decimals problems (multiplication and division). Within each test there are 3 assignments.

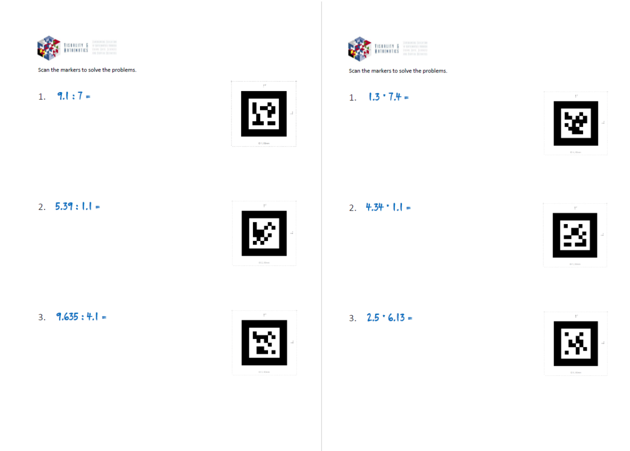

Figure 4: Prepared test sheets as an example of usage in math classes
Source: Đorđe Manoilov, Dejan Todosijević

Conclusion

As result of interdisciplinary work an augmented reality application was designed that can easily be included in regular math classes. It represents a demo project that is still in development. This application helps children to understand operations with decimal numbers in a fun way, to be more concrete to understand entire process of multiplication and division with decimal numbers. For some future version of this application it should be tested with kids to observe how they interact with it, and what can be improved.

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The principle of decimal numbers. How can 10-12 year-olds imagine a newly learned numbering system?

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Abstract

The objective of the project was to find ways that help in imagining the principle of decimal numbers for the target group of 10-12 year olds. In order to improve the understanding of decimals the comparison with analogue three-dimensional principles was used s a method. Divided circles and spheres are forms that build a bridge to the world of numbers. As a result the numbers are linked with sensual qualities that help to understand on a more profound level what decimals mean. The interdisciplinary approach was to link the world of mathematics with the world of arts and design. Both go together successfully when it comes to help to find a richer and deeper foundation for students and teachers in the field of mathematical instructions and art/design education.

Keywords: decimal numbers, divided circles and spheres, 10-12 year olds, interdisciplinary work, mathematical instruction, design education

Introduction

The link between primary school and AHS is often causing difficulties for students. Especially in mathematical instructions the difference in teaching methods is considerable, as the time schedule of primary school is more open, whereas the AHS has 50 minutes units for each subject. Also schoolbooks for maths differ a lot compared to primary school. As a consequence a large number of first grade AHS students has difficulties in maths at that age (Hanisch, Katschnig 2006).

The target group focused on is therefore the 10-12 year-olds. A math teacher working in a Viennese AHS investigated his tests of the last several years in order to find out which field caused the major problems for students. It was the chapter of decimal numbers: on the one hand the principle of decimal numbers itself, on the other hand the calculations done with decimal numbers.

The goal was to enrich the field of maths education with sensual qualities of design and the arts in order to address a diverse range of learning capacities like seeing, feeling, forming and composing. Decimal numbers are difficult to imagine precisely as they are proportions related to the number one. So the introduced cake-principle (divided cylinder or sphere) offers a help in imagining the proportions of one.

Addressing more than one human sense is a long known improvement when it comes to sustainable learning (Steininger 2005).

Material and Methods

The principle of decimals is difficult for students to imagine at the beginning as it adds proportions after the whole number that are not easily understood in terms of size. The conventional way to imagine proportions smaller than the number one in school is often the divided cake. So half a cake means 0,5 quarter a cake 0,25 and so forth.

This here called cake-principle is conventional but useful for the used method. It is partly already introduced in primary school and therefore approachable for the target group. However the AHS first graders are challenged by the fact of partly leaving the real world connection (cakes, pizzas, bread, petals, wheels. . .) and going further to the aesthetics of pure geometry, which is typical for mathematical ways of thinking. So on the one hand connections to real world decimal proportions are established, as will be seen in the photographical part of the used method, but on the other hand those connections are left behind in the clay projects. It is most essential that students make the design projects themselves, if possible. Only using ready design tools as teaching aids would be a problematic limitation. The involvement of arts and design education would make a lot of sense.

As students all have the possibilities to make photos very easily it is useful to open their minds for the variety of cake principles in daily surroundings through the means of making pictures of them. Such picture collections have the quality of the aesthetics of serials and make visible the circle proportions very lively. Arts education or new media would be related school subjects.

The second material used is clay, as it is often used in the area design education of our target group. Forming clay gives the feeling of creating something on your own very much, the outcomes are regarded as the direct product of one's imagination. Josef Frank, the great Swedish architect with Austrian roots, saw clay as the highest level of human creativity, as no other medium is formed so directly by hands without any other instruments (Frank 1931). It is also very suitable to create geometrical forms very quickly. The easiest clay transformation of decimals is the divided cylinder (circle would be imprecise, as clay always has a volume)

The more sophisticated clay project would be the divided sphere as it is found in waterballs, domes or melons (Denny 2010). It can be done by the students themselves, which of course improves the learning effect, or just be used as a ready-made teaching aide.

Results

The result of the students design project linked with the decimal system, is that in producing such design objects they have a very strong personal relation to the newly introduced decimals. Proportions that are felt with all senses come more easily into one's mind, than just numbers. In order to build the important link to the world of decimals it is necessary to add the proper decimal number repeatedly on the design objects, otherwise the bridge is not given and decimals stay apart from the circles and spheres.

Discussion

Mostly the problem of improving the quality of maths education is seen as one that is linked with time problems within the daily routine of the school subject. A maths teacher is not free in choosing his topics but is dependent on the curriculum. Therefore it is important that design tools used for mathematic instructions are most effective and easy to introduce. The cake-principle is such a classic, conventional but useful tool.

The connection to the school subjects of art and design education is obvious. It would be a profit, also for these subjects, to concentrate on the creation of such user oriented design tools. Art and design is not primarily something for the sake of itself, so such projects would improve students awareness of user orientated design, especially as in this case they become the users themselves.

Figure 1: Photo examples of divided circles and spheres
Source: Stefan Wykydal

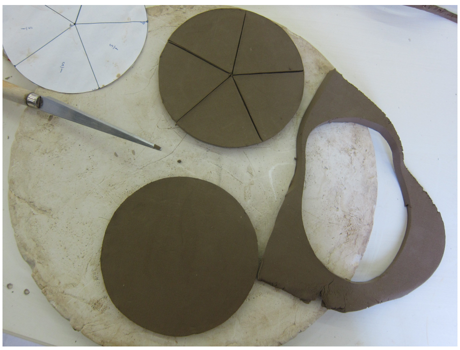

Figure 2: Divided circles out of clay
Source: Stefan Wykydal

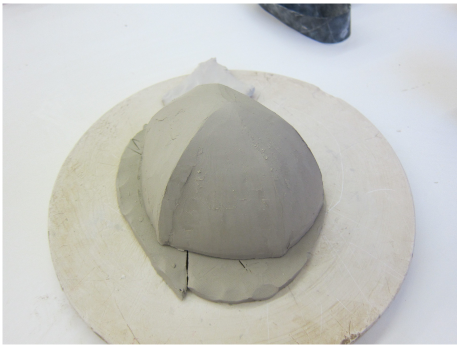

Figure 3: A Dome out of half clay petals
Source: Stefan Wykydal

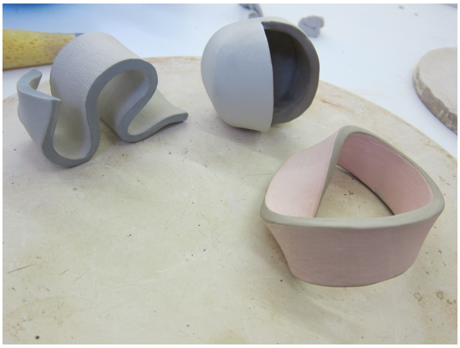

Figure 4: Geometrical objects out of clay, among them a ball out of petals
Source: Stefan Wykydal

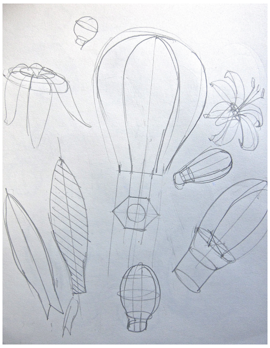

Figure 5: How to make a balloon- or ball like form out of several petals
Source: Stefan Wykydal

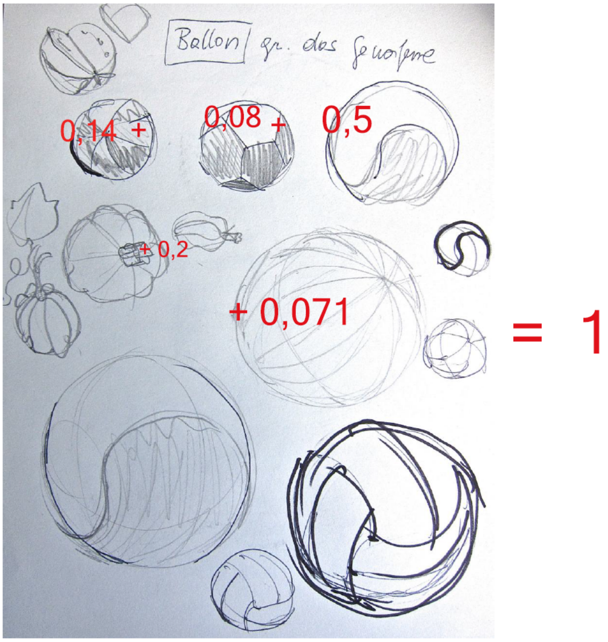

Figure 6: Counting with decimal balls:
the equal surface parts of a water ball, a dodecahedron, a tennis ball,
a pumpkin and a melon go together as one
Source: Stefan Wykydal

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INFINITOP: A card game about approaching Infinity

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Abstract

Some things in math are harder to understand because they are not real and one of them is infinity. There are ways to explain the idea of infinity so that pupils can understand it better. The infinity is not something that can be proven in practical examples but rather to give an idea that we have to imagine to understand. By imagination, infinity couldn't be explained mathematically so easily. Studies have shown that the better way to explain the concept of infinity was made by storytelling, games, and raw imagination of the pupils.

Keywords: Infinity, mathematics, design, card game, interdisciplinary work

Introduction

Pupils in schools today are being told that infinity is an enormous number, a really big value. And the talk about infinity would very often stop there. If the idea of infinity is briefly explained to pupils and then asked to draw something that would represent infinity they would often just write 1000000. So as we can see the understanding of infinity is a problem. It is crucial in mathematics to understand things, how they work, what are they used for, so we can memorize them better and use them better. In math infinity is used for limits, basically, but it is used for other stuff also. Infinity can be explained by other nonmathematical means.

Although it is not proved that Einstein really said it, he is frequently quoted: "Two things are infinite: the universe and human stupidity; and I'm not sure about the universe".

Research

Infinity is all around us

INFINITY IN MATH

In mathematics, "infinity" is often treated as if it was a real number, but it is not the same as a real number. Infinity in math is a "number" bigger than any real number. In calculus, the symbol ∞ called "infinity", is used to denote an unbounded limit. $x \to \infty$ means that x grows without bound and $x \to -\infty$ means the value of x is decreasing without bound. Infinity is used in limits when working with functions. It is also used for integral boundaries, and sequences.

The term "infinity" found its way into everyday life. Although children use the term, it is often really hard for them – and for adults as well – to understand the concept of something that is not only incredibly large, long or wide but in fact infinite.

(Not) Reaching Infinity by Approximation

INFINITY IN POPCULTURE

Infinity is often referenced in arts and pop culture. An almost infinite number of songs deal with infinity in love. A popular example for a kid's movie is Pixar's Toy Story (1995) in which a character named Buzz Lightyear says "To infinity and beyond." Also video games use the concept: for example in the fun racer Mario Kart 8 (2014) for Wii U by Nintendo there is a track that has the physical appearance of a Moebius Strip (Hideki Konno, Kosuke Yabuki)

Widely known are M. C. Escher's Penrose stairs or the Penrose triangle, impossible objects that can be seen as examples for the concept of infinity. If someone would walk upstairs the Penrose stairs – like protagonists of the movie "Inception" (2010) by director Christopher Nolan (Warner Bros., Legendary Pictures) – it would take forever for that person to reach the top, because in fact there is no top that can be reached. This is a good example for the fact that by taking a number of a certain value and making it bigger and bigger will get us close to the idea of what infinity could be, but we actually won't get there.

PICTURES OF INFINITY

Having school kids as target group, it was interesting to find out what they have in mind when thinking about infinity. In the process of the research, an experiment was done with a school class of 25 students at the age of 13–14 years. In art education they were asked to do four drawings in 20 minutes (about five minutes per drawing).The first drawing should show something they have in mind that has to do with the mathematical value of "1". The second drawing should deal with the mathematical value of "500". The third drawing should deal with the mathematical value of "1.000.000.000" and the fourth and final drawing should visualise infinity.

The first drawings (Value: 1) have nothing significant in common – for the most part they show one object. The second and third drawing (Value: 500 and 1.000.000.000) show basically money or stars – things that seem uncountable, but in fact are not endless. The last drawing (Value: Infinite) showed three things: love, outer space or jewellery, using the sign for infinity. There were no significant differences between boys and girls.

ENDLESS LOVE

While the infinity of outer space is still in question, abstract terms like love can easily be called infinite because you can't quantify it as love has no certain mathematical value and therefore has no limitation. While math tends to operate with limits, infinity has no limit.

The childrens' story telling book „Guess how much I love you" by Sam McBratney and Anita Jeram, published in 1994 in the UK (Walker Books Ltd., London), tells the story of two hares. At the beginning of the book the little hare asks the big hare the titular question, "Guess how much I love you?" As the book continues, the little and the big hare use larger and larger measures to quantify how much they love each other. At some point the little hare is getting tired and falls asleep, while saying "I love you to the moon" to the big hare. "To the moon and back again is how much we love each other" responds the big hare while the little hare is already fast asleep and that's where the story ends. If the story was preceded the two hares may have agreed that their love for each other was actually infinite. The book was named in the U.S. named the book one of its "Teachers' Top 100 Books for Children by the National Education Association in the U.S. in 2007.

Approaching Infinity by Playing Infinitop

METHOD

Studies of research have made it clear:

First of all it is hard (if not impossible) to show infinity, but maybe it can be imagined.

Secondly infinity can't be reached by approximation but it helps imagining it.

Telling a story about approaching Infinity seems like a good way to solve the problem in the first place and could be considered as a tool. While it would inspire the imagination of the students, at the same time it would suffer the hands-on experience, which would enable discussions and interactivity.

James Paul Gee derives a set of learning principles from his study and suggests that adherence to these principles could transform learning in schools, both for teachers and for students. One of these principles, the "Psychosocial Moratorium", is that learners can take risks in a space where real-world consequences are lowered (Gee 2013, p. 10). This can be done by a game.

Thus we created the game INFINITOP. Mathematics itself can be seen as a type of game that is played with symbolic shapes and uses certain rules (David

Landy, Erin Ottmar 2013, p. 522). With the game INFINITOP we tried to transform these symbolic shapes into ideas. INFINITOP is a card game that will help students/pupils to better understand the idea of infinity by playing and having fun. It can be played by 3 or more players. Additional cards can be added in the deck by choice depending how many players are playing.

RULES OF INFINITOP

As this game is a prototype, so it's not a final version. Rules may be altered.

Before the beginning of the game each player gets equal amount of cards. Each card has a numerical value. The player who got the $-\infty$ card starts the game by putting that card in the middle of the desk, and creates a middle deck.

The game is played in turns, to the right. Every next player must put a card on top of the middle deck.

If a player puts a card in the middle of the deck he earns 2 points.

If a player has a card that has a smaller value of the card that previous player put on the top of the deck than he can put his card under previous player's card and claim it as his own card. Player who claimed a card can use it latter on for gaining more points. Up to 3 cards can be claimed by a single player in one turn from the deck.

For putting card you will get 2 + 1*(amount of cards you claimed) points. Player/s that got their cards claimed will get -1 point.

Some cards are doubled or tripled. If a player puts a card with a certain value, the other player who has a card with the same value can jump in and put his card on top, even if it is not his turn to play. By doing so the player who jumped in gets 5 points.

If a single player has two cards of the same value he can put both of them in a single turn and earn 5 points.

There are power cards, which values can be chosen by a player or it would depend on some other factors (power cards are different). If a power card with value defined by a player gets claimed, the player who claimed that power card can put another value on the card. Points for power cards are the same as the points for regular cards.

As players run out of cards they are excluded from game. If there is only one player left with cards in his hands he will get 2*(Number of cards he has) points.

Value of the card on top will rise. The game ends when the last card is put on the top. Usually it is the ∞ card.

Values on the cards are not written on the, the values must be calculated or figured out. That way, pupils will be calculating to figure out what is the value of their cards or to find out an interesting numerical fact, thus making the whole experience more interesting. The values of each card will be present alongside the rules, just in case.

When the game is finished the points are added up.

Design While all cards can easily be made by hand, we developed a possible graphic design for the game. The design of the cards is the basis for the success of the game. It is important to find a balance between the challenge to win the game and the skills of the players. If the challenge is too big students may feel overcharged. If the challenge isn't big enough they may become bored (Chen 2007, p 31) For the graphic design of the game INFINITOP it seemed important to create a strong and simple visual identity in order to make it easier for students to feel attached to the game.

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Figure 1: Packaging Design
Source: Clemens-G. Göller (2014)

Figure 2: Examples of regular cards
Source: Clemens-G. Göller (2014)

Figure 3: Examples of special cards
Source: Clemens-G. Göller (2014)

Conclusion

Infinity is sometimes difficult to understand, simply because it is not something that is “real”. But through playing and interacting, everything is easier to learn and understand.

There are infinite numbers of things that someone can learn, and one of them is infinity. The card game should be achieved as open source download and be a game for learning. Further it explains the concept of math, imagination.

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Art in mathematics: the sinilluminate project

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Abstract

The aim of this paper is to present a visual approach to the theory of functions, within the theoretical content, detail graphic examples created in GeoGebra package software and meaningful use of new media tools. This mathematical field of differential calculus is designed as interactive content for e-education and blended learning, since majority of students become faced with a problem of misunderstanding several crucial function's concepts when it comes to exploring and investigating it's properties. They more than often experience difficulties to draw a function graph even they have already have become familiar with its concept at preschooler's level (correspondence between the sets with same number of members, etc). In order to foster students motivation and attention, new technologies and media are involved in way of creating mathematically significant images, whether dynamic or not, as important tool for learning by visualizing. In order to correlate art and mathematics education and making it more attractive for high scholar's, new instructional approach involves art history, techniques, visualizing and learning math content by light painting. Cultural aspects of young people's thinking are considered very thoroughly, therefore light painting as a way of popular and modern youth expressing plays an important role in this instruction.

Keywords: mathematics, visualization, light painting

Introduction

Functional thinking gives a special dimension to mathematics education; it is used to explain the phenomena of nature and science. At the same time, it prepares students for advanced learning and understanding more abstract mathematical knowledge at higher education institutions, using particular images or diagrams in the service of mathematical generalization and interrelationships between modes of thinking and its application in real life. At the same time, it is generally reported that students of mathematics rarely use advantages of visual approaches to support meaningful learning. This issue is especially connected to symbolic representation in mathematical topics, where students become reluctant to engage with visual modes of learning according to (Presmeg & Bergsten 1995, page 58-65; Healy & Hoyles 1996, page 67-74).

It is clearly that visualization can't be used as 'panacea' for each math classroom situation since an intuition about a generalization involves more than observed evidence. For example, although one calculus images supported student's level of mathematical functioning, occasionally these images did more to obscure then to explain (Aspinwall, Shaw, and Presmeg, 1996, page 301-317). This type of imagery can be a major issue in constructing meaning for mathematical concepts. Some of the reasons are: the belief that visual proof is not 'real' mathematical proof, the algebraic mode instead of the graphic or visual is more commonly used on calculus tests and the belief held by both teachers and students that doing calculus is skillfully manipulating numbers and symbols according (Aspinwall, Shaw and Presmeg 1996, page 301-317). On the other hand, a role of visual/nonvisual teacher's model is one of the crucial causes of this problem. The visual teachers constantly made connections between the subject matter and other subjects, sciences and the real world despite nonvisual teachers. As presented, the basic reluctance of students to use visualization in mathematics is the result of the low status accorded to visual aspects of mathematics in the classroom environment.

Aims of the project

As a way of overcoming the anxiety of reading, learning and understanding mathematical literature that includes theorems and proofs, the e-material offers numerous animation of these and similar problem-solving concepts as motivational both educational tool. The aim of created e-material is to provide material which will encourage learners to use visualization, help them to overcome the difficulties and at the same time to be adequate for incorporating in classroom activities. The basic functions e-course is publicly available and created within GeoGebra software. Though it is written in Serbian language, there are no language barriers for the countries in the region. E-course according to (Jezdimirovic, 2013; Jezdimirovic, 2014) contains interactive mathematical simulations based on changing graph functions and it's parameters as in the Figure 1 below.

Since the project combines learning functions (sinusoidal for example) using Light Painting technique it is intuitively called "sinilluminate". Created educational materials are publicly available at `http://alas.matf.bg.ac.rs/~ml08060/`, also in the form of QR codes and text step-by-step instructions. Using mobile or tablet device, this "sinilluminate" project is easily reachable for teacher in order to create and design their own workshop, seminar or any other kind of mathematical educational activity on this topic.

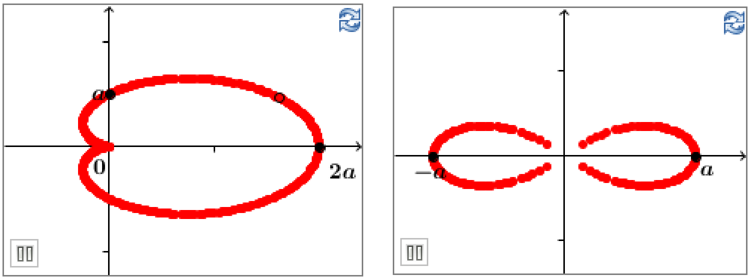

Figure 1: Function examples
Source: Milan Licina, Jovana Jezdimirovic

Visualization in mathematics and GeoGebra

Following Presmeg, visualization includes processes of constructing and transforming visual mental imagery and spatial information that may be implicated in doing mathematics (Presmeg, 1997b) where a visual image is taken to be a mental construct depicting visual or spatial information and a visualizer a person who prefers to use visual methods when there is a choice. There are variety of imagery types, in this approach is combined static and dynamic images created by software package GeoGebra with dynamic images (light painting videos) as an effective educational tool for interactive and self-regulated learning. There are many authors as Dreyfus (Dreyfus, 1991) which argues that the status of visualization in mathematics education should and can be upgraded from a helpful learning approach to a fully recognized tool for learning. There are many effective examples which demonstrate the power of visualization in mathematical reasoning that have since become a regular feature in publications of the Mathematical Association of America.

Computers impact to visual reasoning in mathematics education is undeniable and learner's use of visualization through dynamic geometry software which facilitates visualization processes have been discussed over the years in many papers and created e-course highly supports the idea. In Picture 2 are presented interactive GeoGebra applets which are designed for student's exploring function graphs and investigating function graph changing with moving sliders or buttons which represent certain parameters. In order to foster students' motivation both accomplishment created instruction can be implemented in digital classrooms, for playful activities, classroom's interdisciplinary approach as well as the creative interactive visual workshops, etc. GeoGebra is used for interactive approach in mathematics education through modeling, picking the right function, clicking checkboxes, moving sliders, etc...

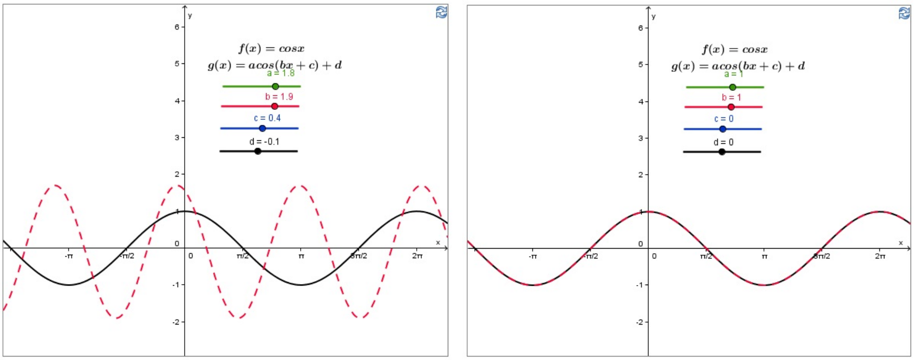

Figure 2: Sinusoidal functions
Source: Milan Licina, Jovana Jezdimirovic

Widening effects of computer technology in mathematical visualization prove that this methodology have affective attributes in mathematical problem solving processes. This approach represents powerful engine not only in mathematical topics such as geometry and trigonometry but also in algebra, analyses, etc. There have also been reported many special advantages of computer software usage that encourages dynamic visualization according to (Hohenwarter and Fuchs 2004).

Techniques of Light Painting and its usage in the mathematical instruction

Light painting as educational tool is chosen because of the todays' people's necessity in new technologies, and numerous advantages as instructional method as learning by visualizing, it's convenience for workshop activities. This video recording offers possibilities for math learning without any language barriers. At the same time, this approach highlights student's interest in new ways to express their creativity, so at the same time fosters students motivation and achievements. Based on the goals in certain mathematical instruction, different light paintings techniques can be used. There are three basic light painting techniques. The first is light painting, the second known as light drawing, and the third known as kinetic light painting or camera painting. Light Painting is using handheld lights to selectively illuminate and/or color parts of the subject or scene and requires a slow shutter speed, usually a second or more. On the other hand, Light Drawing is shining a light source to "draw" a picture by shining it directly into the camera lens and also requires a slow shutter speed, usually a second or more. Light painting can be done interactively using a webcam. The painted image can already be seen while drawing by using a monitor or projector, but you should record your video in order to exhibit created work. It can also be seen as kinetic light painting.

Also, the fourth technique emerges itself, yet without a name, but for the purpose of this project it shall be called Interactive light painting. Many artists recognized new technologies as a possible "canvas" so now it is possible to light paint with combination of hardware and computer patches. Some of the hardware resolutions might be use of Microsoft Kinect scanner, Nintendo Wii controller or Leap Motion device in order to identify and track motion. Otherwise DSLR can be used, as already mentioned, webcam to track points on a video which you want to leave light trails. Both of the principles work well for the users, because they can instantly see light trail that they add only with their bodies. Some of the common computer tools to achieve this kind of effect are Quartz Composer, VVVV open-source toolkit and Processing, visual programming language for visual artists, much of the foregoing information are based on 5 and 6. For the purpose of creating educational workshops or innovative math lessons it is very convenient that a QC can be found at the Apple store under the X-code tools, VVVV is downloadable for free from vvvv.org; and can be used for non-commercial purposes; processing can be found at processing.org, so it is easily reachable for most educators (Ličina 2014).

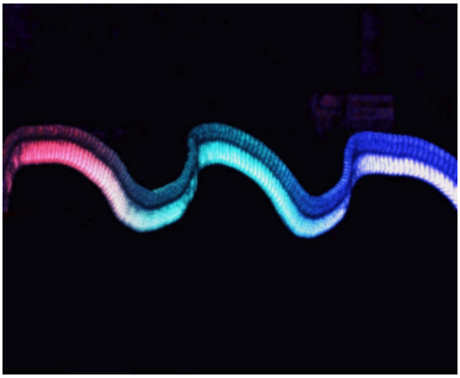

Figure 3: Light painting function example
Source: Milan Licina, Jovana Jezdimirovic

Processes behind prototype for the project "sinilluminate"

Overall artistic expression and technique could be described as a light painting. Because our goal was to illustrate mathematical functions that can also define light moving or any moving at all, this was our obvious choice. Being result of some tests with pocket camera it does not represents highest aesthetical domain of the author, yet illustrates idea that can be upgraded or even transferred to other tools and techniques. We assembled our own light source made out of a flashlight, plastic packaging and some color pencils in order to make it work as a demonstrating tool as in Picture 4. Vienna's Votivkirche proved to be a perfect location for our testing, because it is placed on a good spot for this kind of art-creating technique. Using only videos that we recorded in a guerilla-way, the time has come for us to make it became adequate visual representation of our concept- light illustrations of mathematical functions. Only tool used to create this illustration content was Adobe "After Effects". Process of getting the finished piece depends only of the creators themselves. There is not a defined process of how to get light painting video, but for us, following steps have proven themselves to be the fastest way of getting notable results with hardware that is not made for professional use. Following tutorial is indented for use of "After Effects", but same principle can be used with similar motion graphic softwares.

Figure 4: Light painting tools
Source: Milan Licina, Jovana Jezdimirovic

STEPS:
Getting the JPEG sequence:

- Open AE AFX
- Import desired video
- Make new composition out of imported video
- Select start and ending of the video to define length
- Right klick above the timeline to trim video to desired lenght
- Go to Composition > Add to render queue
- Switch to Render Queue tab
- Select output format to jpeg sequence, name it properly (do not erase #### signs), and select output path

- Click Render

You should now have each frame of your video as an independent JPEG file.

Getting the light painting effects:

- Open new composition (File > New Composition)
- Import the exported frames (make sure that “JPEG sequence” box is unchecked)
- Make new composition from Composition > New composition
- Drag all the frames into newly created composition
- Now, in order from first to the last, move all the frames on the timeline for one frame forward (example, frame 2 one frame after frame 1, I know it is tricky and time consuming, but there is no other way of creating this effect unless you buy commercial scripts or some professional plugins)
- Pre-compose all the frames into single compositions by selecting al the frames and pressing Ctrl+Shift+C
- Name your new comp
- Apply some color correction tools for better visual results (you can access them from the Effects>Color correction tab; Curves, Levels and Hue / Saturation are the most common ones, Photoshop users will find it very easy to use)
- Apply described steps to export your video by choosing between several file types (.avi, .mov, .h264 to name the few)
- Enjoy created content

Historical background of Light Painting

Light Painting, more specifically light drawing, dates back to 1889 when Étienne-Jules Marey and Georges Demeny create the first know light painting “Pathological Walk From in Front”.

In 1949 Pablo Picassso was visited by Gjon Mili, a photographer and lighting innovator, who introduced Picasso to his photographs of ice skaters with lights attached to their skates. Immediately Picasso started making images in the air with a small flashlight in a dark room. This series of photos became known as Picasso’s "light drawings". Of these photos, the most celebrated and famous is known as "Picasso draws a Centaur" During the 1970s and 80’s Eric Staller used this technology for numerous photo projects that were called "Light Drawings". Light paintings up to 1976 are classified as light drawings. In 1977 Dean Chamberlain gave birth to light painting (using handheld lights to selectively illuminate and/or color parts of the subject or scene) with his image "Polyethylene Bags On Chaise Longue" at The Rochester Institute of Technology. Dean Chamberlain was the first artist to dedicate his entire body of work to the light painting art form. The artist photographer Jacques Pugin made several series of images with the light drawing technique in 1979. Picasso and Mili's images should be regarded as some of the first light drawings. Now, with modern light painting, one uses more frequently choreography and performance to photograph and organize. This art form is currently enjoying a surge in popularity, partly due to the increasing availability of dSLR cameras, advances in portable light sources such as LEDs, and also in part due to the advent of media sharing websites by which practitioners can exchange images and ideas.

Much of the foregoing information are in accordance to "Painting with light", LPWA publications.

Conclusion

Aims of the project are fostering students' motivation and achievements in learning functions and mathematics in general, creating publicly available and interactive educational toolkit, innovations in mathematics education, learning by visualizing. Also, one of the goals is to design suitable educational toolkits for workshops, seminars and materials for innovative math lessons in order to make it easy for teachers to incorporate these techniques in their instruction.

In order to involve young people in self regulated and life long learning, educational meterials will be available in a form of QR codes. QR code (abbreviated from Quick Response Code) is the trademark for a type of matrix barcode (or two-dimensionalbarcode). A barcode is a machine-readable optical label that contains information about the item to which it is attached. A QR code uses four standardized encoding modes (numeric, alphanumeric, byte / binary, and kanji) to efficiently store data according to (Denso-Wave, 2012). In this way, interactive mathematics will be available and outside the classrooms using smart devices or tablet devices.

Impact of created educational material to students' motivation and achievements is left to be revealed in the future study. All is needed is to be use in the mathematical instruction!

Acknowledgements. Thanks to International Journal of Technology and Inclusive Education.

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The use of playful tools in math: What are circles made of?

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Abstract

Following one month of interdisciplinary work, this paper presents the process of discovering, designing and conceptualizing a tool that will embody the principles of playful learning and personal growth. We set out to design a tool that will enable the pupils to learn mathematics while developing a sense for visual expression and understanding. Our chosen math problem is the geometry of a circle. The circle itself is a shape which is widely used in hands on techniques as it allows a certain flow. In our approach we will use this shape as a main guideline to create our tool, not only content wise, but also as a base for our tool itself. The process explores math as a visual language, and learning as an interdisciplinary process that begins at an early age. Through explaining the creation process, the hands-on method and the elements that are employed in the tool, this paper will give a clear and coherent overview of the process and expected results.

Keywords: Circle, hands-on, crafts, play, math

Introduction

"I hear and I forget. I see and I remember. I do and I understand." Confucius[1]

„The principle goal of education in the schools should be creating men and women who are capable of doing new things, not simply repeating what other generations have done." Jean Piaget[2]

These quotes are basic premises of our work. When we set out to create this tool, we did extensive research in education theory, and one of the things that struck us most is Jean Piaget's genetic epistemology work, where he speaks about tracing back the knowledge and the knowledge processes to their roots in the attempt to understand and deconstruct different learning processes. Having discovered this, we decided to try to create a tool, which would serve as an aid for children to learn the chosen math content in a visual way, while creating an experience where they can also explore visualization and art, and create their own trajectory of learning.

Our chosen math content was the geometry of a circle. Drawing back on the experience of the members of our greater team and ourselves, we were introduced to a problem that children have while learning geometry. This problem was the lack of physical or empirical understanding of the shapes, their parts, their construction and their mathematical expressions. The inspiration for our tool comes from learning languages, and different tools that we have used and explored in this area. We believe that mathematics can be taught using a combination of both the visual and the mathematical language, which will result in a teaching and learning process with a tangible outcome and newly discovered ways of understanding.

This approach to a great extent relies on the on the principle of less is more, and the way pupils can learn math using very simple techniques, tools and materials. In addition to this, the process completely liberates from the limits of precision and allows the development of visual expression.

What follows, is a description of our tool, as well as visual representation of the process.

The research

The hands-on learning, or experiential learning as J.W. Gentry called it has been a topic researched ever since education theory has taken on the role of providing educational tools comprehensible for as wider audience as possible.

One of the main ideas behind hands on learning is enabling an environment which is non-threatening and can provide the safety required to foster creativity and learning in primary school children. Maria Montessori, one of the pioneers of the hands on approach, created the Montessori Method which set out to achieve

[1] A famous quote attributed to Confucius.

[2] `http://psychology.about.com/od/profilesofmajorthinkers/p/piaget.htm` Accessed on May 22nd, 2014.

exactly this: encourage pupils to search and bring out the best in them while learning and growing in an intellectual and personal way.[3]

Hmelo-Silver, who has a positive opinion about developing critical thinking skills through hands-on teaching, says that "by investigating the subject matter through hands-on activities, students learn both content and thinking strategies" (Hmelo-Silver 2004, p. 235). Hands-on activities use real objects to support multiple modes of communication, linking visual learning to what is being said and discussed (Lee, Penfield and Maerten-Rivera 2009, p. 826). Hands-on activities enable students to discuss and debate, verbalize and explain processes and concepts while working together. An observation of hands-on learning noted that the students demonstrated strong communication tied to working in teams (Bass et al 2011).

Business leaders regularly complain that our education system fails to teach students the 21st-century skills they need for the work world, such as problem-solving, communication, and the ability to work well in teams. In the course of doing a hands-on project, students learn to work well with other team members who may have different socioeconomic backgrounds, different learning styles, and different cultures.

The benefits for educators are also numerous. For example, professional development workshops that stress hands-on learning are significantly more successful in improving teacher confidence in math and science instruction (Basista and Mathews 2002, p. 359). Hands-on activities help teachers cut the time needed for remediation, improve classroom management by unifying students around a common organized activity, and foster a greater interpersonal and supportive emotional connection with students through sharing the process of learning with them (rather than one-way lecturing).

The Method

When we set out to create this toolkit, we decided that we wanted to create a tool that will not only be useful in learning mathematics in a visual way, but that will also create a character building experience for the pupils.

Learning in a playful way is a concept that allows a wide variety of processes to be covered. During the process of making this tool the pupils will go through a process where by employing some basic elements, they will get in contact with building a visual identity, creating their own individual visual preference, different materials, different techniques and hopefully, what they will be left in the end is learnt content and a developed visual understanding of it.

The elements are very basic:

- Crafts, or the process of creating something using your hands;

  Our experiences and our research led us to the notion that crafting is not only a process of creation, but also a process of empowerment. When pupils

[3] `http://www.montessori.edu/method.html` Accessed on May 15th, 2014.

craft their own tools, they are a part of a process of creating a tool that will later help them learn. Being in this process they gain confidence about their learning process and which is fundament personality trait that should be developed during early school years.

- Movement, the element that gives a certain dynamic to the process;
- Playfulness can be experienced or designed in many different ways however movement is almost always involved in the process. Movement allows changes, experimenting, discovering. In the tool that we designed the pupils get to move the different layers, match the shapes with the names and create a playful way to memorize the content.
- Haptics, the element that enhances the sensual experience;

Last but not least, haptics is a very important element when it comes to playful learning. In creating this tool, the pupils have the liberty to use any materials or colors that they desire. In this process, the element of haptics plays the role of facilitation; it transfers the play or the feeling of playing to the senses and aids in the process of memorizing.

The Design

The toolkit is designed such that there are different layers, which represent different parts of the circles. The layers that are following are represented in the way that they would be drawn.

- Background layer – the lowest layer which contains the names of each of the elements, is color-coded to correspond each of them as well and is the largest layer.
- This part should also contain a graphic representation of the tangent.

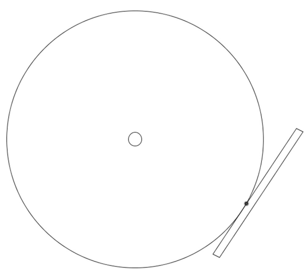

Figure 1: Tangent layer
Source: Klelija Zhivkovikj, Jovana Radenovic

- The circle – The layer, which represents the inner part of a circle.

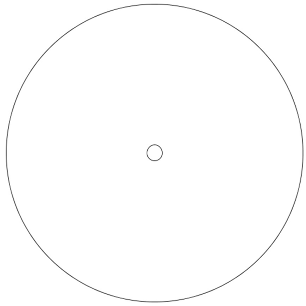

Figure 2: Circle layer
Source: Klelija Zhivkovikj, Jovana Radenovic

- The Arch – the element of the circle, which represents just a part of the outer circle.

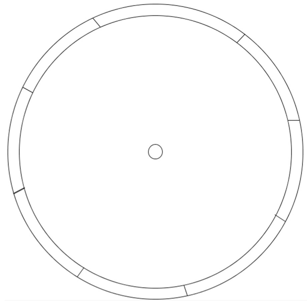

Figure 3: Arch layer
Source: Klelija Zhivkovikj, Jovana Radenovic

- The Circular – or the outer part of the circle.

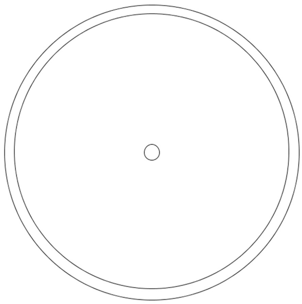

Figure 4: Circular layer
Source: Klelija Zhivkovikj, Jovana Radenovic

- The central angle – the angle that starts at the center of the circle.

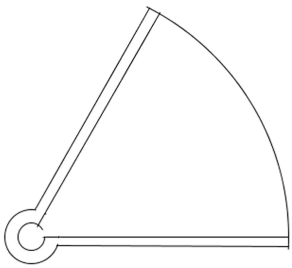

Figure 5: The central angle
Source: Klelija Zhivkovikj, Jovana Radenovic

- The peripheral angle – The angle that spreads from one side of the outer circle to the opposite side.

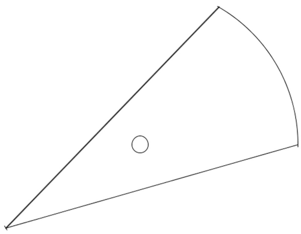

Figure 6: The peripheral angle
Source: Klelija Zhivkovikj, Jovana Radenovic

In addition to understanding the position of the angles, they show a proportion in size: the peripheral angle is always half from the central angle.

- The tendon – A line that connects any two dots on the opposite sides of the circular (outer circle).

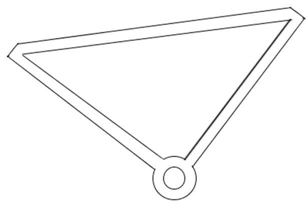

Figure 7: The Tendon layer
Source: Klelija Zhivkovikj, Jovana Radenovic

- The diameter – A line that connects two dots on the opposite sides of the circular, while passing through the center.

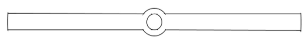

Figure 8: The Diameter layer
Source: Klelija Zhivkovikj, Jovana Radenovic

- The radius – A line that connects the center to any other dot on the circular.

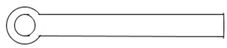

Figure 9: The Radius layer
Source: Klelija Zhivkovikj, Jovana Radenovic

This visualization should also represent a proportion is the size of the radius and diameter: the radius is always a half of the diameter.

This is a visual depiction of all the layers put together.

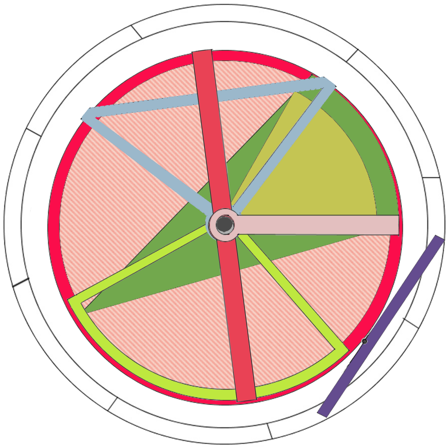

Figure 10: All the layers combined
Source: Klelija Zhivkovikj, Jovana Radenovic

And here is a picture of our first prototype:

Figure 11: First prototype
Source: Klelija Zhivkovikj, Jovana Radenovic

Conclusion

We have presented a brief introduction about the significance of using of different teaching approach. In particular we talked about the importance of usage hands - on approach in learning and teaching process.

Visualization has momentous impact on the understanding and exploration of complex mathematical phenomena. Consequently, we were interested in putting together some ideas about visualization of mathematics by employing tools that combine learning with character building. We consider that the use of educational tools helps pupils to develop conceptual understanding of mathematical ideas by representing the ideas in multiple ways. In connection with this idea our goal was to create an educational tool that will be equally useful to pupils and teachers. We have created a tool that allow learning and teaching in a playful way.

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The Crazy Crochet Circle Machine: A webpage about unfolding a mathematical thought within the three dimensional space

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Abstract

One mesh can be seen as one unit of measurement within a crocheted plane. As well as a mathematical two-dimensional space, a simple crocheted plane consists of elements – called single meshes – which are situated in a net of perpendicular lines, where the meshes are situated on the crossing points. This fact makes it possible to express mathematical thoughts in crocheting patterns and also to experience basic geometry as an educational tool. The task, described in the article, of crocheting a circle is only one possible application of this connection of mathematics and textile arts. The developed webpage can be used as an introduction into this precious activity of building a relationship to one of the participating disciplines. On one hand it is held in an informal layout to interrupt habitual perception of mathematical appearance in society and to underline the creativity of doing mathematics. On the other hand it can open a hand crafter or an artist a different access to create their projects. Either way, it opens borders and tries to open a self motivated learning experience.

Keywords: crocheting, mathematics, tactile exploration, two-dimensional space, pattern, game enhanced learning, education, textile art

Introduction

Since Daina Taimina wrote the book "Crocheting Adventures" which describes how to crochet a model of a hyperbolic plane (Taimina 2009), crocheting settled as an adequate medium to make mathematical thoughts not only visual but also tactile. The hyperbolic space is a mathematical idea that had its problems with getting understood by mathematic students and other interested people before the publication of this book. To crochet a hyperbolic plane doesn't need a lot of crocheting experience. It can be done by each child and therefore also create points of contact to minds of everyone who gets curious about it. It is a marvellous example of a visual and tactile tool for experiencing complex thoughts easily. Lilian Wieser works as an artist on this topic, but she never crated a pedagogical tool out of those thoughts. Together with the teachers Milena Zivotic and Ivana Djokic the idea of using this tool for educational reasons came up.

The collaboration with the programmer Vuk Vasic made it finally possible to create not only a visual but also interactive facilitation to acquire that tool. Every person can explore the path of the researched structure provided pedagogically within the application. The structure unfolds in front of the users eye to let him experiment with it afterwards and to build his understanding of the world out of this discoveries (Ahrens 2011, p. 65).

A mathematical thought unfolds in front of the eye of a curious user

The Crazy Crochet Circle Machine developed itself starting from the idea to create a program that supports the user in crocheting a circle mathematically which is described below.

Mathematical Solution of Calculating a Crocheted Circle

The used solution of calculation is based on interpreting a crocheted single mesh as the basic measurement of all calculations. Because of the reality of a crocheted mesh, which is as high as wide, we can set one mesh as 1 in each dimension of its connection with other meshes above, under and beside itself (which also are the properties of a two dimensional space). Another property of a mesh, which is chosen as a base of the calculations is its softness due to the woolen material, which allows us to slightly change the exact results of each step. The material forgives the hand crafter a mathematical inaccuracy which is at the same time necessary and useful for applying mathematics to crocheting, because of technical circumstances.

In crocheting of course it is not possible to divide a mesh into parts, so we are forced to find mathematical solutions within the natural numbers. And there is another technical characteristic, which is necessary to build a circle and is described below.

To start the project, one connects a chain of six meshes at its beginning in order to achieve a ring. The number of six meshes are chosen because of the formula $c = \pi d = 2r\pi$.

So a circle with the radius (r) of one mesh has a circumference (c) of 6,28318... meshes. We round it to 6, because of the need to apply our calculations to the real circumstances of crocheted meshes, which can't be produced partly.

After that sixth mesh of the first row, the second row begins. It is defined similar as in the first row so it is based with $r = 2$.

Second row: $r = 2 \rightarrow d = 4 \rightarrow c = 4\pi = 12,56637... \rightarrow 12$ meshes

Now it is getting necessary, how to connect those calculations by realising it as a circle within the technical possibilities of crocheting. Therefore one can use the technique of doubling a mesh of one row in the row below, which can be achieved by crocheting it like shown as a graphic pattern in Fig. 1.

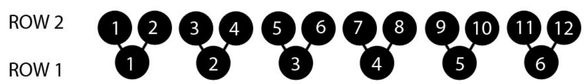

Figure 1: Pattern to connect the first row with the second row of crocheting a circle
Source: Wieser 2014

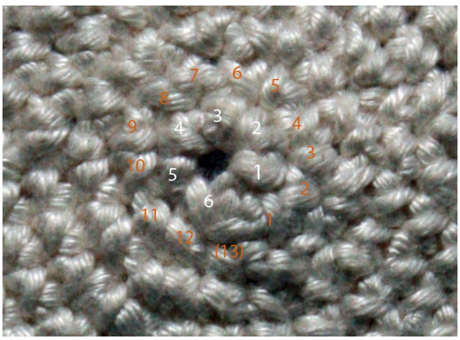

Figure 2: A photograph of a realisation of a crocheted circle with inserted numbers to show its mathematical content of its first and second row
Source: Wieser 2014

By calculating the third row, there has to be taken care of more mathematical correlations as for example finding the greatest common divisor. The steps where taken as follows.

Third row: $r = 3 \rightarrow d = 6 \rightarrow c = 6\pi = 18,84955... \rightarrow 19$ meshes

In this row it gets necessary to connect the second row of 12 meshes with the third row of 19 meshes. Therefore one has to find their greatest common divisor. 19 is a prime number, so the only divisor of both numbers is 1. Hence it is accurate to use the value as follows: $19 = 18 + 1$.

18 and 12 can both be divided by 6, 3 and 2. The mathematical facts needed to go on: $18 : 6 = 3$ and $12 : 6 = 2$.

For this reason the following crocheting pattern were chosen.

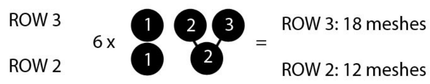

Figure 3: Crocheting pattern of the connection of the second with the third row
Source: Wieser 2014

It is more recommendable to divide those patterns into the smallest parts as possible to achieve an as possible round crocheted circle. They should be spread the most even as possible through the whole row, so the circle doesn't have corners at some points of one row.

For showing the manner of enlarging the circle, this calculations can go on until any number of radius one want's to realise, the calculations of the fourth is presented here.

Fourth row: $r = 4 \rightarrow d = 8 \rightarrow c = \pi 8 = 25,13274... \rightarrow 25$ meshes
third row: 18 meshes
fourth row: $25 = 24 + 1$
$18 : 6 = 3$ and $24 : 6 = 4$

So the crocheting pattern can look as follows:

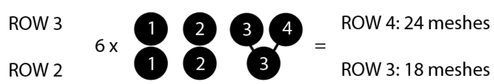

Figure 4: Crocheting pattern of the connection of the third with the fourth row
Source: Wieser 2014

The Problem About Rounding for Computer Application

The part of just taking a number that is usable to find the greatest common divisor with the number of the row before brought the team to the problem of expressing this thought in the logic of programming language which actually can be solved with floor functions.

How to Make the User Curious

This way of thinking can lead to a deeper understanding of all the three containing disciplines: art, handcrafting and mathematics. It is a visual and also tactile way to understand, that mathematics can be used creatively for aesthetic decisions to realise a certain shape through a real given structure of a certain technique.

But how can a person get curious about this precious activity presented in the developed application? How can a person be led through the phenomenology of such an activity?

The team decided to connect the application with a an interested audience through a webpage that leads the user not only through this mathematical solution of calculating a crocheted circle step by step, but also through a tutorial how the old Egypts thought about calculating this constant number that expresses the ratio between a radius and a circumference of a circle (Beckmann 1982), and which later were indicated with the Greek sign Pi. (Lehmann 2004). So the tutorial also describes a tactile way to understand this old idea with only some rope and a piece of chalk inspired by, what the Egyptians did 4000 years before in sand. For everyone it feels like a moment of enlightenment, following the old Egyptians on this trace. Using this mathematical constant afterwards will lead to a deeper understanding of its content than only pushing the button on a common calculator. The program then is integrated as The Crochet Circle Calculator, but it will provide the user a structure to help him or her calculating independently instead of doing it automatically.

Positioning of the Interdisciplinary Collaboration Within the Project

As already known in interdisciplinary collaborations the misunderstandings due to a different use of language (not only in terms of English as a foreign language, but also as a different use of language professionally) and also a different habit of developing projects inside of each discipline led also in this project to a big question mark at the beginning of the collaboration. (Mangelsdorf 2013)

But the team held on the project in a manner of a metaphor used by Corinna Barth. It is called the double slit experiment or interference pattern. It is a term of physics that describes the experiment which proved that light consists of waves and not of rays. This way of thinking about light is not easy to accept for a daily habit. So the two slits of the double slit experiment bring something to the surface, that was not expected by the habitually perception, like two disciplines sending waves and creating an interference pattern. (Barad 2013)

So The Crazy Crochet Circle Machine is therefore the interference pattern of mathematics, textile arts and programming and the proposal of the webpage is to lead into unexpected thoughts about the understanding of reality.

Results

The whole composition of those two applications should facilitate to understand the connections and relations within this interdisciplinary project very informal and through the method of asking questions which require each other. Like that the user can not only get answers of some of these questions, but also get an idea of how to connect this experience of using the application to further curiosity respectively further research. It grew to a sense giving tutorial, which goes beyond pure handcrafting but also doesn't get stock in theory, which very often closes possible connections to some human individuals.

To finally round the whole project and to not forget the programming part in this ranks of curiosities provided by programming language, also Ada Lovelace, as the first person who wrote a computer program and also connected its functionality with textile fabrication, has her little spot within the Crazy Crochet Circle Machine, but that has to be explored independently by visiting the webpage by oneself.

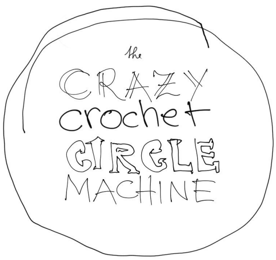

Figure 5: Logo of the Crazy Crochet Circle Machine
Source: Wieser 2014

Figure 6: Example of the layout for the webpage
Source: Wieser2014

The link to the webpage can be found at the Visuality and Mathematics hompage,www.vismath.ektf.hu.

Acknowledgements: Ruth Mateus-Berr, Michael Schreiber, Milena Zivotic and Ivana Djokic

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Mathician – How to combine music with mathematics

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Abstract

Aim of the project was to overcome the difficulty of teaching mathematics to schoolchildren in an understandable and true-to-life way using interdisciplinary method and develop knowledge. The goal was to find a simple and user oriented way to teach mathematics in combination with music and music theory principles. Following the guidelines and principles of an ordinary C major scale, a computer application was developed, where mathematical formulas can be tipped in on a keyboard simulation. Every key on the keyboard has its mathematical equivalent, therefore if the Pythagorean theorem $a^2 + b^2 = c^2$ is played correctly on an ordinary computer keyboard, a triangle appears on the screen.

Keywords: Maths, Music, Application, Education

Introduction

Mathematics and music are closer to each other than one would think initially. The logic steps from one note to each other in between a scale is based on and explained with mathematics and also refers to the strictness and tight organisation which builds the base of music theory. Provoking a break up of this strict organisation, our aim was to dismantle each individual tone and translate it into a simple piano keyboard.

In order to interchange research interests and general preferences and skills, it came to our mind that there has to be a way of finding a combination between math and music, since one of our common interests is music. Further, the two completely different amounts of knowledge and understandings of mathematics made the process even more exciting. For the design education student, who describes himself as "weak" when it comes to mathematics, there was an even greater interest of finding solutions to teach and understand mathematics better.

After a lot of time spent on brainstorming and researching useful combinations between art and mathematics, we found ourselves very satisfied with the different resulting ideas and approaches which lead to our final idea. Interesting theories such as "The Four Colour Theorem"[1] or the interesting compositions of Emily Howard (Petri - Preis 2013) were used to develop and construct our idea and then put together into a pedagogical concept.

The combination of an IT expert using computer programming tools with the input of design thinking based methods regarding problem solution, construction and artwork built the interdisciplinary concept of this work.

Materials and Methods

Mathematical formulas are as necessary for the subject as breathing is for life. In primary school these fundamental skills are taught as basis for upcoming mathematical operations. In order to find out how to integrate mathematical formulas into music, we used the musical equivalence of mathematical formulas: notes.

After researching different scientific theories, many hours of brainstorming and experimenting on the connections between music and mathematics, we invented a method to translate mathematical formulas into music scales by simply giving every note a mathematical correspondence. Therefore we tried to invent a guideline/legend that explains which note corresponds to which mathematical expression (see Figure 5.).

The Application

The application is written in C# programming language. The Open-source library of the Stanford University for midi controller was used during the development of the application. The next section shows the Use Case diagram:

[1] http://www.mathpages.com/home/kmath266/kmath266.htm

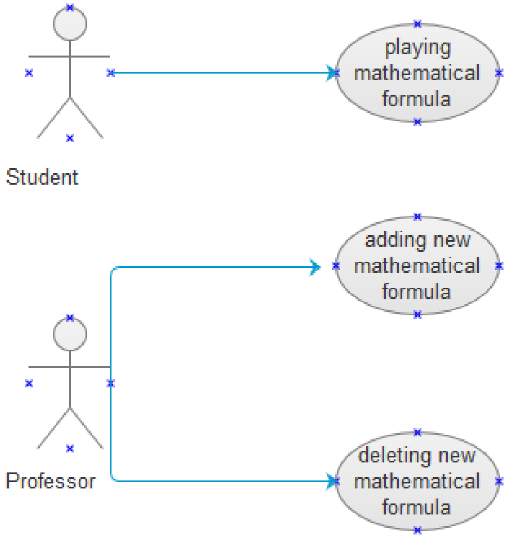

Figure 1: Use case diagram
Source: Filip Popovic, Julian Griss

When the application starts, the home screen appears. It includes a virtual midi keyboard, a box for showing the played formula and a checkbox to select different musical instruments.

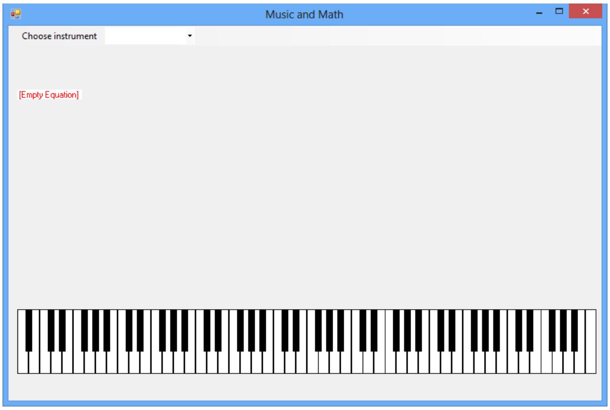

Figure 2: Home Screen
Source: Filip Popovic, Julian Griss

After playing the desired formula, the display shows a graphical representation of the mathematical formula, as it's shown in the following figure:

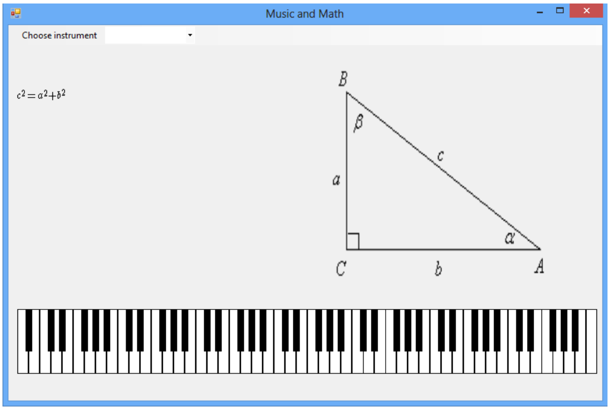

Figure 3: Pythagorean Theorem
Source: Filip Popovic, Julian Griss

By clicking the checkbox “Choose instrument”, it is possible to choose the instrument which the user wants to play.

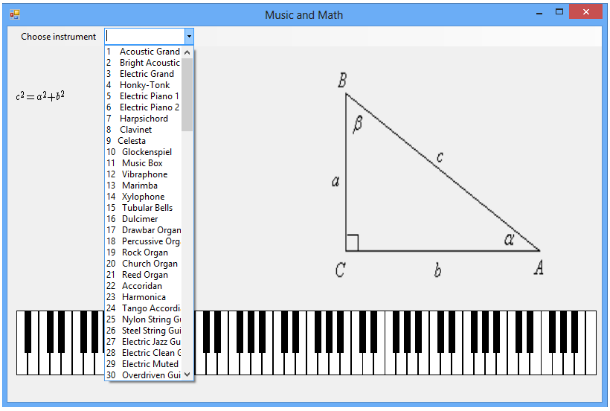

Figure 4: Choosing instruments
Source: Filip Popovic, Julian Griss

In order to allow the user to play the desired mathematical formulas properly, it is necessary to know how the tones are transferred to the keyboard. The following table shows the legend for the mapped tones:

Tone	C_4	D_4	E_4	F_4	G_4	A_4	H_4	C_5	D_5	E_5	F_5	G_5	A_5	H_5	C_6	D_6	E_6	F_6	G_6	A_6	H_6	C_7
Mathematical expression	c	a	b	O	S	V	ENTER KEY	+	-	*	/	=	(	)	π	r	H	2	3	4	5	6

Figure 5: Keyboard and mathematical expressions cross-reference
Source: Filip Popovic, Julian Griss

The Design

Having in mind our target group, we decided to stick with the classic look of a window application and add some tools. The final idea was to have a simple-looking, retro design with a catchy name (see sketch 2.). Instead of having a toolbar (to press STOP & PLAY, such as discussed during the process, see sketch 1.) the user now has to press an enter key which enlarges the amount of tones.

Since there is no USB Midi needed, the integrated keyboard makes our application more user friendly.

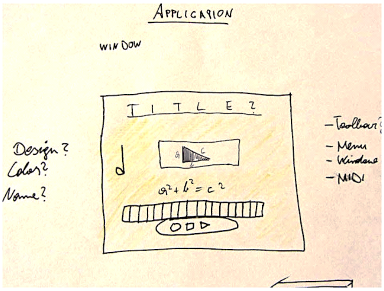

Figure 6: Sketch
Source: Filip Popovic, Julian Griss

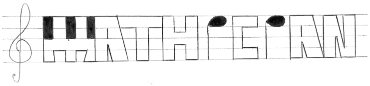

Figure 7: Sketch
Source: Filip Popovic, Julian Griss

Results

Our Result is a simple but useful practice tool for children. By having a concrete and detailed plan and content it is actually possible to program an application in a short time. The major part of our discovery is the deep connection between music and math, which can be found in the strict organization and course of a music scale. The spacing between every single tone is mathematically explainable. Also, artists made the aesthetics of musical notes and music pieces for their use. The visualization with the help of software design and music makes it possible to learn the content with two senses: seeing and hearing. This function of the tool helps to actually remember basic mathematic formulas.

Conclusion

By connecting two sciences with each other we could fulfil our wish of helping to understand mathematics better by creating a visual and interactive tool for children. The learning process does not only include a better understanding of mathematics, of programming tools and of design thinking, but it also helps to understand that there are connections between science and art which can be discovered and reconstructed. In contrast to other works within this area, which concentrated on aesthetics and theory, the produced learning application is an interactive learning game based on these aesthetics and theories.

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3D modeling in Rhino 3D program for website, Cocoon shape 3D model

Aleksandar Celar

Sint Lucas school of art

3D modeling in Rhino 3D program for website

The idea was to give young people a closer look to math and try to show them as fun. The first part of the workshop was to create a website -Mathematics in Brussels. On the website, there are photos of mathematical objects in Brussels with their explanation and location as well as a 3D model with a short animation. My job was to make the animation Rhino 3D program. With the help of Professor Rinus Roelofs mastered the basics of the program and made several animations of mathematical shapes that are now on the website. The photographs are screenshots of animation.

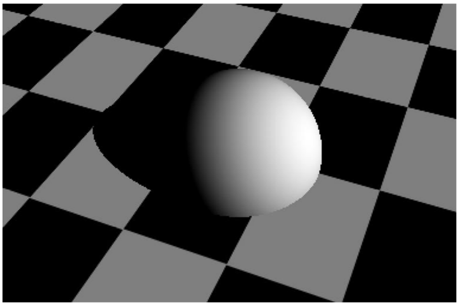


Figure 1: Half sphere

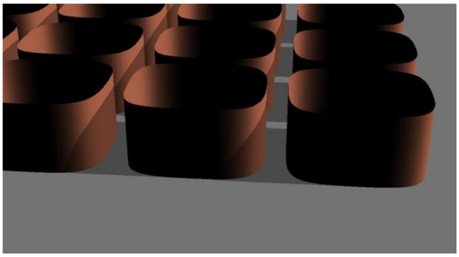


Figure 2: Pattern

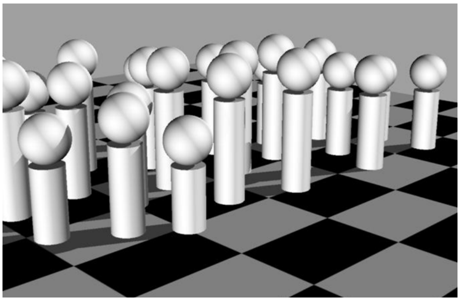


Figure 3: Monument

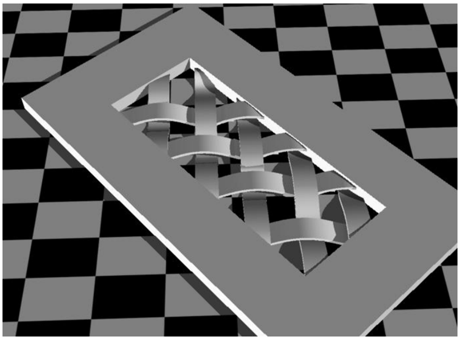


Figure 4: Net

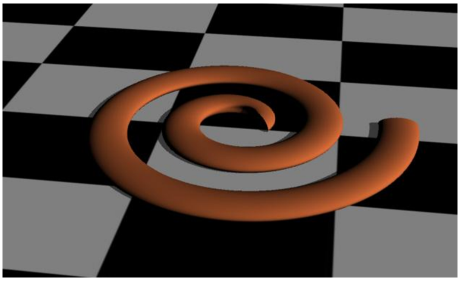


Figure 5: Spiral

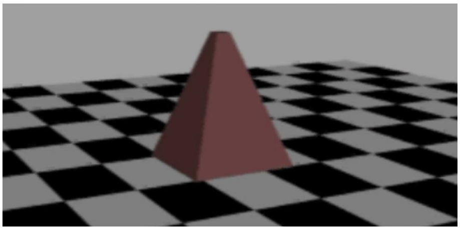


Figure 6: Pyramid

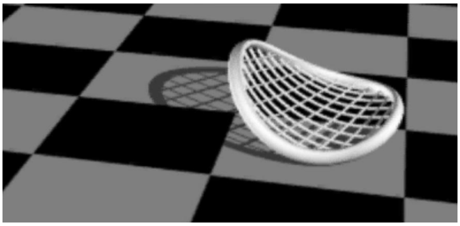


Figure 7: Hyperbola

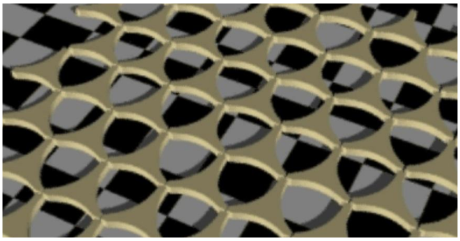


Figure 8: Pattern

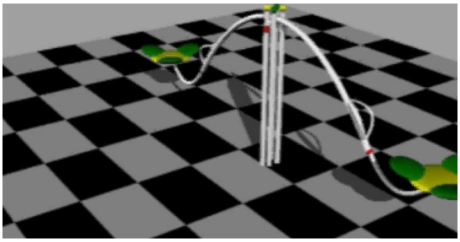


Figure 9: Kids toy in park

Figure 10: Symbol

Workshop with kids – Cocoon shape 3D model

The second part of the workshop was to create 3D geometric shapes of circles, squares and triangles. I made a design of PVC pipes and wrapped it over PVC transparent foil. So we made a 3D objects from simple 2D mathematical forms. I called this installation cocoon form installation, inspired by butterfly and spider webs.

Figure 11: Construction

Figure 12: Students

Figure 13: Final installation

Figure 14: Final installation

Conclusion

This workshop for me was very good because I learned the basics of modeling and animation in Rhino 3D. In addition, I have met a lot of young people with whom I could share experiences related to art and architecture.

Professor Rinus Roelofs and his works were very interesting and inspiring to me. I would also like to thank Gudrun and Dirk for the support and patience they have shown in working with us.

Balloon and Straws

Jelena Mataija

The main purpose of this project was to see mathematics in different ways we used to. In the workshop we tried to show mathematics to children from the age of 8 to 18 and by my opinion, we succeeded.

The workshop that I was doing was the balloon and straws workshop. The children where quite intrigued to see and build mathematical objects like Tetrahedron and Dodecahedron from balloons. They figured out so quickly how to build object presented to them, and what was more amazing, they quickly started to improvise and build new objects.

Figure 1

Every child loved to build the Tetrahedron and Icosahedron from the balloons, which were the easiest to build, but the enthusiastic ones tried and succeeded in building the Sierpinski's Tetrahedron and Dodecahedron which is by far most complicated object to build because it takes a lot of team work.

Figure 2: Helping a girl to make tetrahedron out of balloons

For me, the straw workshop was most fun. The great thing about straws is that everyone, no matter the age, can build and create various shapes and objects. Children very quickly learned what they could reach with different connections and how the number and type of connections can influence on the object they where making.

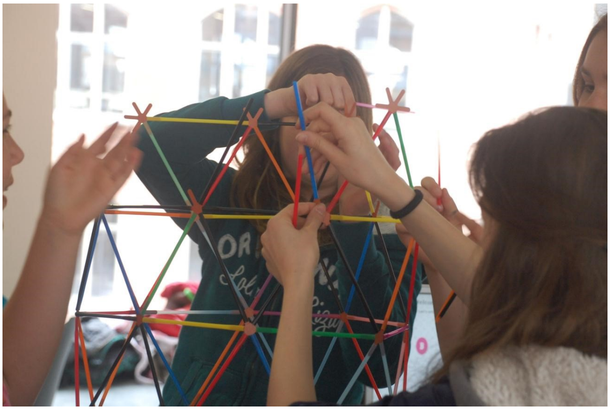


Figure 3

Most interesting thing, for me, was to see how the children of different age think different ways and how they build objects that where not in the explanations.

Figure 4: Girls age 15 making Octagonal Antyprism

Figure 5: Kids age of 8 building they own shape

Working with children was a big challenge and great experience which I would recommend to everyone. The children learned a lot in these two days from us as we learned a lot from them. I am glad that I had this opportunity to be a part of this project and that I intrigued children to think about mathematics in different way.

Coffee Sticks

From 17th April till 17th May, I was, alongside with 9 other Serbian students, a participant in Tempus project in Brussels, Belgium. The workshop I was working on was named “Coffee Sticks”, and my job was to make and explain the creation of stick bombs. The workshops were held on 14th, 15th, and 16th of May, 2013.

Stick bomb, which I made from simple coffee sticks that can be found in almost any office, is pretty simple to make, and pretty simple to explain. The pattern for it was taken from Leonardo Da Vinci’s sticks (Figure 1).

The pattern is explaining how to make the coffee stick bomb. By following the pictures, anyone can make, with a little effort, a stick bomb.

The principle works like this: when you pull the stick on the end of the line (on the picture, it is showed as red stick), chain reaction pops up all the sticks, making them move like a cobra. With a simple dose of mathematics and physics, it becomes a spectacular sight.

Children worked with the sticks, making a bomb patiently, and then releasing it with enjoyment, followed by the cheers of the crowd, also enjoying the sight.

All of them tried. Some were faster leaners then others, and before anyone noticed, they started helping each other, giving advices and helping with the construction.

At first, they were confused, wondering what they were making, after showing them how it works, they were very happy and amused, wanting to make one themselves.

There were, of course, a lot of unintentional blow-ups. We all laughed together and made more and more lines of bombs. Teachers were also intrigued in making it themselves, but gave up pretty fast.

When the children were on the lunch break Ljuba (a co-worker of mine) and I made a big coffee stick bomb, and let one of the children set it fly.

In my opinion, the whole workshop was a huge success. Children really enjoyed it the whole time, so we let them take some sticks home, so they can play with them.

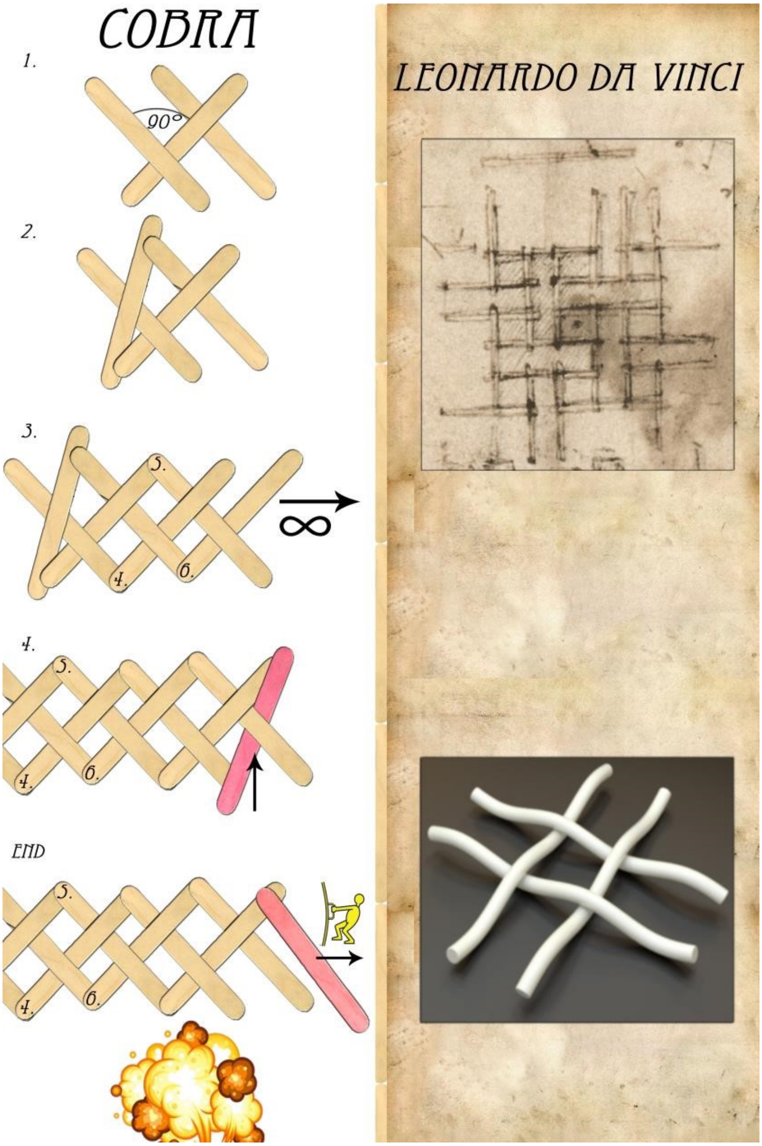


Figure 1

Figure 2

Figure 3

Smoke Gun and Leonardo Sticks

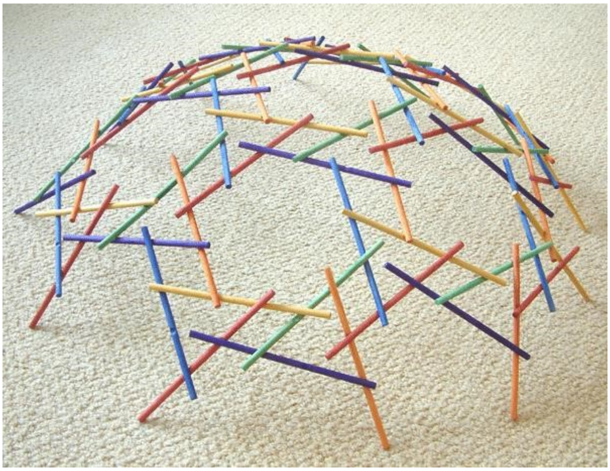
The workshop I was working on was the Smoke Gun and Leonardo Sticks.

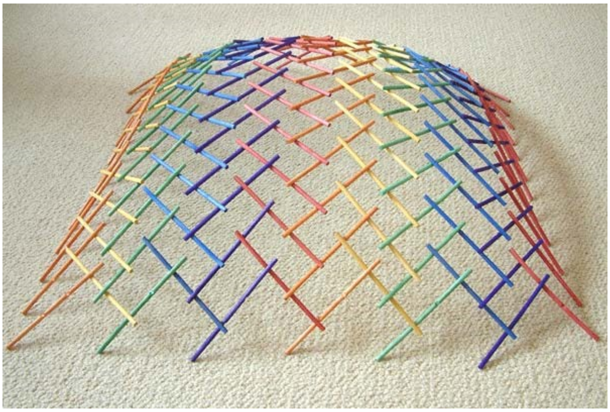
The Smoke Gun is basically a cardboard box with a hole on one side. Through the hole you put smoke inside of the box. When the box is full of smoke you hit the box on the sides and the air bullet shoots from the hole in a circular smoke shape.

The workshop with the Smoke Gun worked in a way that kids interested in it gathered around the table that had 3 Smoke Guns, and they shot the air bullets at the cups placed on another table few meters away from the Smoke Guns. The air bullets travelled the distance before they hit the shape built with plastic cups and shatter it. The group of younger kids from the first day were mostly interested in the Smoke Gun. The boys were handling the guns, and the girls had fun building different shapes from cups for them to destroy. But the roles changed from time to time. The place was filled with the sound made from the Smoke Guns and the children laughter and enjoyment around the table.


Coiffure Mariya
BEL Voyages

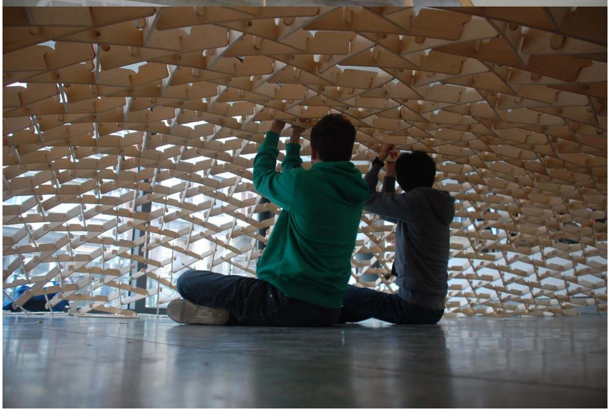
Leonardo Sticks have one Rule. Rinus Roelofs developed a simple way of describing the Rule that governs the building of all the structures. He called it "+ - - + ", or the "plus -minus -minus -plus" Rule. This means that each stick in a structure is connected at four points to four other sticks, and the way they connect depends on which side -top or bottom -the other sticks are on. A stick's end must be over the notch of another stick, and each notch in the middle must be under the end of another stick. So another way of reading the Rule is "over -under -under -over". A useful reminder is that the notches in a stick must always face up, to receive the ends of other sticks on top. Rinus used this Rule to build not only the three or four patterns that Leonardo stretched, but over one hundred additional patterns. In all of them each stick observes the + - - + rule, but the results can be very different A few of these patterns are included in the kit, but there are lots of others.

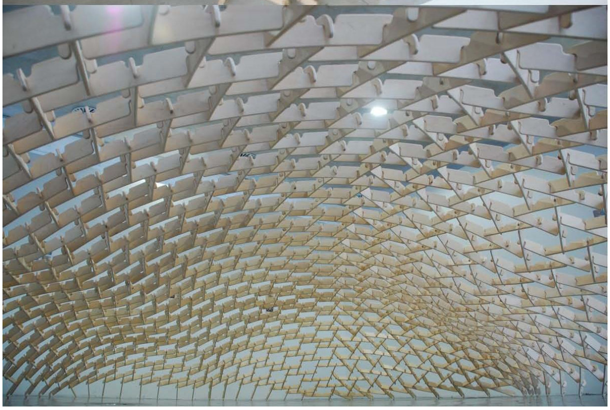




The workshop with Leonardo Sticks is best to start on the floor, which is exactly what we did during the Tempus Project workshop. I took a huge area of the room, brought the big Leonardo Sticks boxes there and opened them, took some out and started some patterns. Few kids gathered around me and watched me expand the shape, following the pattern they realized how to do it. Rinus Roelofs himself helped as well when it came to explaining and showing the children the patterns and a basic how to build the dome. The teachers also helped a bit when it came to language differences, making it easier to communicate with some kids. Finally one by one, they took out more Sticks from the box and continued the pattern, when the shape started to rise up higher and higher making a shape of a dome, more kids gathered and joined the building process. I tried explaining to all of them how to do it if they were stuck or just couldn't figure out how to start the pattern at all. Surprisingly most of them had a lot of fun with this and wanted to see the whole thing through, endlessly making it larger and larger. Eventually the dome became so big that they could go inside of it and stand, which they did. Sadly I just couldn't explain to them that there are two shapes of the Sticks, one that make the construction rise, and one that makes it go lower a bit. That was the downfall of every shape in the workshop, eventually it got so big that the pressure was too strong for it to hold on some side, so it just started to crumble at one point. Personally I was always a bit sad to see the dome crumble on its own. Impossible to fix the damage dealt from too much pressure.

Luckily the children didn't mind that at all, actually they couldn't wait to start a new shape.





Conclusion, I had a lot of fun and I know the kids that spent time on my workshops had a 'blast' as well. At first I thought it was going to be a lot of hard work and a bit of a bother but I'm glad I was wrong. I had the most amazing time on the workshops during my stay in Brussels.

PolyPuzzle

Katrina Radic

The concept of making mathematics more interesting to children is a new way of looking at mathematics in general. I think it is important to show children that mathematics can be very creative and intriguing, thus the workshops were a big challenge and an interesting experience.

Through a geometric-based 3D puzzle game, named *PolyPuzzle*, in which children could make various geometric forms – even little animals and toys – we succeeded in showing children that mathematics can be very creative and fun. The children reacted very well to the workshop – it was fascinating to see how children of different ages reacted differently to the puzzle. In the end, they all succeeded in making something of their own, thus bringing mathematics closer to them.

Three boys ages 10-12, enjoying the workshop and making various geometrical forms.

Boys 13-15 years of age, experimenting with the puzzle.

The children could choose to make various objects – from geometrical objects to little animals (turtles, bats, frogs)...

Even the youngest child enjoyed the workshop.

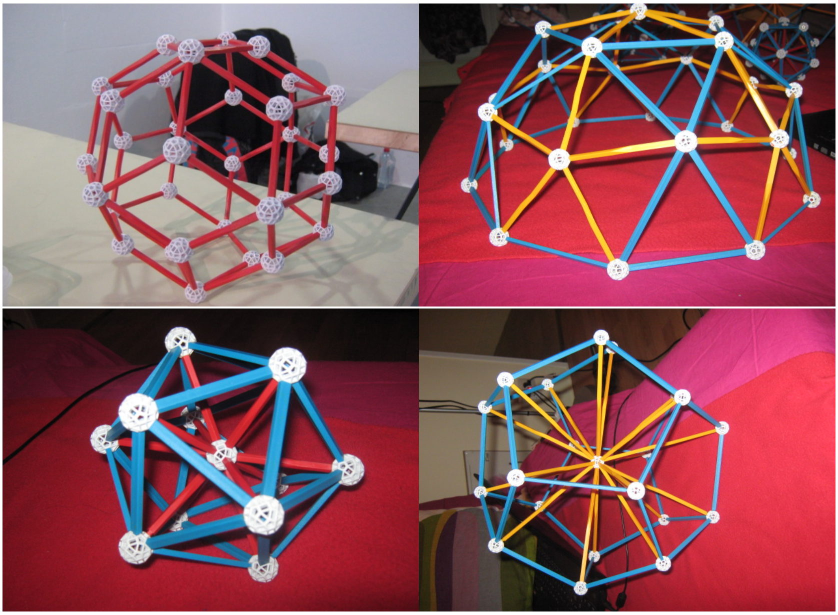
In the end, the workshop was, I think, a success – we brought mathematics closer to children and showed them that they are neither complicated nor hard – we have showed them the creativity and playfulness of mathematics, reflecting the fact that it really can be fun and interesting.

Zometool and Ice-Cream

This workshop is organized in purpose of the visualisation of the mathematics and for presentation of mathematics to children in a creative and interesting way. In cooperation with the professors and assistants from Sint-Lucas School of Architecture, we have organized the workshops using a zometool and ice-cream.

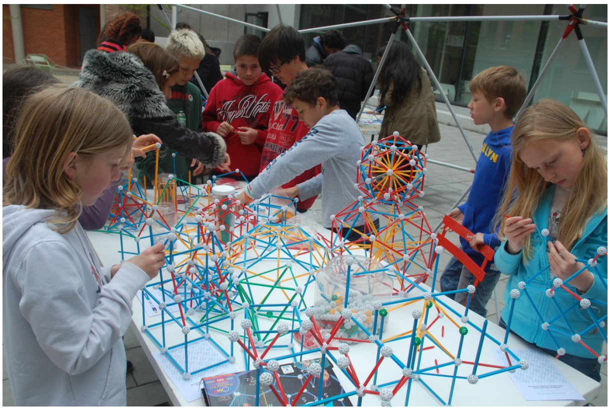
ZOMETOOL

Zometool is fun and educative tool, which is used for the visualisation of mathematics and it is used for making 3D shapes. It consists of the small balls and sticks in several colours, shapes and dimensions.



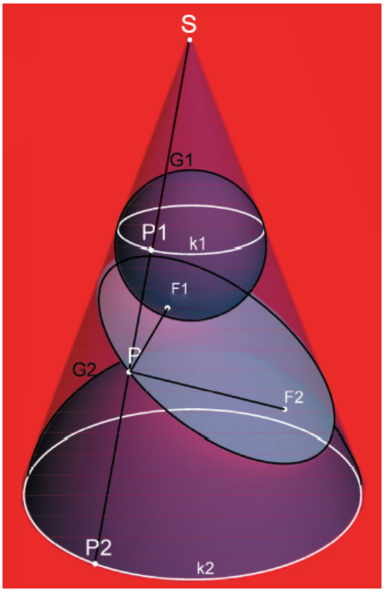
In the first day of the workshop, we were working with the children of age 6-7 years. We had prepared models in advance, such as octahedron, tetrahedron, dodecahedron, cube, icosahedron... and the purpose of the workshop was to show to children how they can make the models on their own. They also had a chance to learn the names of the mathematical shapes. Children experienced that mathematics can be fun and that there is no need to be afraid of the teacher or from exams.

On the second day, we had visits from the other elementary schools - children aged from 8-10 years and from high schools - children aged from 15-18 years. On the third day it was the day of the open door- people from the general public could come in and make models.



ICE-CREAM

The Ice-cream workshop is the workshop that is inspired with the Dandelin theorem. More about the Dandelin theorem you can find in the Wikipedia website. In geometry, the Dandelin spheres are one or two spheres that are tangent both to a plane and to a cone that intersects the plane. The intersection of the cone and the plane is a conic section, and the point at which either sphere touches the plane is a focus of the conic section, so the Dandelin spheres are sometimes called focal spheres. The Dandelin spheres were discovered in 1822. They are named in honor of the Belgian mathematician Germinal Pierre Dandelin, though Adolphe Quetelet is sometimes given partial credits.

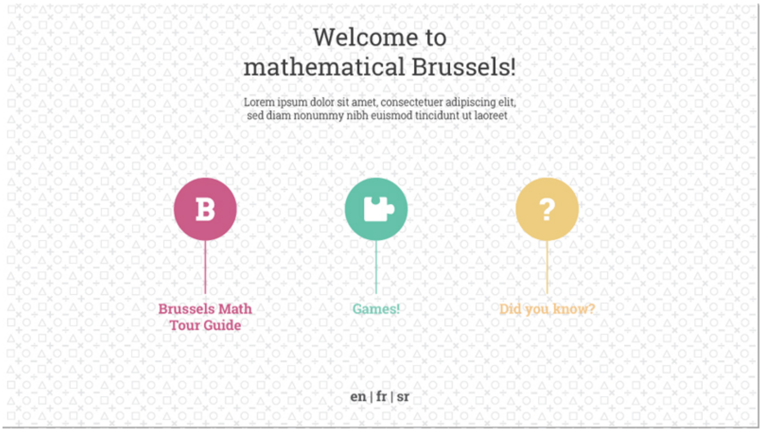


The difference to the original version of the theorem, we have replaced a cone (cup) with a glasses in the form of cylinders for this workshop. Children have explored,in a very tasteful and interesting way,the mathematical forms and their mutual relations. The children enjoyed playing with us and that the workshop has been successfully completed.

Website project

Katrina Radic and Milena Nicic

Our task was to design a website that would bring mathematics closer to children. The website consisted of a mathematical tour of Brussels and a mathematics-based game.

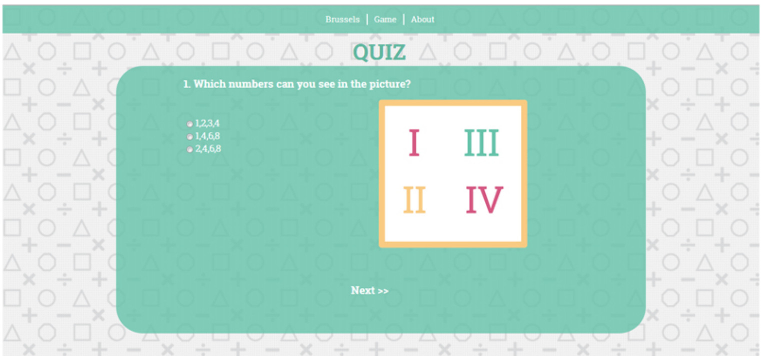


1 Mathematical tour of Brussels

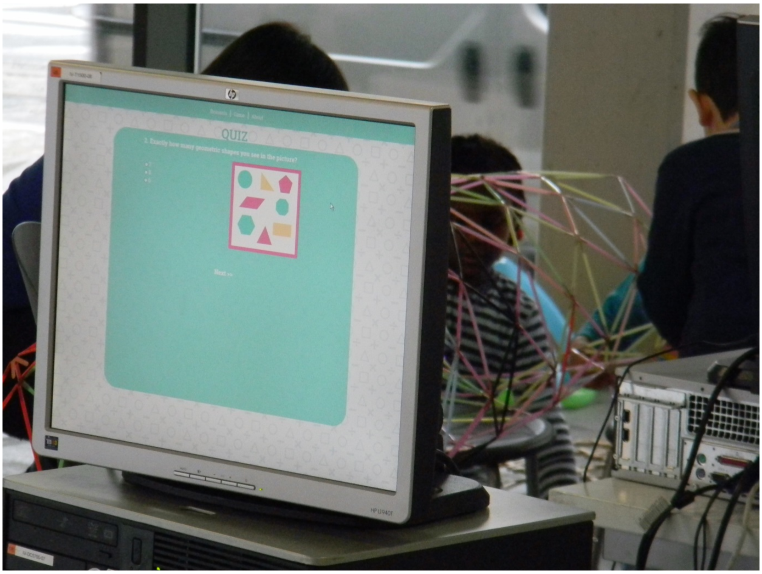
The mathematical tour of Brussels was designed in a way of bringing maths closer to people, by practically viewing it and exploring it, showing that mathematics are all around us. The mathematical tour consisted of pictures of mathematical objects which were all around Brussels (details on buildings, streets etc. Illustrating their mathematical explanations, and a map showing were the objects were located in the city.

2 Quiz

We also created a quiz for children to play, and test their mathematical knowledge with fun in an engaging way.



The designing of the website was made completely user friendly, with bright colours and simple commands, playful patterns and able to use with a quick response, the website was let alone simple, creative and interesting bringing mathematics closer to children and adults.



Our website is a fun to use project, in which we could demonstrate our creativity and design skills. It also became a challenge, since it had to be child-friendly and engaging to both young and mature students. We found that the most interesting part, was the mathematical tour itself, finding the objects in Brussels and integrating them into a website-based experience.

The Final Report – TEMPUS Project / Ghent 2014

Irma Puškarević

As part of the TEMPUS project Visuality & Mathematics we were chosen to take participation in one of the courses that took place in Belgium. We were given the opportunity to stay in Ghent one of the most beautiful and intriguing cities for the period of one month. Our task was to familiarize with the study course at Sint-Lucas School of Architecture which would eventually help us develop a workshop proposal that will combine both visual arts and mathematics. This workshop would be our final project on the course and its aim was to engage children in mathematical thinking through visually interesting elements.

The beginning of the course

Upon our arrival we were introduced to the course aims and were given an introductory lecture by professor Huylebrouck. The next day we met professor Roelofs. Firstly he gave us a task to assemble a shape out of the cut paper and later on showed us how to make these 3D models using a software Rhinoceros. It was not easy at first because I come from the 2D world but with some practice my colleagues and I managed to work the software out.

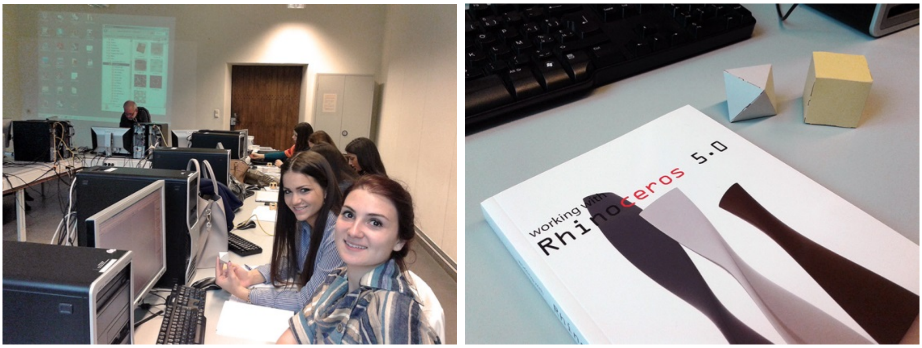


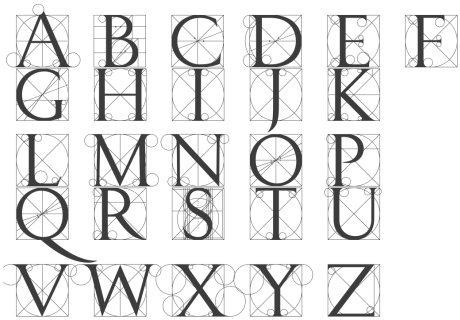
Figure 1: In the class with professor Roelofs

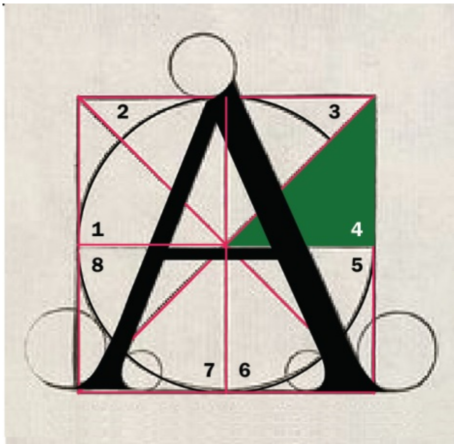
The school which was set in an old monastery added to our overall impression of the place which from the very beginning was very positive. It was interesting to watch Flemish students working on their assignments in the common room during our breaks. We met some of them during the course and they were very friendly and helpful. It is always interesting to meet new people and share knowledge and experience especially if they are from a different country.

Figure 2: Enjoying the fine weather at the campus

The workshop proposal

Apart from our meetings with professor Roelofs, we also had the meetings with professor Huylebrouck on regular basis to consult on the workshop. Initially, we tried to develop an idea for the workshop that will have elements of graphic design (in some way; which is my area of study) and mathematics (the area of study of the two girls that had been in my team). We found inspiration in Pacioli's letters that Leonardo drew.





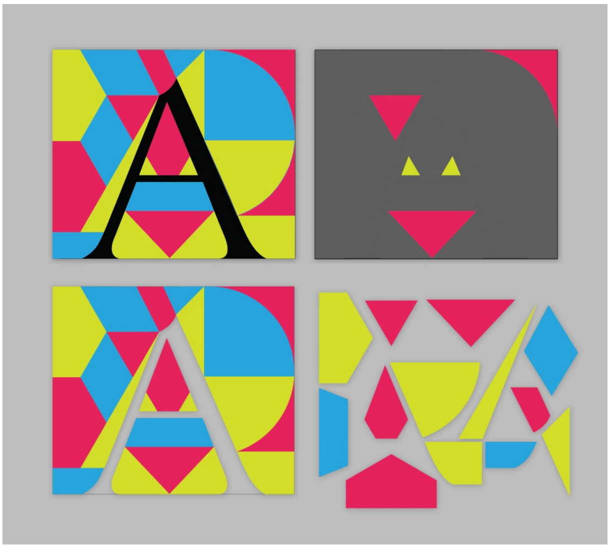


Figure 3: Following Leonardo's example we put the letter in the square frame and drew geometrical shapes around it. This puzzle set up was approved with suggestion to make the pieces of the puzzle bigger i.e. to combine some shapes so there are fewer of them.

The next step was the making of the puzzle. We have decided it was going to be done on the cutting machine and we would use would for the elements. It was required to go to Leuven for this assignment because there is a laboratory that let's students use the cutting machines free of charge. In the end we had all the letters of Leonardo's name made into the puzzle.

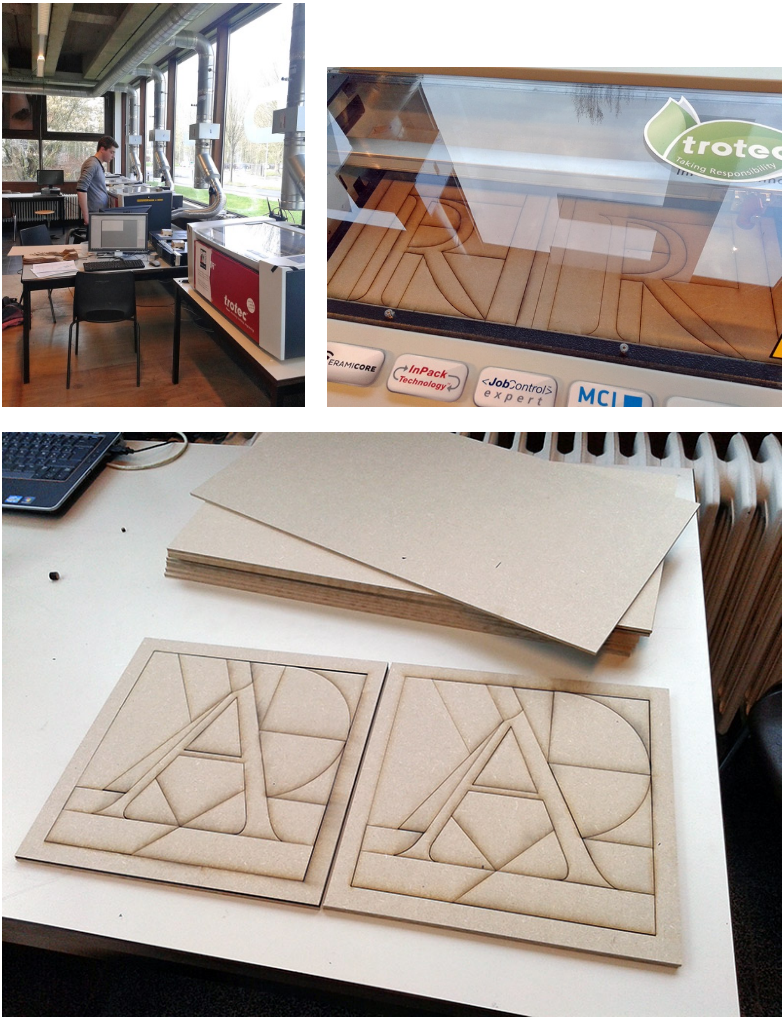


Figure 4: The making of the Leonardo's letters at FabLab in Leuven

Considering our initial idea was to have the elements in colour because we thought it would be more interesting for the children this was not possible at the time so we decided to paint some elements. There were two types of the letters: easy and hard. To distinguish them visually one type of the letters would only have the letter painted and the other type all the elements painted except the letter. The latter version was the easier one because we glued the letter onto the board.

Figure 5: The final outcome of our puzzle

Visual Identity

Additionally visual identity for the workshop was developed. By some strange chance I was the only graphic artist in the group. My idea was to have clear and simple design. I used three colours, red and blue with frequent black. This was a children's workshop so I did not want to make it too serious. I chose Hiruko typeface to express casual and playful atmosphere.

Figure 6: The basic visual elements of the identity

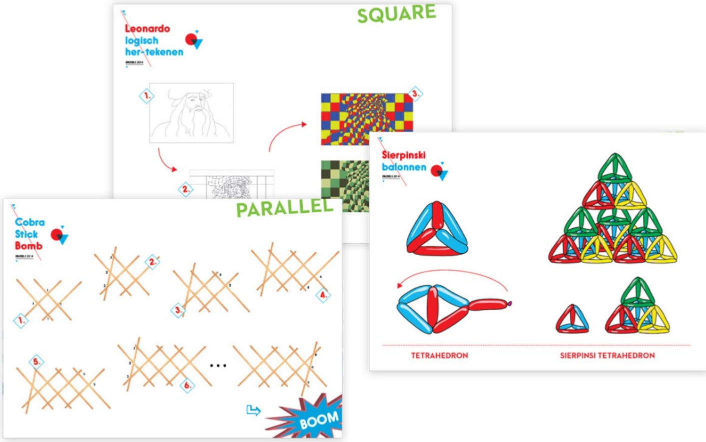


Figure 7: Some of the posters for the workshops

Figure 8: Leaflet with all the information about the workshops that was handed out at the entrance of Munt Punt

Figure 9: Participants of the workshop wearing t-shirts made for this event

The workshops event @MuntPunt, Brussels

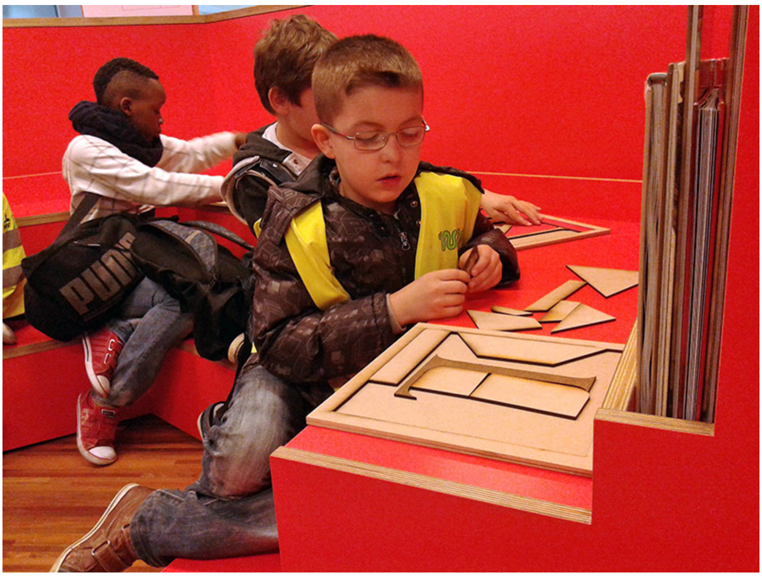


Figure 10: The children had fun time during our workshop. To motivate them additionally we had cut small Leonardo letters and gave each one as a present if they finish the puzzle. It was lovely watching them try to make their way to the final solution. Some children were so determined to make it on their own that they didn't want any help from our part. Their excitement was much greater when they reached the solution than that of the other children.

Children seemed to enjoy other workshops as well. It was fulfilling to see their lit up faces while they were trying to make out the way elements should be put together.

Figure 11: Children enjoying themselves during the other workshops

The traveling part

Apart from studying we have visited a lot of sights in Ghent that were of interest, some of them being The Book Tower which had been designed by the famous Belgian architect Henry van de Velde, the Museum of Contemporary Art S.M.A.K. currently exhibiting the works of Richard Jackson, The Ghent Alterpiece by Van Eyck brothers, Ghent Design Museum. We have also visited other cities (Brussels, Bruges, Ostend, Antwerpen, Leuven) and their historical or otherwise interesting sights. The country is very rich in places to see and things to do and this has made our stay much more pleasant.

Conclusion

We have reached the aim of our stay successfully. One month was enough to get acquainted with the project but it would have been nice if we had one more month to reach more distinguished results. The project that involves people from different study areas and engages them in one common purpose is part of the progressive way of thinking and similar things should be done in the future. I would like to say special thank you to our host, professor Huylebrouck. He made our stay more authentic and interesting and was attentive when we needed help. Also professor Roelofs should get praises as well. I had met him before during the summer school in Eger and was already familiar with his work. We were very lucky to have had these people on the course with us.

Tempus Student Exchange – Final Report

Dušan Tatić

Faculty of Electronic Engineering, University of Niš, Serbia

Introduction

This paper describes our activities during short study visit at Sint-Lucas School of Architecture as a part of the TEMPUS IV. Project, Visuality & Mathematics: Experiential Education of Mathematics through Visual Arts, Sciences and Playful Activities. The goal was to create the tools that can expand and modernize field of mathematics and science. We developed the AR Leonardo Drawings augmented reality mobile application that can improve posters of Leonardo da Vinci mathematical figures. Also, we participate the meeting Leonardo da Vinci + mathematics where we had math art workshop. Our task at workshop was to make Stick bomb with coffee sticks, that resembles on one construction of Leonardo da Vinci.

Lectures

The purpose of the seminar was to develop innovative tools and methods that will make mathematics more appealing for young people. Knowledge and ideas are derived from lectures and workshops that were held at Sint Lucas School of architecture in Gent. Attended math-art course related to the Leonardo da Vinci mathematics, which was held by professor Dirk Huylebrouck, were inspiring and useful. At lectures we could see how mathematics and ordinary life are connected. Also, we saw how mathematics can be fun if it is connected with art. Workshop about math and art was done in specialized 3D modeling software tool Rhinoceros 3D. Mastering the class, Mr. Rinus Roelofs, showed us techniques and methods for creating 3D models of mathematical objects. Also, special animations with these models were created using Grasshopper plugin. Our task was to create mathematical models of Leonardo da Vinci drawings and to animate them. Scenario was to connect unfolded parts of mathematical figures (like evaluated cube, octahedron and etc.) shown on Figure 1. Created animations of Leonardo drawings inspired us to go further and create mobile application based on augmented reality called AR Leonardo Drawings.

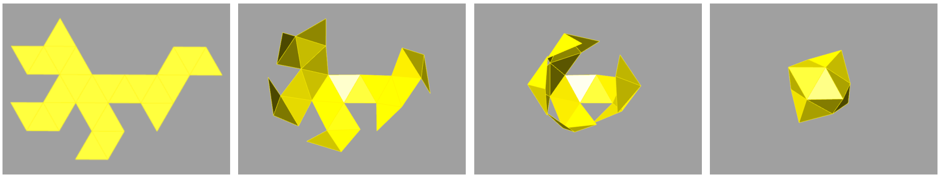


Figure 1: Rhinoceros 3D animation using Grasshopper plugin

AR Leonardo Drawings

Rapid growth of technology increases expectations of people to get an easier approach to the information they need. They expect to see more information about subjects of their interest using multimedia systems. Therefore, there is a need for creation of attractive and innovative multimedia systems that will provide detail information and that will be easy to use. Creating a direct interaction with a learning subject a system becomes more attractive. This can be done by using the technologies of augmented reality (AR).

Augmented reality (AR) is a technology that combines a live view, captured by a camera from real world, with computer-generated images. Thus, reality is augmented by additional multimedia information presented on the screen of a particular device, such as smartphone or a tablet PC. The user's perception of the world is, in this way, augmented by additional multimedia information (audio, video, and 3D objects).

During our study visit we created mobile AR application using this AR technology. The AR application is based on the Android technology and helps to improve the posters of geometrical figures of the Leonardo da Vinci. Poster consist of several pictures (mesh, shaded and unfolded) of the same geometrical figure (Figure 2. a). With AR application recognition and tracking of the unfolded part of geometrical figures are implemented. When the camera of the mobile device is pointed towards poster we get the real world image augmented with the animation that shows connection of all unfolded parts of the geometrical figure (Figure 2. b). The models and animations are created in Rhinoceros 3D like elevated cube, octahedron, etc.

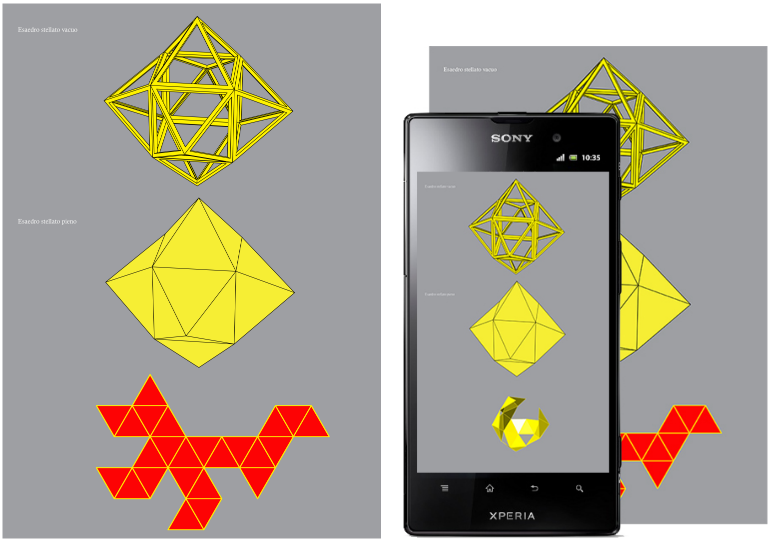


Figure 2: Posters a) real scene b) augmented reality application

Leonardo meeting

At the end of our study visit we were participants of the meeting called Leonardo Da Vinci + mathematics. The meeting was divided in two parts: practical workshop with children and presentation of invited speakers about Leonardo da Vinci mathematics.

For the work with children we needed to make mathematics more interesting and appealing. The goal was that children do Leonardo da Vinci mathematic trough game and play. Also, we had to make intuitive posters for Belgium children where we described in a pictures every step of the game (Figure 3. a). Our task on the workshop was to make Stick bomb that consists of coffee sticks that reminds of a similar construction of Leonardo da Vinci (Figure 3. b). Long structure in form of snake fires when first or last stick is released. During these game children learned more about parallel lines and kinetic system.

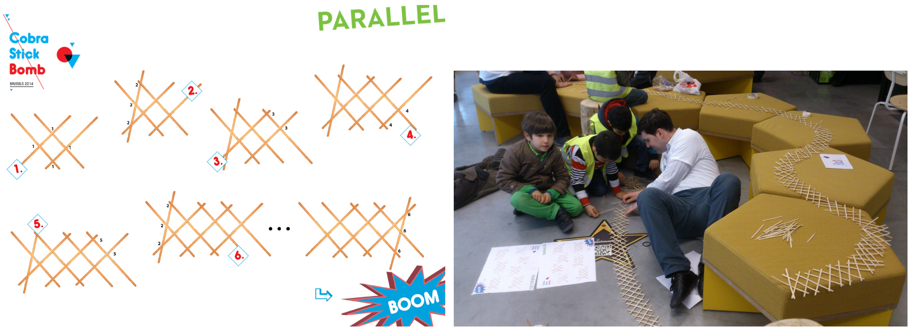


Figure 3: Leonardo Da Vinci + mathematics a) poster b) work with children

Conclusion

Knowledge gathered at the exchange helped me to expand my research interests and improve a creative process. Learning new methods and tools how to combine mathematics with art was great experience and inspiration for my further research. I enjoyed doing investigative and innovative work, which leads to practical applications. Lessons that I have attended, gave me a wider perspective how mathematic education and other disciplines can be improved. Also, it was a pleasure to be in the largest and oldest School of architecture in Belgium where central thing is creative act of designing.

Sierpinski Tetrahedron

Jovana Kovac

Sint-Lucas School of Architecture, Gent, Belgium
Tempus Project 10.03.2014. – 09.04.2014.

The main goal for this year's Tempus Project was to have a lot of workshops in Brussels, with kids. My workshop was called 'Sierpinski Tetrahedron'.

Basically, the kids and I had to make a tetrahedron with two balloons. When they make four tetrahedrons, they have to connect them into one big tetrahedron. Then, when they make four large tetrahedrons, they have to connect them, and it will make even larger tetrahedron. This step repeats, and the idea was to make a huge, tall tetrahedron out of balloons.

The children that I have worked with reacted very well to this workshop. To them, it was fun and they enjoyed it. For me, I think that the fact that children understood what they had to make, even though we do not speak the same language was magnificent.

In the end, some of them have succeeded in making mathematical figures with balloons, which was the main goal for this workshop. It was a great experience working with children and trying to make them familiar with mathematical figures and shapes.

Figure 1: Sierpinski tetrahedron ('making of' process)

While I was attending lectures at Sint-Lucas, I have also made some basic 3D models and 3D animations in Rhinoceros 3D modelling program. I have also tried to help one of the students there to make a fractal tree model in Rhinoceros, and therefore solve a problem with intersection of branches.

During my stay in Belgium, I've learned a lot, in a fun way. Professor Dirk Huylebrouck's presentations were very funny, but also educative. Also, I would like to thank him a lot for his patience, and also because I got to know Belgium a little bit more because of him. Professor Rinus Roelofs taught me a lot about Rhinoceros 3D modeling program, which I find very useful .

I'm very thankful for this amazing experience I've had.

Leonardo's bridge

During our stay, we had several lectures, held by Dirk Huylebrouck and Rinus Roelofs. Professor Huylebrouck presented us Leonardo da Vinci's work from another perspective, which was quite refreshing and different, and pointed out some mistakes that he made, even though Leonardo is considered to be one of the brightest minds in history. With professor Roelofs we had a basic tutorial on how to use Rhinoceros program and create mathematical objects, make animations out of them and even create our own objects and shapes. The way he explains and teaches made everything much easier and more interesting.

We held Leonardo's bridge workshop. Point of the workshop was to show a true model that Leonardo da Vinci drew, and which would actually work in real life. All models presented in museums (Figure 1) are wrongly made, without poles that can hold the bridge and the ropes that keep the tension, and by that, those bridges are not possible to make and use. It is not known exactly how Leonardo wanted it to be used, but that what is known for sure is that the bridge is designed for military purposes, for it to be easily made, assembled and crossed over.

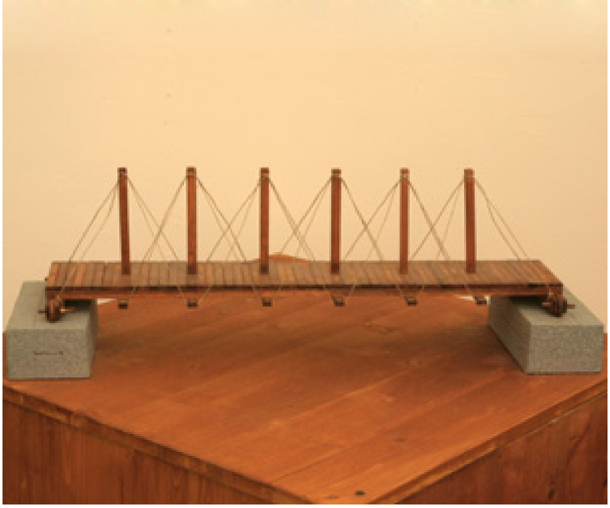


Figure 1

We made our own model, first modeled in Rhinoceros program, and then, with professor Roelofs help, we laser cut it. A few models were made, one for demonstration, and others for people to assemble it and try different stuff so they can see how the bridge works. Model was made based on original drawing of Leonardo da Vinci (Figure 2).

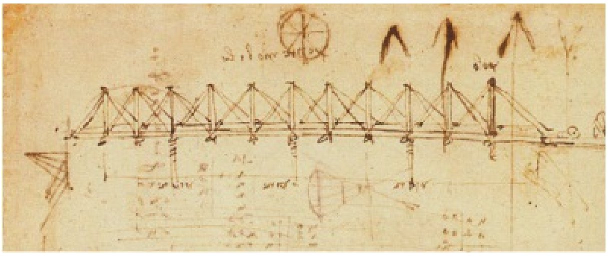


Figure 2

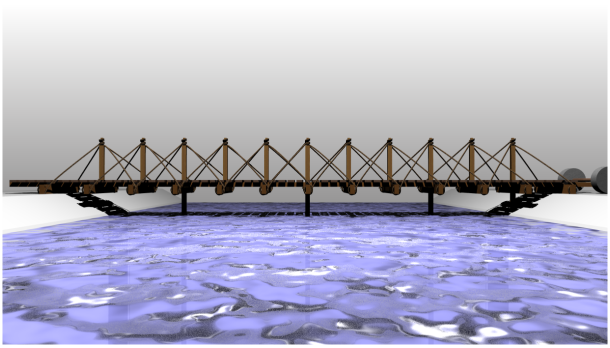
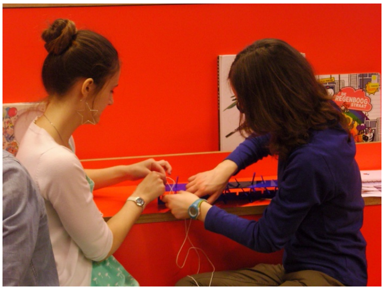


Figure 3

Image above shows our model in Rhinoceros program (Figure 3).

During two days of workshops, we had two groups of students, one of students 7-10 years old and other of students 17-18 years old, and surprisingly they found this subject quite interesting. They had the chance to assemble the bridge themselves, and see how the bridge works. Many of random bystanders wanted to see what the workshop was about, try to make their own bridge and hear the story behind it. We think that the workshop was a success.



We want to say thanks to Laurens Luyten for helping us understand the engineering side of the bridge, Rinus Roelofs for helping us making the real, printed model of the bridge, and Dirk Huylebrouck for helping us organize the workshop and showing us the best parts of Belgium.

Short term stay at Sint-Lucas school of architecture – Final Project

Miloš Stošić

Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade

Introduction

During the short term stay at the University of Ghent we obtained lots of useful information on how teaching mathematics and related subjects can be improved by visuality. Prior our visit it was clear to us that scientific subjects are not easy to teach, especially if the topics are hard to present. We were pretty much aware that, consequently, it is very hard to attract children and young people to science, since it is usually presented as something very difficult and complex.

In Ghent we visited school of architecture Sint-Lucas and had a chance to attend math and 3d modelling classes. This visit was very informative, and we were able to see how classes are organized and held and what kind of equipment teachers were using for their presentations. On whole curse we had math classes with professor Dirk Huylebrouck, very interesting and with a lot of examples. We've got those presentations, that will be very useful in our studying. Very important tool that I have been using on this university is Rhinoceros. That is the tool for creating 3d models and animations, and can be very useful in a field that I am occupied.

Through all what we had seen and experienced one problem often rises – presenting 3D objects in an image. Sometimes this image can be quite descriptive, but usually it lacks a lot of information. If only one plane is taken into account for object description, then whole image of an object is coarsely simplified.

The result of our engagement in this project has two parts. First one is that we created an application that will help teachers presenting mathematics and mathematical models in a more dynamic and visually appealing manner by incorporating some of the 3D modeling and video material. Application is based on augmented reality and it is made for Android operating devices (tablets and mobile phones). 3d models and animations are made in new program that we learned on this University, Rhinoceros, with big and devoted work with professor Rinus Roelofs. Augmented reality part of our application is made using Metaio technology. Metaio is a SDK (Software Development Kit) that is used to connect computer made data with object in real world and all those data connected are shown on display of android device. Also, application can be made for phones that use iOS operating system. Second part of our engagement in this short-term stay at Sint-Lucas school was participation on workshop in Brussels. We used coffee sticks and made cobra stick bomb effect. Overlapping the sticks we created structure of snake. Pulling the last stick the snake begins to develop in form of burning fuse. The final goal was collapsed structure constructed of plastic cups. Also, we tried to reach world record of 11.000 sticks. Whole organization of workshops was of our professor Huylebrouck, who transfer his experience to all of us. He's very charismatic, clever and versatile person.

AR Leonardo drawings

First page of application, shown in Fig. 1 explain purpose of application, give some more explanations and have options for starting augmented reality part of application.

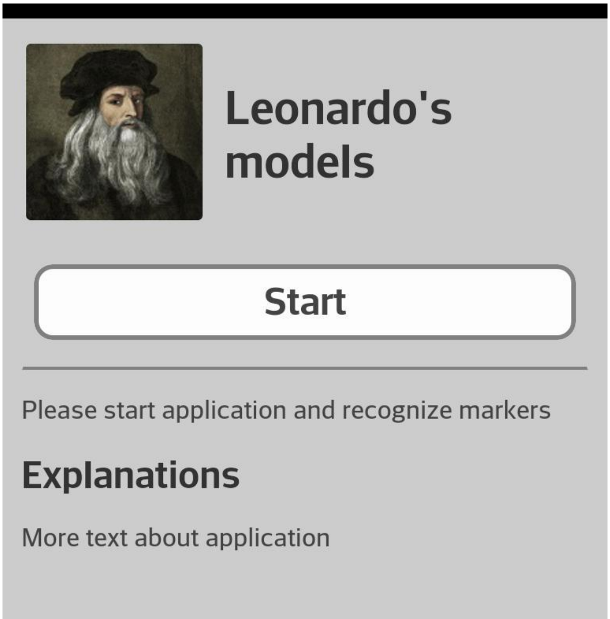


Figure 1: Home Page of application

Purpose of application is that will help teachers presenting mathematics and mathematical models in a more dynamic and visually appealing manner by incorporating some of the 3D modeling and video material.

This part is intended to help explain the term of an object net. By definition a net is a two-dimensional figure that can be folded into a three-dimensional object. When presenting an object in its unfolded form, some people have a problem of visualizing how the net would look like or, contrariwise, which net belongs to which object. In order to help visualizing the connection of 2D and 3D representation of the same object, we choose two solids –Evaluated cube and octahedron. Their nets served as markers (Figure 2, 3) and by scanning them corresponding video is show.

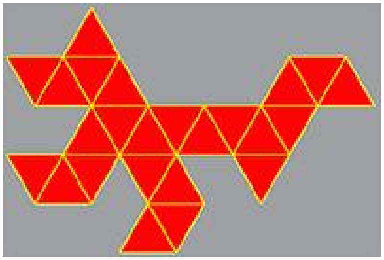


Figure 2: Evaluated cube's net

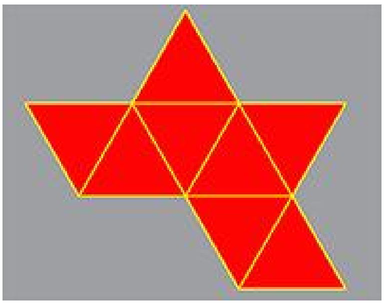


Figure 3: Octahedron's net

First we had to create mentioned 3d models, evaluated cube and octahedron using Rhinoceros. Then, we connect created model with Grasshoppers, plug in for Rhinoceros that is using for making animations. With some functions we make net of object and connect every part of the net with correct kind of relation for this part.

With Grasshoppers we exported animation frame by frame, and by using some tools for connecting images to video file we create animation.

Video that is created we use in our Android & Augmented Reality application. We recognize nets of objects and show videos that describe way of inflexion of all parts of net. Finally when application recognize appointed marker, objects nets, on the screen is shown video animation of corresponding net, Figure1 and Figure2. Markers are places on posters, like on Figure 4.

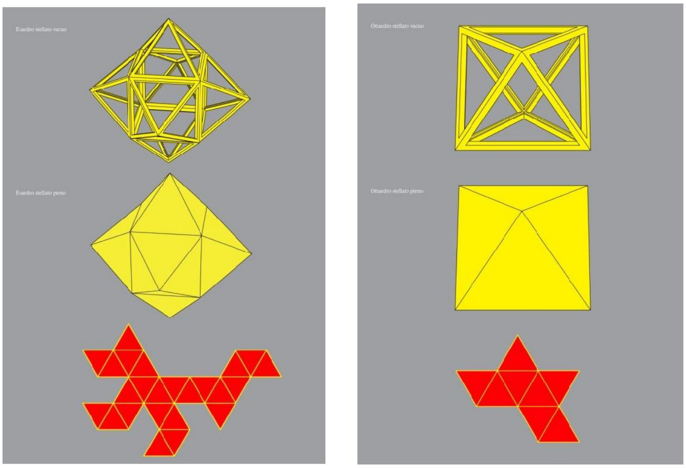


Figure 4: Examples of posters

Figure 5: Final result of application Evaluated cube video animation

Cobra stick bomb

Second part of our engagement in this short-term stay at Sint-Lucas school was participation on workshop in Brussels. We used coffee sticks and made cobra stick bomb effect. Overlapping the sticks we created structure of snake. Pulling the last stick the snake begins to develop in form of burning fuse. The final goal was collapsed structure constructed of plastic cups. Also, we tried to reach world record of 11.000 sticks.

On this workshop we made some little games for kids that came. When kids make a snake, on the end we put a castle of plastic glasses and on developing snake the tower is broken.

Using coffee sticks following steps of tutorial on Figure6.

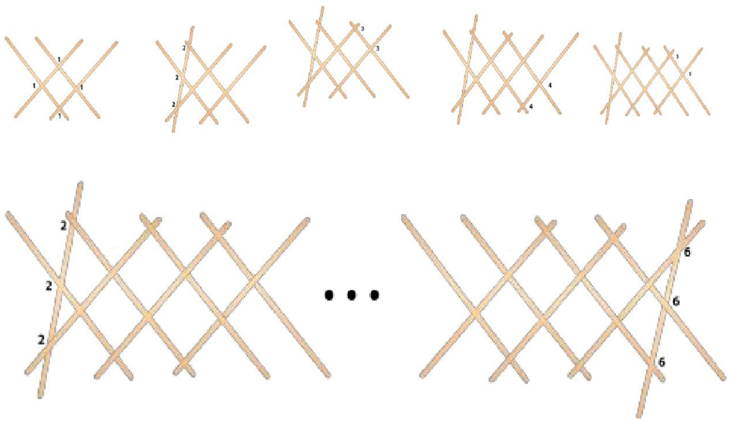


Figure 6: Tutorial of making stick bomb

The last goal for our workshop was to try to pass world record of more than 11.000 sticks. But, the type of coffee sticks was not that quality good enough for making record. Also, every time when we try do to something big, with more than 1000 sticks, some kid come to us, and pull over one of the sticks, or walk over them.

This workshop was very good experience for all of us, and we were happy that those kids went home with smile on their mouth.

This kind of projects should be organized more often because it makes very good base for students to continue to expand the scope of knowledge.

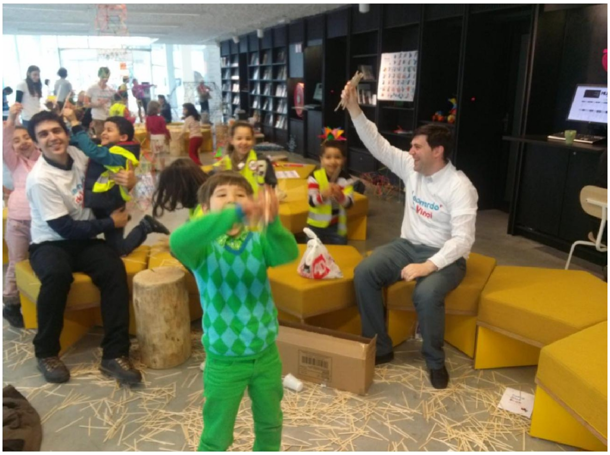


Figure 7: Workshop with Flemish kids

Figure 8: Workshop with Flemish kids

TEMPUS IV. project – Visuality and Mathematics Report

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Courses and inspiration for workshops

I had opportunity to participate in TEMPUS project Visuality & Mathematics for one month. During that period I was attending courses in Sint-Lucas School of Architecture in Gent (Belgium). The institution is amongst the oldest schools of architecture in Belgium. The course was taught by professor Dirk Huylebrouck and professor Rinus Roelofs, where they prepare us for workshop which we had to do in Brussels, 4th and 5th of April.

There were eleven Serbian students (ICT College of Vocational Studies, Belgrade Metropolitan University, University of Novi Sad, Mathematical Institute of the Serbian Academy of Sciences and Arts) and also three Flemish students of masters studies in architecture and all of us work together on our workshops.

Professor Huylebrouck made a few talks and presentations about mathematics and Leonardo da Vinci inventions related to math. There was a lot of interesting and unknown facts about Leonardo and his work. I will mention here some of them, which I like most. First one is about Polyhedra on an equilateral hyperboloid (Figure 1). In behind, one can notice that there was a lot of geometry and also number theory. After all, for me, this was an interesting talk, different from the every-day math I do. Also, he made talk about golden section how and where one can find and recognize it, but also about mistakes and illusions about it.

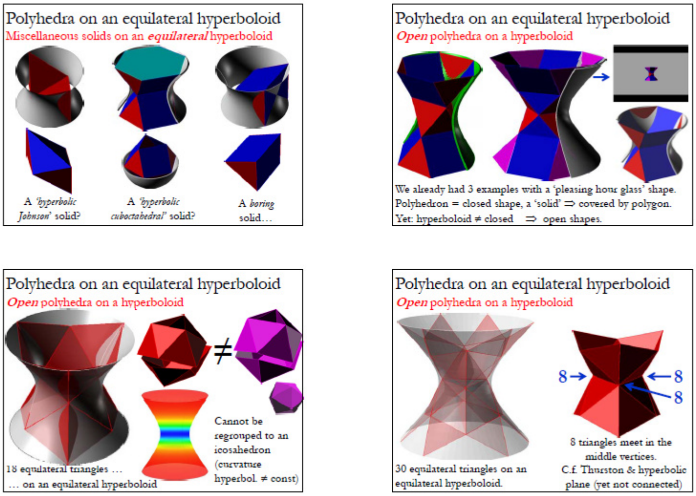


Figure 1: A part of professor Huylebrouck's talk about polyhedral

Also, professor Huylebrouck's talk answered the question – is math fun? For sure, yes, it is. All of these gave us a lot of ideas, how to make new workshop for children this year, which was our main goal.

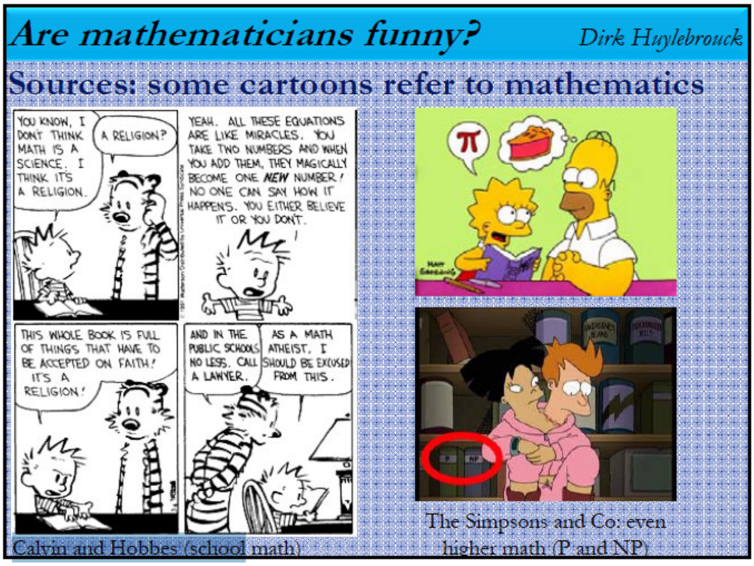


Figure 2: A part of professor Huylebrouck's talk "Are mathematicians funny?"

Beside this, I had opportunity to learn something completely new and really useful. Professor Rinus Roelofs presented us software "Rhinosceros" for drawing and 3D modeling of mathematical figures and objects and showed us interesting examples - how to make a cube, a node, etc. We had classes in computer classroom, where each of us had computer where he/she can try to do the example. Before making 3D figures in "Rhinosceros", we had to make such objects from paper. Also, I saw 3D printed objects and I was fascinated what can be done with 3D printer (for example, Figure 3).

Figure 3: 3D printing

Preparing the workshop

The main goal of the project was to make interesting math art workshop for children from elementary and high school. We wanted to show new ways in teaching mathematics: methods, software, games, toys, materials. With the help of this experimental view of mathematics education, we believe, we can make it easier and more joyful for children to learn mathematics.

As I already said, both courses, by professors Roelofs and Huylebrouck gave us the idea what to do. Together with my colleagues Danka Lučić and Irma Puškarević, I was preparing workshop called "Leonardo letters". Where the idea comes from? As geometric shapes and letters are actually one of the most important things that children learn in school, we decided to combine them.

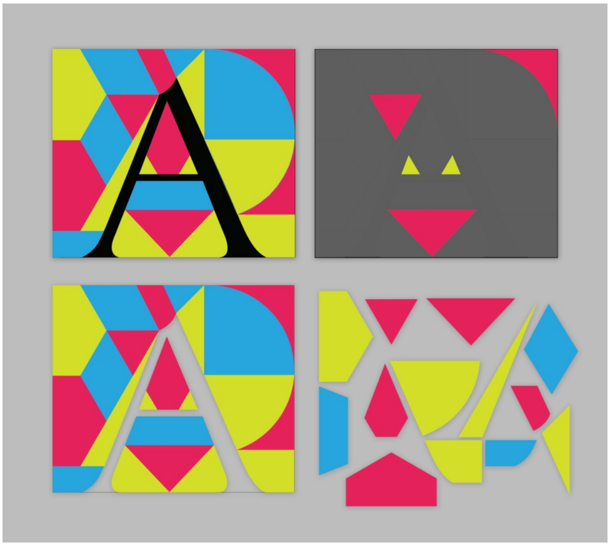


Figure 4: Leonardo letters – colorful puzzle

As one can see, we wanted it to be colourful, and in that way more interesting to children. But there are some technical details, because of which we were not able to make exactly same puzzle like in Figure 4. Because we had limited amount of time, it was easier to make a wooden puzzle, and cut its pieces with laser cutter. This was also one new experience for us – doing the laser cutting. It was fun and interesting.

We decided to put letter in the middle of the square, and then to make its anti-form with different geometric pieces. Then, we noticed that there could be two levels for making the letter puzzle: easy one, where the letter is fixed in the centre of the square and child has to put parts of the puzzle around it, and difficult one, where one has to make a "hole" for the letter in the middle of the square and to put letter into it, at the end. Of course, workshop had to be related to Leonardo da Vinci, so we decided to use Fra Luca Pacioli letter font (Figure 5.).

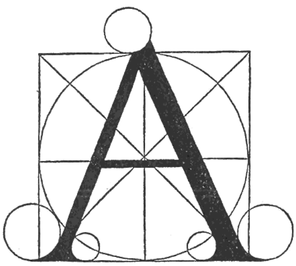


Figure 5: Fra Luca Pacioli letter

Fra Luca Bartolomeo de Pacioli (1445–1517) was an Italian mathematician, Franciscan friar, collaborator with Leonardo da Vinci. He constructed his characters geometrically, and used them in his well-known book Divina Proportione. The subject was mathematical and artistic proportions, and the book was illustrated by Leonardo da Vinci.

So, at the end, this is how our Leonardo letters were look like:

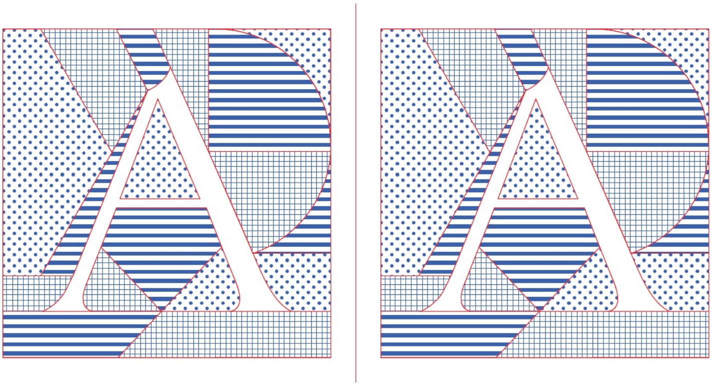


Figure 6: Final shape of "Leonardo letters" puzzle

In the next picture it is shown how our laser cutting was going.

Figure 7: Laser cutting

There are few more thing which we had to manage – to make posters (Irma was in charge of designing it, but everyone had to send her material for his poster), also leaflets and T-shirts. In the next pictures are final preparations for the workshops.

Figure 8: Final preparations for workshop day

Figure 9: Leaflets

Workshop

I have no experience in teaching and explaining things and math to young children. But, at this workshop it was easy job for me, because I did not have much to talk. Our letter puzzle was speaking for itself. Three of us, Irma, Danka and me were there, if children need any help, but they mostly figured out what they have to do. For some of them, the youngest who were seven years old, the easy level was enough (but there was also some who succeeded with the difficult one, with our help). The difficult one was challenge for older children, and for their teachers also. Following pictures describes atmosphere during the workshop:

Figure 10: Place for Workshop

Figure 11: Children doing the puzzle

Figure 12: Child doing the puzzle

Conclusion

Finally, having been a part of such nationally diverse and interdisciplinary project was rewarding in so many aspects. Working in a team was amazing and at the same time a learning experience. I have learned to listen to others ideas and thoughts and combine them with my own. What's more, I've got an insight into completely different way of thinking, one of an artist and successfully implemented it with my mathematical way. Also being able to experience the city of Gent and to get to know the culture and language is something I will cherish always. In the end, it was my pleasure to be part of this project and I have enjoyed every second of it.

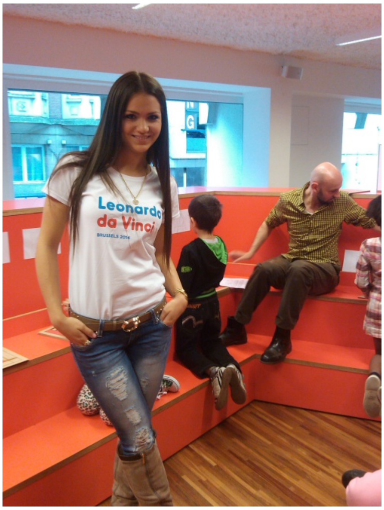


Figure 13: Workshop

Straws

Ana Rodic

Belgrade Metropolitan University

For this workshop I used straws and connectors to make mathematical figures and shapes from it. Workshops lasted two days. First day of the workshop there were children form primary and high school. Second day came children and adults. It was in Muntpunt in the centre of Brussels.

Figure 1: Children makings octahedron, icosahedron, etc. using the manual

The idea was to make different mathematical shapes like tetrahedron, octahedron, icosahedron and octagonal anti-prism from straws using the correct connectors. Workshop went excellent because it was fun for kids and they learned that mathematics can be interesting.

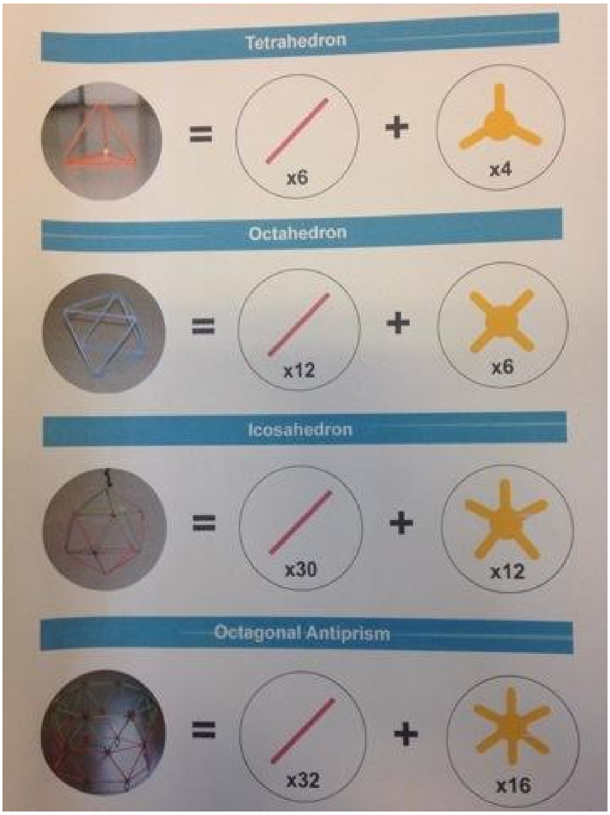


Figure 2: Manual for making different mathematical figures

The children reacted very well to the workshop – it was fascinating to see how children of different ages reacted differently to the straws. In the end, they all succeeded in making something of their own, thus bringing mathematics closer to them.

Figure 3: Kids are having fun doing math

Figure 4: Mixture of fun and learning

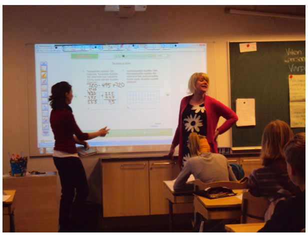
Tempus Short Study Visit Report, May 1–31, 2013

Kristóf Fenyvesi

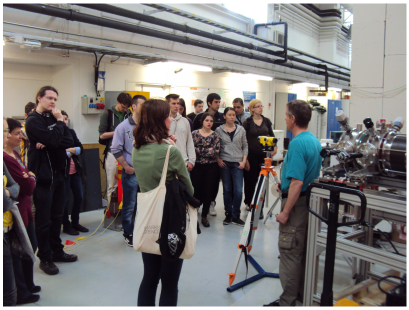
University of Jyväskylä

In the framework of the Tempus project, we hosted 10 Serbian students in 2013 May at the University of Jyväskylä. The visiting students accomplished an intensive seminar, together with Finnish and international students, and took part in a lot of programs connected to the main topics of the Tempus project.

During their 1 month stay the students got familiar with the Finnish culture, the school system and mathematics teaching.



They met with Finnish specialists of experience-centered mathematics education. Studied remarkable examples of scientific visulazition in natural scientific research and got important experiences about how to collaborate effectively in interdisciplinary teams in international context.



Members of the Study Visit program:

1.	Miljana Radenković	ICT, Bg	ICT
2.	Nemanja Nikolić	ICT, Bg	ICT
3.	Jelena Kričković	ICT, Bg	ICT
4.	Ivana Brkic	Metropolitan	BMU
5.	Nikola Filipovic	Metropolitan	BMU
6.	Miona Celebic	Metropolitan	BMU
7.	Djordje Manoilov	El.f. Nis	MISANU
8.	Srdjan Ivanovic	El.f. Nis	MISANU
9.	Milos Stosic	El.f. Nis	MISANU
10.	Ivana Tomic	FTN NS	UNS

Program of the intensive seminar:

Visual culture, Mathematics, Education: Experience-centered Education of Mathematics through
Visual Arts and Visual Art Education through Experience-centered Approach to Mathematics

The course provided an outline of the theoretical background and practice of experience-centered education of mathematics through arts, sciences and playful activities; and art education through the experience-centered approach to mathematics. How to integrate pedagogical results of using art, science, and play-centered learning into mathematics teaching programs? How to integrate experience-centered mathematics education into art teaching programs? How to expand the set of tools used for develop a learner's perception, aesthetic sensibility together with increasing mathematical, logical, combinatorial, and spatial abilities, structured thinking skills, motivate collaborative problem solving, interdisciplinary and inter-artistic approaches? How to organize math-art events in your school? The participants of this course became familiar with the basic ideas and main activities of such organizations like the Experience Workshop Math-Art Movement, a movement advocating experience-centered mathematics education, the Bridges Organization, the world largest mathematics and arts community, and the Symmetrion, a world association for Symmetry Studies.

Other educational programs:

May 3., 17 PM: Museum of Central Finland. Guidance by Ulla Pohjamo, Phd art historian. Address: Alvar Aallon katu 7, 40600 Jyväskylä
May 6., Seminar
May 10., Visiting the Palokka School's math classes on elementary and secondary level.
May 11., Optional program: boat trip on lakes of Jyväskylä.
May 13., Seminar
May 14., visiting the Particle Accelerator of the University. Presentation on visualizations in Physics by Norbert Novitzky (researcher of the CERN, `http://home.web.cern.ch/`)
May 18., Optional program: Uptown Night – Spring Cultural Festival in Jyväskylä
May 20., Seminar
May 21., Visit at the Cygnaeus High School.
May 22., Optional program: Study trip to Helsinki. Visiting Heureka, KIASMA and the Opening of the math-art exhibit by the painter Katalin Haász in the Hungarian Cultural Centre:
May 23., 'The Smartest Person in the Room Is the Room.' Visiting the Sepän High School's Thinking Lab.
Visiting Sirpa Turpeinen curator of art & science exhibits in the town's Art Museum.
May 24., The Great Jyväskylä AalTour: visiting the Campus buildings designed by Alvar Aalto and visiting the Aalto Museum. Guidance by Satu Kähkönen, Phd art historian.
May 27., Seminar
May 28., Tempus Students meet Tempus Management at the Campus. Student presentations.

Conclusion

The students successfully accomplished all the tasks they were given and they gathered a lot of information and new experiences and developed a lot during the study visit. Their study journals, essays and reports are attached to this report.

Tempus Short Study Visit Report, May 1–31, 2014

Kristóf Fenyvesi

University of Jyväskylä

In 2014 May, we hosted 9 Serbian students at the University of Jyväskylä. Similarly to the 2013 program, the visiting students accomplished an intensive seminar, together with Finnish and international students, and took part in a lot of programs connected to the main topics of the Tempus project.

The Tempus Intensive Seminar "VISUAL CULTURE, MATHEMATICS, EDUCATION: Experience-centered Education of Mathematics through Visual Arts and Visual Art Education through Experience-centered Approach to Mathematics" provided an outline of the theoretical background and practice of experience-centered education of mathematics through arts, sciences and playful activities; and art education through the experience-centered approach to mathematics. With a number of guest lecturers we sought answers to the questions of how to integrate pedagogical results of using art, science, and play-centered learning into mathematics teaching programs? How to integrate experience-centered mathematics education into art teaching programs? How to expand the set of tools used for develop a learner's perception, aesthetic sensibility together with increasing mathematical, logical, combinatorial, and spatial abilities, structured thinking skills, motivate collaborative problem solving, interdisciplinary and inter-artistic approaches? How to organize math-art events in any school? The participants of the course became familiar with the basic ideas and main activities of such organizations like the Experience Workshop Math-Art Movement (www.experienceworkshop.hu), a movement advocating experience-centered mathematics education, the Bridges Organization (www.bridgesmathart.org), the world largest mathematics and arts community, and the International Symmetry Association (www.symmetry.hu), a world association for Symmetry Studies. Course participants took part in study visits of local schools and other Finnish institutions of education and culture.

Among our guest lecturers, Fulbright visiting teacher from Chicago, Jeanne Zoellner-Gonzalez talked about how she and her colleagues are working on the establishment a math culture in their Chicago primary school's classrooms. The

LukiMat Project, a digital math-education project for the education of children with special needs was introduced by Jonna Salminen. Stimulating ways of popularizing quantum physics by card games was introduced by the physicists Márton Vargyas. The main concepts of linguistic landscape in education research were introduced by the linguist Tamás Péter Szabó. Juggling and mathematics was connected not only in theory by the Aalto University mathematician and juggler Harri Varpanen. Markus Hähköniemi, mathematician and education researcher talked about the GeoGebra software's classroom implementation in Finnish schools and also held a practical workshop for the students. Erasmus Staff fellow, Gábor Zsolt Nagy from the Budapest University of Technology and Economics talked about ICT and digital media use in education, and introduced several projects including developing curricula and courses for blended learning, learner support with the use of ICT, Open Educational Resources and their use in education and ICT enhanced teaching and learning. Finally, the course was closed by the physicist Prof. Emeritus Matti Leino, who talked about the role of visualizations in the teaching of physics. In the framework of the course, Professor Raine Koskimaa discussed mathematics embeddedness in and the implementation of culture computer games in education. Kristóf Fenyvesi introduced experience-centered methods in mathematics education and compared PISA 2012 results with the results of the Tempus Attitude Survey 2013.

During their 1 month stay the students got an insight into the Finnish culture, the education system and mathematics teaching. They visited two local schools in Jyväskylä's Palokka and Seppälä district to meet with Annuka Talja and Arto Hjelt and their teacher colleagues, who introduced them the Finnish education system in action. In the Art Museum of the city, students met with the curator, Sirpa Turpeinen who introduced the museum's educational projects and the museum's main concepts on museum pedagogy.

Meanwhile students also had time for recreation programs, to enjoy Finnish nature – the group took part at a canoe & sauna tour.

It was an exceptional month, full with great experiences not only for the students but for their teachers as well.

Members of the Tempus Study Visit 2015 program:

- Vladimir Francisti
- Marija Pešić
- Jovana Vujičić
- Ametović Anja
- Tasić Nikoleta
- Slavisa Radovic
- Aleksandra Arsić
- Aleksandra Stevanovic
- Jelena Mataia

Chaos Theory in Max Ernst's Paintings

Nikola Filipović

Chaos theory is the field in mathematics that studies the dynamical development of elements that are highly sensitive in their initial conditions and develop through certain changes. This leads to the variety of possible results in the final condition of their culmination. Meaning that small differences in initial conditions yield widely diverging outcomes for such dynamical systems and minor changes during the developing process could completely change the rest of the developing process. In this way we cannot predict the outcome. Even if the future behaviour is completely determined by the initial condition, with no random elements involved, the prediction of the final outcome can be uncertain or even impossible sometimes. The effect of making the same experiment more times and making some minor changes (willingly or unwillingly), thus getting completely different results is called the Butterfly effect. Mathematically, chaos can be achieved by the simple iteration of certain equations. In the real world, however, it is still in theory stage. Chaologists propose several possible causes of chaos:

- The value of a control parameter is increased to a point where chaotic behaviour sets in.
- The nonlinear interaction of two or more separate physical operations.
- Ever-present environmental noise affecting otherwise regular motion.

Although extreme sensitivity to initial conditions alone does not necessarily lead to chaos, it is a very important feature of chaos, for it is the factor that makes long-term predictions meaningless.

It is very easy and helpful to implement chaos in the art, but the relationship between the chaotic structures and conscious interference can vary. Even though the chaos theory has been stated in the end of the 19th century, artists had already known that by making their objects scatter more randomly, they could get much more natural looking environments, making their paintings more dynamic. However, these methods hardly had any link to the chaotic creation of the elements since these artists consciously constructed and controlled the randomness in which the elements would scatter within the paining. It was only that when Surrealists started developing their subconscious techniques to define an image that the randomness began to emerge on a bigger scale.

There have been many successful attempts to add chaos itself to the art. The most successful one of all was Benoit Mandelbrot who created the world of fractals. Fractals are a very mathematical approach to art and sometimes people question whether it can be called art. It involves patterns of seemingly same shapes that repeat eternally. It is just a matter of taking an object, add a mathematical function to change it's appearance, and apply that function to every new object that appears again. Playing with fractals can lead to very interesting patterns and images. It cannot be created by an artist, it needs to be generated by a computer, therefore most of the people don't consider it an art. It is also very often that you see fractal patterns in nature, since the same forces and weather conditions are shaping the landscapes and mountain ranges for thousands of years, just like the same function is being applied to the given object in a fractal.

It was not until Max Ernst started experimenting with new techniques in order to fully neutralize the human effect in the assembling of an image, so the chance creation can kick in. He had frequently worked in "controlled accidents" that allowed chance elements into his compositions. Along with collage, Ernst added frottage, grattage, and decalcomania to his repertoire and, in each subsequent technical experiments, he allowed chance an ever-greater role. Over the years, his intentional control and force of hand loosened, blurring the line between activity and passivity, eventually arriving at the astonishing level of delicacy shown in such works as Europe After the Rain. His paintings from the late 1930s up to the time of his internment were among his most daring and imaginative experiments. Works from this period in decalcomania such as Swamp Angel and The Robbing of the Bride remain some of his most intriguing and enigmatic.

Max Ernst was a German painter who has been very involved in experimenting with various mathematical ways of creating the base, background and the structure for his paintings. He was born in Bruhl near Cologne in Germany and was the third child to the catholic family. His father Phillip was always a strict disciplinarian and had which inspired in Ernst a penchant for defying authority. He enrolled in the university of Bonn to study fine arts and by the year 1912 he had met with the paintings of the famous painters like Picasso, Van Gogh and Gauguin. He produced his own work and held a few exhibitions before WW1 broke out. He was mobilized and sent to France to fight on the borders. Only after the war had he been released and, influenced by the horrors of war, he started his painting career. He wrote in his journal: "On the first of August 1914 M.E. died. He was resurrected on the eleventh of November 1918." Ernst started off as Dada movement painter experimenting with collage and working with fellow artists like Hans Arp and Tristian Tsara. It was only when the Dada movement started to fall apart and the artists scattered around the world, that he started experimenting on his own and creating his own ideas of art. He was very keen on the surrealist way of viewing the inner world of subconscious and the Freudian theories of the psychoanalytic and he painted in the manner of the most other surrealist painters, but very soon he developed his own means of expression. Ernst continued living as a French citizen and by the time WWII broke out, he was infamous among the Nazi party in Germany for his

Controversial paintings and was sent to the detainment camp. He managed to get out of the camp by his very good friend Paul Eluard and was migrated to America, with the help of Peggy Guggenheim, along with several Surrealist artists. There he married Peggy and had a short marriage with her. A year later he married Dorothea Tanning and moved to Arizona where the built a house and he had a lot of time to devote to his painting, so he created some of his most famous paintings there. There he developed his final technique, Oscillation, which gave birth to the abstract expressionism, which Ernst helped Jackson Pollock to develop. He wrote a book about his paintings called: Beyond painting that gave him public acknowledgement and financial success. In 1953, he returned to France and continued to work there. Max Ernst died on the first of April 1976 and left a fruitful career of experimented art that helped develop different painting stiles.

Max was also a practitioner in mathematics, even in his earlier paintings where he didn't use the automatic techniques that he developed. Both in the terms of terms of content, and techniques, mathematics and geometry can be traced as the base for some of his paintings. It He met with Christian Zervos, a Greek art critic and publisher of the Parisian "Cahiers d' Art" magazine, and gave him series of models representing mathematical surfaces that struck his imagination. As curious objects, reminiscent of algebraic surfaces, appear in several of Ernst's paintings, it is quite possible that he himself made attempts to incorporate them in his works. In "Design in Nature" (1947), a painting heavily influenced by Geometry and carried out using geometric technique, Ernst provides an allusion on the mathematical nature of the world's design. In the background, a variety of shapes and curves give birth to living forms among alternating areas of light and dark. In the colourful "Feast of the Gods" (1948), a collection of surfaces and solids emerge amongst intense colour rays of light, supposedly forming the monumental figures of the gods.

Besides geometry, there can be scene presence of algorithms in some of his works, like the painting "Phases of night". The lush landscape is illuminated by the light of the strange shaped moon and a sinusoidal curved shape casts its shadow. The same trigonometric patterns can be seen in the dotted line that spreads through the lower part of the picture. The most surprising feature of the painting is however the emergence of explicit mathematical symbols next to an. There's an equation that involves the symbol of the heart and the imaginary unit i, or square root of -1, a peculiar number that was invented to aid in the solution for the cubic polynomials in the closed formulas, during the 16th century. One of the greatest mathematicians of all time, Leonhard Euler made a connection of the exponential and trigonometric functions with the numbers containing "i". This connection inevitably leads to what may be considered as the most remarkable equation of Mathematics and which appears in "The Phases of the Night" as the power i^i of complex analysis.

These are the techniques that he developed:

Frottage:

The first technique in painting that he discovered was the most automatic since the painter had least influence on the outcome of the image. The technique consists of taking a piece paper and putting it on the piece of wood with a texture on it. Then rubbing a pencil over the piece of paper, by doing so, Ernst got very interesting textures which reminded him of strange objects, sites, furniture and many other stuff. Max Ernst developed this technique in 1925, when he was lying on his bed in a hotel and looking at the floor. The surface of the floor reminded him of strange shapes and forest ambient textures so he immediately tried to copy them and that's how frottage was born. He regularly shifted the paper while rubbing. Ernst experimented with this technique a lot only with pencil or a crayon because that was the best way for the fine textures to emerge, so this technique resembles drawing rather than painting. After that, Ernst added any type of textured surface, like the glass, fishbone, etc and began on making a composition with them so to get and initial textured background in the image that he later drew the shapes that inspired him the most on the paper. In 1925 he put together a selection of frottages for the portfolio Histoire naturelle. The thirty-four prints in the series were reproduced here as lithographs in order to further underscore the impression of their pictorial reality compared to the original frottages.

Grattage:

This is the further build up to the technique of Frottage and one of the most underrated paining techniques that Max Ernst developed. This technique involves any textured surface, like the piece of wood or the fishbone etc, a canvas and a lot of paint. He put a canvas over the textured surface and added paint to it. He used another canvas to press the paint to the initial canvas in order for the paint to form the texture from the items bellow the first canvas. Then, while the paint was still wet, he took the scalpel or the pallet knife and took off the layers of flat colour above it, so the textured forms emerged on the first canvas. Ernst developed this technique only two years after Frottage, in 1927. It was at this time that he mostly painted forests and wilderness. He used amazing textures that he got from the textured assemblage to create tall trees and shape the creatures in them, but he mostly just indicated their form, rather than describe their details making them completely merged with the background. For him, the forest was the presentation of the human mind. It was under the influence of Freud's theories in which, Ernst was most interested. He sad that forests are like labyrinths: dark, cold and haunting. In almost every paining in the forest, he added a figure of a bird. Ernst said that it was his alter ego in the paintings and he had been very attached to the birds since his childhood. In his early days, he was playing with his parrot Loplop and one day, when his parrot died, he started grieving for him, but five minutes later, his father announced that his sister was born, making it one of the happiest days of his life. Since than he, put Loplop almost everywhere in his paintings. Inner and outer world, the subconscious, fantasy and reality are linked together in Max Ernst's pictorial worlds. This is a characteristic feature of his entire oeuvre.

Decalcomania:

The third technique that Ernst used in his paintings came a bit later, in the late 1903s. This technique offers the largest amount of variation and randomness in the beginning form of the painting. It was, actually developed by another Surrealist artist, Oscar Dominguez. Max Ernst continued to work in this technique and brought it to fame with his surrealist images. It includes a very smooth surface like glass or a smooth table on which he put a thick layer of paint and pressed a sheet of canvas over the painting which gave him very interesting tones and variations in the painting's structure. The colours merged together to form a smooth gradient with different levels of shades and *valers*. Since the paint was wet, it produced thin bubbles of air, rivulets and branching of paint which emerged to the surface when the canvas was lifted from the glass and added much more randomness to the texture. When Ernst looked at that surface he immediately began to imagine different rock formations and human like creatures in them. With this technique he got the most dynamic paintings of all. Not only that he could get the dynamic range of shapes, but the randomness in which they could appear was extraordinary. This technique at it's base completely overlaps with the chaos theory. Only then can the painter bring artistic stamp to the image and dress it in his own style.

The foundation to the similar artistic procedure can be traced to the work of Leonardo da Vinci. He said that when you looked at the wall spotted with stains or with a mixture of stones, if you had to devise some scene, you may discover a resemblance to various landscapes, beautified with mountains, rivers, rocks, trees, plains, wide valleys and hills in varied arrangement." By adding paths and horizon lines, Ernst was able to create the impression of fantastic, lush landscapes and scenes looked amazingly detailed and surrealistically themed. In this stage of work he had a lot of paintings that depicted male figures with bird heads.

One of the things for which I find Ernst's decalcomania amazing is that the link between the conscious and subconscious is so thin that you start to wonder whether he painted some parts with his own will, or it was the result of the automatically random process of smooth glass gradient in the beginning. The thing is that Ernst created amazing geological maps in his paintings, full of monsters and fossils, that in term cannot be distinguished from the randomly added paint when in combination with other objects in the same image. Another lovely thing is not just the way it's painted, but the way that one has to look into it to whether there really is a painted face, or whether he's just imagining it because of the chance shapes that emerged in the initial state. All around his decalcomania paintings, there is a creepy sense of potentiality of the objects, landscapes and faces in the painting.

This is the technique in which we can sense the presence of fractals in the structures of the objects and these paintings are about 30 years before Mandelbrot.

Decalcomania is the most interesting technique to me not only because it allows a completely natural way of creating background, it also allows the greatest extent of artist's interaction with the painting after the decalcomania part is over. I guess that by using the chaos theory in the initial state, the artist can give himself the opportunity to use his creativeness in a much different way than just painting on a blank sheet. The artist starts combining his ability of association with the creativity making the rest of the conscious painting process more dynamic. This technique is hardly used in this world nowadays and it can be a great help for children to practice their associational abilities and creativity.

Oscillation:

The fourth and final new technique that he used was completely had chaos theory in it as a base for creating his fist abstract paintings. It included a pendulum like structure. A thin thread was tide to the can filled with paint on one end and it was tied to some object that can hold its wait on the other. He put a piece of paper below the can and poked a hole on the lower part of the can so the paint can drip steadily. He then pushed the can back and forth so that it started swinging above the paper and randomly dripping the paint over it. The paint created concentric circles exponentially overlapping each other, lines and dots. This technique allowed him to create pictorial worlds of multidimensions that led to the beginning of abstract expressionism. This technique employed fully-automatic function – the law of gravity – that controls the paint, leaving the artist's hand momentarily out of the act altogether.

This technique has been further developed by Jackson Pollock. It did not find a lot of usage in the art afterwards because of its lack of attractiveness to people who don't understand art and to artists who don't understand math. Anyway, it is a good and fun experiment for any artist who struggles to move on from the intuitive old fashioned painting.

For me, Max Ernst is an extraordinary painter who was interested in much more than fields of study than just paining. He studied mathematics and was heavily involved in the books of Freud, as well as the history of German paintings. Most astounding fact is that he was evolving his painting techniques with the flow of the other surrealists and was always one of the most popular people among that group, but still managed to keep his own ways of expression through experiments that he did with his art and techniques that he developed. He had phases in his life where he painted chaotic and disturbing themed paintings but in such settled way that it is always pleasant to look at them because they represent a very high form of art.

I don't know whether he was familiar with the chaos theory or the butterfly effect, but it can be certainly seen in his paintings, which show that he was involved in the other aspects of mathematics.

Short term stay at University of Jyväskylä – Final Project

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[b]University of Novi Sad, Faculty of Technical Sciences, Department for Graphic Engineering and Design, Novi Sad

During the short term stay at the University of Jyväskylä we obtained lots of useful information on how teaching mathematics and related subjects can be improved by visuality. Prior our visit it was clear to us that scientific subjects are not easy to teach, especially if the topics are hard to present. We were pretty much aware that, consequently, it is very hard to attract children and young people to science, since it is usually presented as something very difficult and complex.

In Jyväskylä we visited lots of schools and had a chance to attend math classes. These visits were quite informative, since we were able to see how classes are organized and held and what kind of equipment teachers were using for their presentations. What was common to all of these schools is that they are trying to follow the technology changes and to incorporate the most modern equipment into their classrooms. From the early age children are surrounded by technology, so using laptops, tablets, interactive boards and math software in school looks natural to them and it was adopted easily. Presenting some geometrical problem with GeoGebra software, for example, saves a lot of time during the class since there is no need for further explanation if something is clear at the first sight. In that way, teacher can skip some complex explanation by just showing an image or a model.

Through all what we had seen and experienced one problem often rises – presenting 3D objects in an image. Sometimes this image can be quite descriptive, but usually it lacks a lot of information. If only one plane is taken into account for object description, then whole image of an object is coarsely simplified.

Hence, as a result of our engagement in this project, we created an application that will help teachers presenting mathematics in a more dynamic and visually appealing manner by incorporating some of the 3D modeling and video material.

Application is based on augmented reality and it is made for Android operating devices (tablets and mobile phones). Augmented reality part of our application is made using Metaio technology. Metaio is a SDK (Software Development Kit) that is used to connect computer made data with object in real world and all those data connected are shown on display of android device. Also, application can be made for phones that use iOS operating system.

It connects 2D and 3D presentations of an object and works on a simple principle – user scan printed or presented marker and on his tablet or a phone he is able to see the 3D representation of certain object. Application contains 3D model of himmeli decorations, model of cube and pyramid. Those models were made in software 3DS Max and they are exported in fbx format so Metaio can recognize them. All the markers are drawn in Adobe Photoshop CS6.

Application contains map which is made using Google Maps Android API v2. This API allows developer to offer interactive, feature-rich maps to users of Android application. Application is extendable and consists of few options which will be shortly explained as it follows.

Option 1. Using an image of an object net to present the object in 3D.

This part is intended to help explain the term of an object net. By definition a net is a two-dimensional figure that can be folded into a three-dimensional object. When presenting an object in its unfolded form, some people have a problem of visualizing how the net would look like or, contrariwise, which net belongs to which object. In order to help visualizing the connection of 2D and 3D representation of the same object, we choose two solids – cube and pyramid. Their nets served as markers (Figure 1) and by scanning them corresponding object is shown in 3D space. By moving the device, user is able to rotate and examine the object from each side.

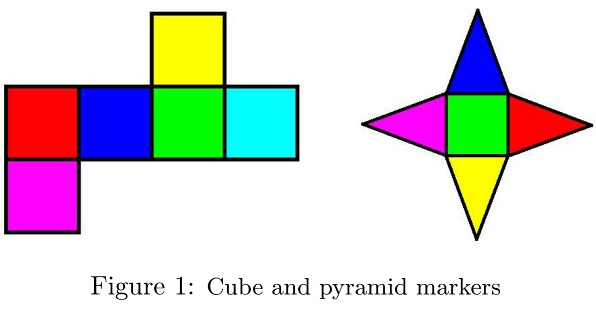


Figure 1: Cube and pyramid markers

Sides are coloured with different hues, so user can monitor how the net is folded and which of the sides are connected together in the 3D object. We believe that this kind of presentation will help people practising their ability to visualize the models and connect 2D and 3D representations of a same object. Model corresponding to the cube net is shown on Figure 2.

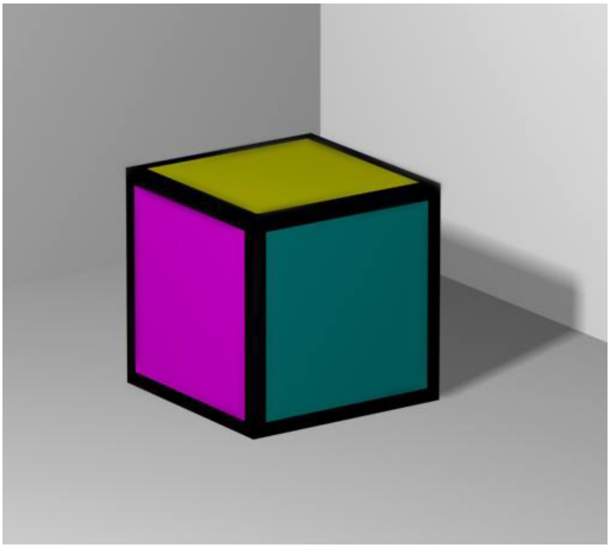


Figure 2: 3D model corresponding to the cube net

Option 2. Using marker to present any object in 3D.

Second part of application is intended to help visualising 3D objects. It use the marker which can be printed in a book, on paper or presented on a screen to show some object in 3D. Similar like in previous case, if user rotates the device, model is rotated too. This will allow user to examine the object from each side and to get a better view of its position in space. In this part we wanted to include something we learned during our short stay. Since we had a lecture about labyrinths we decided to use the labyrinth sign as a marker (Figure 3a). As a 3D model, we made himmeli, a traditional Finnish Christmas ornament which consists of decahedrons, geometric solids with 10 faces and straight edges (Figure 3b).

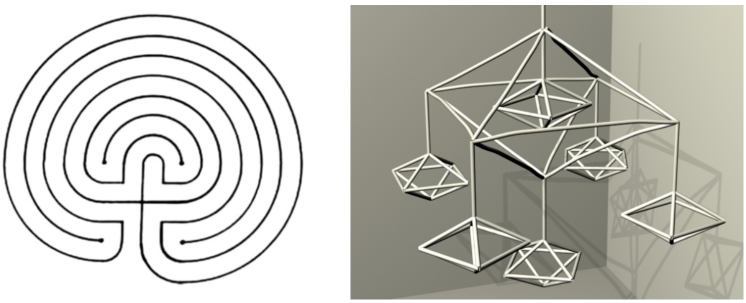


Figure 3: (a) Marker in a form of a labyrinth, (b) 3D model of himmeli

This marker and object are used only as examples and can be replaced by any others. When teaching geometry, for example, teacher can scan circle symbol to present the sphere. Markers can be placed in a math books so that students can scan them on their own when learning the topic of interest. They can also be placed on a screen and scanned during the presentation.

Option 3. Using marker to open video or animated movie.

Since some of the topics cannot be efficiently explained by static images, we also made it possible to open a video or animated movie by scanning the marker. This part follows the main principle explained previously. In this case we insert an image of Jyväskylä Bridge and if scanned it opens the video about the Jyväskylä University (Figure 4).

Figure 4: Image used as a marker and corresponding video

Additional options. Maps and radar.

As additional options of our application we added map and location finder. These are used to help determine current location and to show the closest way to desired education facility.

Application is still in a demo phase, but we believe that once finished it will be very beneficial to both teaching and presenting purpose.

Grid paint

Radenković Miljana, Nikolić Nemanja

Preface

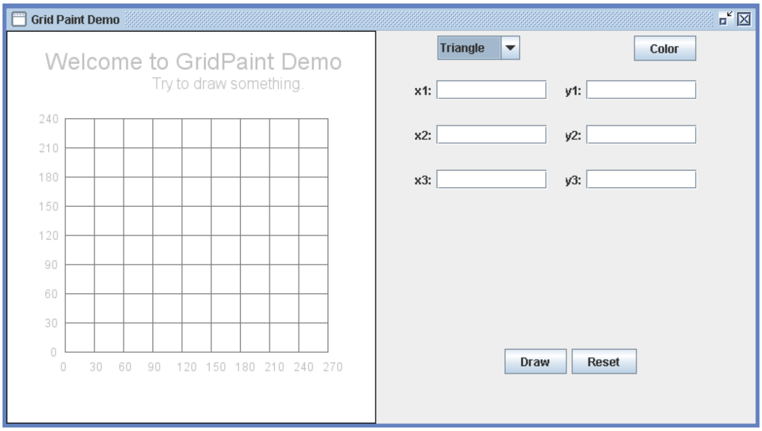
Grid Paint was developed in Java using NetBeans IDE 7.1.2. The goal was to make a connection between art and our programming education. In doing so, this program can deepen understanding of geometrical principles, and also being fun in the process of learning. What we are presenting is the first version of a working prototype, so the user interface isn't optimised for the demography we are aiming for. Nonetheless, we hope that you find the program usefull.

Making of Grid Paint

The core concept of the program is to create a similar environment to that of a game such as Draw Something, where the user is asked to draw an object (a house, a tree, hills, a spoon etc.), but the main difference the ability only to use geometrical shapes placed on a grid canvas. Sadly, lack of time has prevented us from implementing the game logic, only the core program works at the moment.

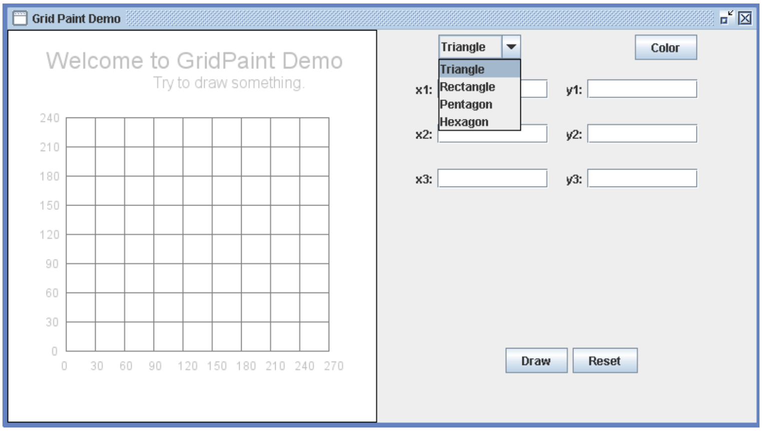
Firstly, since it is a simple program, we opted to use Java as it is multiplatform programming language and its set of libraries efficiently helps with the solving of our development problems.

Program has two main parts. The drawing canvas and the user interface.



The canvas (on the left part of the screen) is the drawing area where, depending on users' choices, the figures are painted.

The user interface (on the right side of the screen) is the area where the user chooses what kind of figure he wants to draw, on what location and what color he wants to paint it.



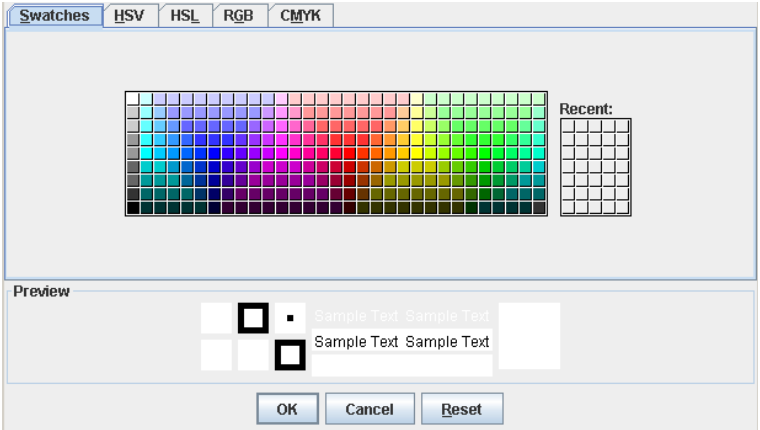
The prototype has four figure types: a triangle, a rectangle, a pentagon and a hexagon. Figure type can be picked from the drop down menu as seen above.

The text fields represent pairs of coordinates, in other words points, of the figures the user wishes to draw. Picking a choice from the dropdown menu alters this panel of information, adding or removing pairs of coordinates depending on the figure.

Clicking the Draw button gathers the information above it and calls the method for drawing on the canvas.

Clicking the Reset button erases everything from the canvas in order for the user to start over.

Clicking the Color button calls a color chooser component, which is implemented with the Swing library of Java. It is a finished component which we didn't alter, and its documentation can be found on the oracle website: `http://docs.oracle.com/javase/7/docs/api/javax/swing/JColorChooser.html`



Visual Mathematics

Jelena Krickovic

Visual way of thinking is considered to be fundamental for mathematics, physics, art and many different scientific disciplines. In art it is always like that, but in mathematics or physics with advanced uses of formulas people start to think like machines and they forget that formulas are there to describe something that is visual in the first place.

In my spare time I give math classes to kids who go to middle and high school.

This course helped me find new ways of explaining certain math problems to my students.

Some middle school students have problem understanding concept of adding and subtracting integers, although it is expected from them to understand that easily.

There are methods that can make this easier for them like: if I loan you 5€, and you give me back 3€, how much do you owe me? This method involving real situation helps kids understand integers.

But I think that is the wrong method. Why not use visual mathematics? I like the method of integer line where all positive and negative numbers are presented. When you are adding positive number you go to the right and when you are adding negative number you go to the left on the integer line from the number given. This way kids understand the concept of negative numbers. My students don't have any more problems with integers when I explain that to them this way, which is a good example of visual mathematics.

Visit to the Finish Secondary school helped me a lot in my work. There is where I found about a program called geogebra. This program has many useful features. One of them is the presentation of the surface of parallelogram. It is visually shown that parallelogram that has one side a and height h has the same surface as rectangle with same dimension, and kids can see animation where part of parallelogram is moved so that rectangle is formed. By seeing this kids will understand this problem.

Not only that, but they can also see how a rectangle with dimensions a and b actually consists of 2 right-angled triangles. Then they can see and understand the formula for calculating surface of a triangle. I try to draw that for them but it is much easier when you have program to do that for you.

Geometry in 3 dimensions is a big problem for primary school students. Although you can try to present a 3D object on the paper it can never be as clear as actually seeing a 3D object. Students have problem understanding those pictures so they usually just learn formulas by heart so they would get good grades without which is wrong thing to do.

It is my opinion that there should be a 3D models making class. Students would make models by themselves so that way, they can see what each object is consisted of. It is very important to visually experience something like that so they would memorize it.

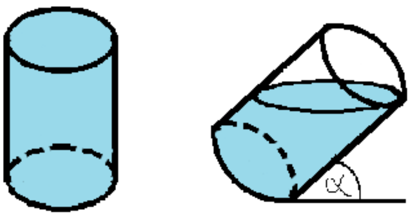
It is a lot easier and more useful for kids to deduct on their own formulas for surface and volume when they know what an object looks like then to memorize formulas they cannot understand. Since I stared using these methods that are more time consuming but involve thinking, the results of my students have improved a lot.

The more things we manage to visualize in math the easier will students gain knowledge and math would be a lot less 'scary' science.

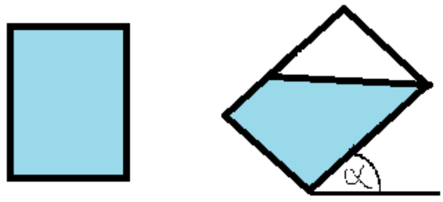
This is one problem where you can visualize and change perspective so that you can solve it in an easier way.

Learning math through games is very interesting thing. Students will like that and they will play these games even more, thus learning more. By interacting with games children will learn much faster.

Problem 1. The glass shaped as cylinder with height b and radius of basis $a/2$ is filled with water and put on the table. Then the glass is tilted at angle of $\alpha = 60°$ (as shown in figure). What is the ratio of volumes of water that poured out and stayed in the glass?



Solution: We can solve this problem by calculating volume of the liquid that stayed in the glass and volume of the empty part of the glass. The problem is that formulas are complicated and there is an easier way to solve this problem. We just need to realize that this problem is similar to the problem shown in the next figure.



Now it's a lot easier to calculate the ratio of surfaces since we only need a formula to calculate surface of triangle and rectangle. Since $\alpha = 60°$, and we take that shorter is a and longer side is b, the surface of the triangle is $a^2/\tan(60°)$, and the leftover surface is then $ab - a^2/\tan(60°)$. This way we get the wanted ratio:

$$\frac{a^2/\sqrt{3}}{ab - a^2/\sqrt{3}}$$

or shorter

$$\frac{a}{\sqrt{3}b - a}.$$

My staying here in Finland and studying visual mathematics will improve my work with students but what is more important the experiences I had here will affect my future specialization and improvement in my studies.

Pythagorean Theorem based on Finland education system

Aleksandra Arsić[a], Aleksandra Stevanović[a], Slaviša Radović[a], Vladimir Francisti[b]

[a]Faculty of Mathematics, University of Belgrade
[b]Faculty of Science, University of Novi Sad

Contacts: arsicaleksandra7@gmail.com, staleksandra@ymail.com, radovic.salvisa@gmail.com, vladimir.j.francisti@gmail.com

Pythagorean Theorem in elementary school

We got a great opportunity to be a part of TEMPUS project "Visuality & Mathematics: Experiential Education of Mathematics through Visual Arts, Sciences and Playful Activities" at University of Jyväskylä in Finland. We chose Finland in the hope of discovering Finland's educational secrets. During one month staying here, we attended predicted courses, visited schools where there were organized round table discussions with Finnish teacher, so we learned and discovered a lot of Finland'S educational system. We also exchanged experiences with teachers from Chicago from which we could compare the advantages, flaws and similarities between educational systems of Finland, America and Serbia.

The Programme for International Student Assessment (PISA) is a triennial international survey which aims to evaluate education systems worldwide by testing the skills and knowledge of 15-year-old students which is repeating on every third year. To date, students representing more than 70 economies have participated in the assessment. From 2001 year, when the results of the first PISA survey were made public, the Finnish educational system has received a lot of international attention.

Until today Finnish results from PISA testing are one of the best in Europe. Some of the reasons for that are following aspects:

- Their schools routinely provide tutoring for weak students.

- Each school has a social worker ("koulukuraattori").
- Substitute teachers are often provided when the teacher is ill.
- Teachers are seldom on strike.
- The methods used for teaching mother tongue are solid. Finnish first graders learn to read first by learning letters, then syllables, then words, then sentences.
- Schools have more autonomy than in many countries. For example, schools can dismiss teachers if they are not satisfied with their work.
- The profession of teacher is more respected than in many other countries.
- Transition from low to high grades of the Finnish curriculum is smoother than in many countries.
- Finnish students have a free canteen at their disposal.

We also think that educational system is not only responsible for their success one of the reasons is their mentality and culture. Finnish people have very high consciousness about importance of knowledge and education. Parents provide great support for schools and they are involved in school activities.

Our experiences from Finland led us to new ideas, made our knowledge richer and opened our mind up to be more innovative and creative. It motivated us and also helped us a lot in realizing our final project. For final project we made a lesson which is directly connected to the school curriculum in Serbia, but the teaching methods are based on Finnish education system. Class is designed to teach students how to apply their knowledge to real-life situations.

Preparation for class

School:	
The subject:	Mathematics
Class:	VII
Date of implementation:	28.05.2014.
Teachers:	Aleksandra Arsić, Aleksandra Stevanović, Slaviša Radović and Vladimir Francisti

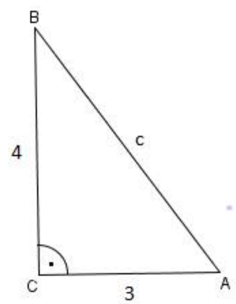
Class Data

Teaching topics:	Pythagorean theorem
Teaching units:	**Pythagorean theorem**
Type of school class:	Teaching new material
The objective of a lesson:	Introducing students to the Pythagorean Theorem Proving these theorems Determining the length of the third page of a right triangle (with the other two known) using the Pythagorean theorem
Educational tasks:	The students should: • adopt the formulation of the essence of the Pythagorean theorem and understand and understand the evidence; • learn how to determine the length of the third page of a right triangle when the other two well-known; • know how to apply the Pythagorean theorem to the computation, and the constructive tasks, especially in solving practical problems; • expand vocabulary of the Pythagorean theorem in English; • develop listening skills and understanding of the song in the English language; • application of the Pythagorean theorem in physics.
Functional tasks:	The students should: • develop the habit of correct labeling and drawing a right triangle; • develop the ability of observation and logical reasoning; • Encourages conversation in English; • develop logical thinking; • develop the skills of the adoption of logical conclusions.

Educational objectives:	Students should develop: • concentration; • ability to persistent and hard work; • gradual and systematic approach to work; • accuracy, precision and neatness in the work; • a positive attitude towards mathematics and appreciation of mathematics as an area of human activity; • self-esteem and confidence in their own mathematical abilities; • develop intellectual abilities; • socialization for group work; • understand of the importance of mutual cooperation and help; • developing a cooperative, systematic and precision; • get the motivation to learn through listening to music.
Key terms:	Pythagoras, the Pythagorean theorem, right triangle, you say, hypotenuse, the other two sides squared, the square on the hypotenuse.
Forms of work:	Group, frontal, paired, individual.
Teaching methods:	Dialogic, illustrative, pilot, work on the text.
Teaching aids:	Board, chalk, accessories for Geometry, paper with tasks, computer, projector, meter, ruler, calculator.
Correlation:	Math 6th grade (Types of triangles) Math 6th grade (Perceptions of congruence of triangles) Math 7th grade (square root) English language, informatics, history and music culture

Lesson

The introductory part of the class: (10 minutes)	Repeating about triangles using the quiz (attach 1). Short story and interesting facts about Pythagoras (attach 2).

The main part of the class: (30 minutes)	Introduction about Pythagoras theorem by interactive GeoGebra applet (attach 3), which will give the idea to students about relationship between squares of the right triangle sides. Definitions and formula of Pythagoras theorem.
	The Pythagorean Theorem or Pythagoras' theorem is a relation among the three sides of a right triangle. It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. The theorem can be written as an equation relating the lengths of the sides a, b and c, often called the Pythagorean equation: $$a^2 + b^2 = c^2$$ Picture 1 (attach 4) Visual proof of the theorem (attach 5). Exercises: 1st task will be solved by teachers with students' discussion. **Calculate hypotenuse length of the right triangle of which sides are shown on picture.**  *The length of the hypotenuse is 5.* 2nd task should check did the students understood the concepts of the Pythagoras theorem, solved by students **The top of the tree is broken at the height of 8 meters. The top of the broken part touched the ground 6 meters from the three bases. What was the tree height before breaking?**

	 *The height of the tree is 18 meters.* 3rd task is real live problem which have to be solved by applying Pythagoras theorem **The dimensions of football field are 120 and 90 meters. Marko should cross it from one to another opposite corner. What is the length of the shortest path?** *The length of the shortest path is 150 meters.*
The final part of the class: (5 minutes)	Listening "The Pythagoras song" (attach 6) and filling out the papers (attach 7). Give the students homework: Using Internet find a different proof then given by teacher. Draw your own right triangle and show that Pythagoreans theorem it is true by cutting and calculating the square areas constructed over the triangle's sides. Prepare a short presentation for next class.

Attachments

Attach 1: Quiz

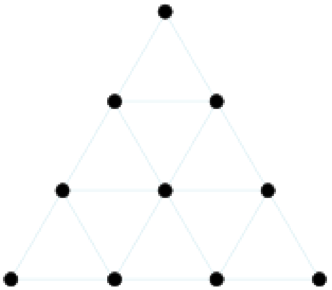
Attach 2: Pythagoras

He was born on the island of Samos, therefore the name Pythagoras of Samos. He was an Ionian Greek philosopher, mathematician, and founder of the religious movement called Pythagoreanism.

The Ionians were one of the four major tribes that the Greeks considered themselves divided into during the ancient period, alongside Dorians, Aeolians and Achaeans.

Most of the information about Pythagoras was written down centuries after he lived, so very little reliable information is known about him.

According to Pythagoras, a perfect number was 10. The "tetractys" or tetrad was also very important object for Pythagoras and his followers. It is a triangular figure consisting of ten points arranged in four rows: one, two, three, and four points in each row, which is the geometrical representation of the fourth triangular number.

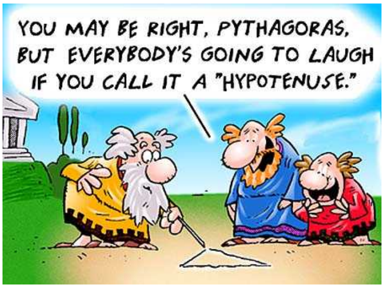


The “tetractys” represented the organization of space:

1. the first row represented zero-dimensions (a point)
2. the second row represented one-dimension (a line of two points)
3. the third row represented two-dimensions (a plane defined by a triangle of three points)
4. the fourth row represented three-dimensions (a tetrahedron defined by four points)

Attach 3: GeoGebra applet

Attach 4: Picture



Attach 5: Visual proof of the theorem
`http://upload.wikimedia.org/wikipedia/commons/6/65/Pythag_anim.gif`
`https://www.youtube.com/watch?v=CAkMUdeB06o`
`https://www.youtube.com/watch?v=uaj0XcLtN5c`

Attach 6: The Pythagoras song

Attach 7: Papers for filling out

There's a question starting in my mind How to find missing _______in right _______Finally I start listening in class And I learn it was easy, use the formula, There are three sides in _______ triangle Side A, B, and C is _______ It's the _______ side between A and B _______ the right angle, _______ degree The notes in my class reminds me of _______ They keep me thinking that I really know it all The notes in my class they help me solve it I can't help singing

Pythagorean theorem: A squared _______ B squared. Is _______C squared and add. Get the square _______, it's the last. Baby I have reminder for you. f missing _______ is A and B_______ it from C Still do exponents, add and _______ root

That's the way you do it Pythagorean theorem

The notes in my class reminds me of _______ They keep me thinking that I really know it all The notes in my class they help me solve it I can't help singing

Pythagorean theorem: A squared _______ B squared. Is _______ C squared and add. Get the square _______ it's the last

Put your heart in everything you do Count your blessings and focus more Turn your book, study and solve Practice, be patient, you will learn it all

Pythagorean theorem: A squared _______ B squared Is _______ C squared and add. Get the square _______, it's the last

Conclusion

Numerous tests show that the Finnish education is one of the best in the world, especially in the PISA results. During the month of stay in Finland, we had the opportunity to attend mathematics classes and school premises, which for us is a valuable experience. It is a great pleasure to have the opportunity to talk with math teachers and to gain a more detailed picture of what is actually Finland education. Motivated thus, we have created a lesson, which subject is the Pythagorean theorem. The teaching approach is different from the traditional, which is common in Serbian classes. The material is correlated with other subjects, such as English language, informatics and music culture. Also at the beginning of the lesson the students will be exposed to some historical information about Pythagoras. During the class they will watch a short video material and use mathematical software GeoGebara. Class is going to end with a song, through which they will repete what they have learn during the class.

Organizing classes and lessons in this way enables that students learn by experiencing education materials. Combinations of multimedia materials can help students to learn new topics with different learning approaches. Students with different learning habits have opportunities to see, listen, experience, and discuss about new material which can greatly enhance their learning process.

Thus, development and implementation of teaching method "learning by experience" that includes presented components will be in the best interest for all who are involved in educational process.

Although the results in Serbian schools does not show signs of improvement for some time, we believe that this type of lessons can make a difference and move the learning approach from traditional to modern, which is closer to the students and easier for them to accept new knowledge.

Mathematics Field Test

Jelena Mataija

ICT Collage

1 Introduction

First of all, I should tell that this idea is extended from the last year Tempus Project from Brussels Belgium, where I have been, along with other Serbian students and artist Rinus Roelofs from Netherland, build the web site where mathematical objects are found and explained in everyday life. The web site can be found on this URL http://bgsajt.com/vismath/. The idea behind this web site was to draw attention, not only to pupils but also to other people, that mathematic is all around us.

The main idea of this Mathematics Field Test is that pupils have the opportunity to explore and see the mathematical object, figures and numbers in everyday life, such as buildings, traffic signs, sidewalks, etc. During the test, pupils will have tablet devices with previously installed application for the test, along with notebooks and pens. By logging into the system each pupil will get the map with locations and routs how to get there and of course tasks for each location. Once the whole test is finished each pupil can see his/hers result and answers which will be marked green if correct and red if not along with correct answer and explanations.

When it comes to teachers, first of all, every teacher will be able to create tasks and pinpoint the locations on the map as they want and therefore to create routs for each test. Second, they will be able to monitor progress on their own tablet device for every pupil and to see on the map where each pupil is. Finally, teachers will see the results and answers for all of their pupils along with the correct answers.

In further text, the application will be divided into two separate views. First view will explain what teachers have to do to create the tests and what will they be able to see and do in the application, the second view will explain what pupils will do during and after the test.

2 First view - Teachers

In this section of the paper I will be explaining the functionality that teachers have in the application and what the teachers themselves are required to do in order tor their tests to work along with the process of registration and creation of classes and tests on the system.

2.1 Teachers registration on the system

When teacher decide to join the platform than he/she first has to register on the system. The registration is very simple; teacher just has to fill in fields such as Name, Last Name, Date of Birth, Town, Country and E-mail.

2.2 Teachers options on the system

After successful registration, teacher will get his/hers place on the system where he/she can:

- Create classes
- Fill classes with pupils and their information
- Create tests for classes
- Monitor pupils aware during the test
- See all results and answers for every pupil
- See the statistics which depends on the items he/she choose to compare

2.2.1 Crating Classes

This option gives teacher the opportunity to create classes. Each class has certain fields which has to be field, such as class name (one teacher could teach more than one class), name of the school (one teacher could teach in more than one school) and class grade.

2.2.2 Filling classes with pupils information

This particular option gives teachers the possibility to fill in the classes, which he/she previously has created, with information of the pupils such as pupils' Name, Last Name, E-mail, etc. After submitting the information, all pupils will get in addition to their information field with their credentials for logging on the system, which are automatically generated in the database.

2.2.3 Creating tests

In order to keep application simple for professors, creating tests will contain two parts:

1. Creating the Test
    - Test Name
    - Class Id (chosen from dropdown menu)

2. Creating the tasks for the Test
    - A frame from google map with location
    - Picture of the object which will take part in the task (to make easier for pupils to find the location)
    - Explanation of what will be the task on that particular object
    - Text of the task
    - Picture of the object for the task (highlighted section of the object for the task)
    - Time needed for finishing the task
    - Correct answer
    - Explanation for the answer

After submitting the test, teacher will have, on his/hers profile, the information about test and tasks along with Test Id which pupils have to choose before starting the test.

2.2.4 Monitoring pupils awareness during the test

This option will contain map of the area where test will take place along with coordinates of each pupil (marked with different colors and their names) in order for teachers to know where all of his pupils are and what tasks they have finished so far.

2.2.5 Results and answers for every pupil

After pupils have finished their tests, teachers will be able to see all the results and answers for each pupil separately, where answers will be marked green if correct or red if not along with correct answer for every task in the test.

2.2.6 Statistics

Teachers will be able to compare results and create statistics which will depend on the items he/she chooses from dropdown menu. Everything will be shown from the graphs and teachers will have liberty to choose the graph type for showing statistics.

3 Second view – Pupils

In this section of the paper I will be explaining the functionality that pupils have and step by step what is happening during the test. For better understanding of this view I will split it up into separate sections:

- Choosing test and logging on the system
- Process of doing the test
- Seeing results and answers

3.1 Choosing test and logging on the system

Before pupils got the chance to do the test they firstly have to fill in the test number, which will be provided by their teacher, secondly pupils will have to log in with their credentials, which will also be provided by their teacher.

3.2 Process of doing the test

Once logged in, pupils will get the message that the test is starting and after clicking on OK button. When test starts, pupils get the map with their current location along with the first location from the test and the route to it. By using the provided navigation, pupil comes to the first location and when application calculates that the pupil is at the right location application sends the notification that he/she is at the right location and the picture of the object that he/she has to find and after clicking on OK button pupil will receive the text for that task. Once pupil finds the object from the picture he/she clicks on OK button and application shows the text for the task along with highlighted section of the object from picture and the timer starts to countdown. Pupil will have the text field to enter his/hers answer and click on Next Task button. The task can be finished either when time ends or pupil clicks on Next Task button or if pupil clicks on I Don't Know button, either way pupil will receive the notification with next location and route to it and his answer will be inserted into database. This process will be repeated as many times as the number of tasks and on the last task there will be only the Finish button instead of buttons Next Task and I Don't Know.

3.3 Seeing results and answers

Once that test is finished pupils will be able to see his/her result and all of his/her answers which will be marked as green if correct and red if not along with correct answers and explanation for each task. Furthermore, pupils will be able to send an e-mail to the teacher and ask them anything regarding the test and tasks themselves.

4 Conclusion

As can be seen and conclude from the text, the main idea of this Mathematics Field Test is that pupils have the freedom to explore and see the mathematical object, figures and numbers in everyday life, such as buildings, traffic signs, sidewalks, monuments, posters, etc. This application not only could be a good way of exploring the city, but also it can be a great physical activity for pupils. When we consider the fact that for most pupils math is a terror, than we could say that this could be one way for breaking this illusion of math, because this application is like a modern version of treasure hunting.

I cannot help but notice that one important question arises from this article and that is whether teachers will be willing to undergo additional work of registering on the system and making the tests themselves? Unfortunately, I believe that most professors would never consider perform this type of test, but also I hope that there are teachers who are enthusiastic and that they would find this application interesting and that they be willing to undergo additional work.

Mathematics in Visual Art

Nikoleta Tasic, Anja Ametovic

Introduction

Our topic will introduce you to the usage of mathematics in visual culture. We will try to explain which forms of mathematics are usually used in art, who used them and why they are important.

When we say the word “mathematics”, people usually get scared. Unfortunately, the first ideas are related to hard equations, big formulas or functions that are impossible to understand. However, mathematic is not used just to annoy children and to make them miserable. It could be really fun. If we looked a little bit closer, we would see that math is used everywhere. In our opinion, the most interesting fact about mathematic is that it is wildly used in art.

We should know that mathematics and art have a long historical relationship. Even the ancient Egyptians and ancient Greeks knew about the golden ratio. In that time math was deeply studied and appreciated. Later, especially in the Renaissance, artists were interested in perspective and perfect proportions. Cubism brought us geometrically shaped objects and modern art showed us how mathematic is used in for example, op art.

In general, all of the master works wouldn’t be that great without the knowledge of math. On this occasion, we would like to introduce you to the most important representatives of different periods in art. We will try to explain why that field of mathematics is studied, why it was important to the painter and what good had it brought to the future art.

Ancient Greece and Egypt

The ancient Egyptians and Greeks were familiar with the golden ratio (in mathematics, two quantities are in the golden ratio if their ratio is the same as the ratio of their sum to the larger of the two quantities). Many artists and architects used the golden ratio as the definition of the perfect proportion and tend to adjust their paintings and sculptures to the golden rectangle. The presence of the golden ratio was mostly found in the ornaments on the Sumerian and Greek vases, Chinese pottery, but maybe the most famous examples would be Egyptian pyramids. Archaeologists found that the height of a pyramid built by Egyptian pharaoh Khufu, divided by half of its base length, makes a ratio of 1.619, which is almost equal to the golden ratio. This suggests that the Egyptians knew about the connection between mathematics and art.

When it comes to Greeks, Parthenon (a temple dedicated to the goddess Athena) is the most famous example. The main decorator of the temple was Phidias, well-known ancient sculptor who was introduced with the golden ratio. In fact, its symbol was named after him -'phi', which is a letter from Greek alphabet. The shape of the temple makes a golden rectangle – a shape whose sides are in the ratio of 1.61. Not only is the complete façade in this proportion, but also the height of the columns and the temple itself and the length of a triglyph compared to the height of the frieze. Also, many buildings of that time have the floor plan of this proportion.

In ancient statuary the mathematical approach is mostly found in works of Polykleitos the Elder, whose male nudes were anatomically close to perfection. His work is not as famous as Phidias, but his approach towards sculpture is more interesting. He even wrote a canon to document the way to represent a perfect human body. Many subsequent artists were following the instructions he noted in this script.

In Canon, the concepts of geometry (especially golden ratio, proportion and symmetry) are thoroughly explained and connected with human form. Among many others, the fact from Canon that most of us know is that the ideal body should be eight heads high and two heads wide.

Byzantine and Islamic art

In the beginning of new era, the appearance of mathematics in art changed a bit. It is mostly found in Byzantine and Islamic mosaics and ornaments which are highly elaborated geometrical structures. Those patterns are usually present in churches and mosques, as part of wall decoration. They appear in diverse geometric shapes and sometimes repeat periodically, which gives them the opportunity of infinite growth. One of the most important geometric properties present in those ornaments is symmetry, both mirror and central, which is often present in circular shapes.

Piero della Francesca

Piero della Francesca (1415-1492) was an early renaissance artist from Italy. He was an expert mathematician and geometer and authored many books on solid geometry and the field of perspective. His famous books are "On Perspective for Painting", "Abaco Treatise", "Regular Solids", etc. He was extremely interested in perspective and this was apparent on many of his paintings. His work influenced other artists, for example Leonardo da Vinci. He was a supporter of linear perspective. Linear perspective is a type of perspective used by artists, in which the relative size, shape, and position of objects is determined by drawn or imagined lines converging at a point on the horizon. In a simpler language, objects are painted as the eye sees them (objects are smaller as their distance from the observer decreases). The point was to gain an illusion of the natural depth. Before this became popular in art, artist had sized objects and figures according to their thematic importance. Important paintings of Piero della Francesca are: Flagellation of Christ (1460), Double portraits of the Dukes of Urbino (1466), The history of the true cross (1466), Brera Madonna (1474), etc.

Leonardo da Vinci

Leonardo da Vinci (1452-1519) was an Italian Renaissance polymath. He was a painter, sculptor, mathematician, musician, anatomist, geologist, cartographer, etc. He studied linear perspective to the every detail and combined it with his studies of light. He imagined rays extending from the body he was observing toward one of his eyes. In his imagination he could put a piece of paper between the eye and the object. The rays would then pierce through the paper. Using this method he could get the important points for the proper perspective on the paper.

He was also interested in geometry. His most outstanding polyhedral accomplishment is the illustrations for Luca Pacioli's 1509 book "The Divine Proportion". These are the first illustrations of polyhedra ever in the form of "solid edges." The solidity of the edges lets one easily see which edges belong to the front and which to the back,

One of his most important works is Vitruvian Man. This drawing which is in ink and paper is called The Canon of Proportions. The drawing is based on the correlations of ideal human proportions with geometry described by the ancient Roman architect Vitruvius. Vitruvius described the human figure as being the principal source of proportion among the Classical orders of architecture. Other artists had attempted to depict the concept, with less success.

Mona Lisa, which represents his most popular work, shows us that Leonardo's perspective was deeply studied. The difference between the left and right background creates the illusion of perspective and depth. This painting is also a representative of the golden ratio. If we draw a rectangle which starts from her right wrist to left elbow and ends in the very top of her head and divide it into smaller golden rectangles, we will be able to draw a Golden Spiral. The edges of every golden rectangle intersect the focal points of Mona Lisa – chin, eye, nose and upper corner of her mouth.

In the Last Supper, Leonardo created a perfectly symmetrical composition. He did intensive studies on how the characters should be arranged at the table. He managed to create a perfect balance between the characters and the background. In the composition we can also notice a presence of the linear perspective.

Albrecht Durer

Albrecht Durer (1471-1528) was a German Renaissance painter, engraver, printmaker and mathematician who made a distinctive contribution in use of geometry in visual art. His most famous theoretical works are “Four books on measurement” and “Four books on human proportion”. The first book of the series on measurement is about linear geometry; the second moves onto two dimensional geometry, the third explains how those principles are used in real life, mostly architecture and typography; the fourth unites the themes of the first two and elaborates three-dimensional forms. The main subject in the whole series is linear perspective. Durer also had a great interest in typography. His aim was to construct mathematically perfect Latin alphabet.

The first book of the series on human proportion shows five most usual types of human figure, both male and female. In this chapter, the bodies are divided in fractions of the total height (e.g. the average height equals eight heads). In the next book, the subject is expanded to eight different body types. The third book explains how the proportions of the figures can be modified, mostly using a simulation of convex and concave mirrors. The last one is devoted to the theory of movement.

The most famous work by Durer which depicts his knowledge on geometry is Melancolia I. In this engraving we see a sad and worried man lost in his thoughts and a strange geometric body beside him. This is called a rhombohedron and it is rotated so that it could be inscribed in a sphere, but only from this point of view.

Hans Holbein

Hans Holbein the Younger (1497-1543) was a German artist who worked in a Northern Renaissance style. His famous painting “The Ambassadors” is somehow related to mathematics. A painting is very interesting because of many details which caused a long debate that lasts for centuries. However, for our topic, the most important symbol on the painting is anamorphic skull positioned on the bottom. On the first sight, this stretched thing looks like a mistake. However, Holbein had used anamorphosis to paint the skull. This is a distorted projection or perspective requiring the viewer to use special devices or look from a specific angle to reconstitute the image. If a viewer looks in the painting from an accurate angle, he will be able to see a skull. The painting was meant to hang in the stairwell, so that persons would be able to see that skull reminding them on a death while walking down the stairs.

Andrea Pozzo

Andrea Pozzo (1642-1709) was an Italian Jesuit brother, baroque painter and architect, best known for his illusionistic frescoes in Sant’ Ignazio church in Rome. The ceiling, while flat, observed from the marked point on the floor, seems endless and decorated with three-dimensional statues. It is very hard to determine where the surface actually begins. When seen from another point of view, it is clear that the vault is even, and the paintings appear slightly deformed. Another interesting fact about this masterpiece is the absence of the dome, which is impossible to notice viewed from the inside. On the place where the planned dome should have been built, Pozzo painted an illusionistic one. It is very difficult to distinguish whether it is real or not. This kind of perspective is called illusionary perspective, and it could be achieved through size, color, lighting, and linear perspective. Many other painters such as Titian used this kind of illusion in their works (e.g. the subjects 'escape' from their frames).

Georges Seurat

Georges Seurat (1859-1891) was a French post-impressionist painter and also an inventor of new genre – divisionism (pointillism). He painted with tiny dots of different color side by side in order to form a picture. In the pointillism, the color dots are blended in a viewer's eye, instead of mixing colors with a brush. Essentially for this discovery, Seurat had used knowledge from chemistry more than mathematics. However, he used a golden ratio in many of his paintings. In all the paintings analyzed, the golden ratio lines are simply based on the height and width of the canvas to eliminate any possibilities that the dimensions analyzed were based on arbitrary points. In the following paintings he used that principal: "Brother at Asnieres", "Bridge of Courbevoie", sketches for "La grande Jatte", "The Parade", etc.

Pablo Picasso

Pablo Picasso (1881-1973) was a Spanish artist, known for co-founding the Cubist movement. He tried to paint a 4D painting on the 2D canvas. For example, on his painting "Demoiselles D' Avignon" we can see disordered and somehow disturbing perspectives that present female faces displaying simultaneously a side and a frontal view. Important thing in cubism was to represent multiple viewpoints by combining different perspectives and looking from different angles. Cubism represented a base for the abstract geometric art. His most important paintings are: "Guernica", "Three Musicians", "Seated Women", "The Dream", "Portrait of Dora Maar", etc.

Piet Mondrian and Kazimir Malevich

Piet Mondrian (1872-1944) was a Dutch painter who was important contributor to the De Stijl art movement group. He evolved a genre in art called neoplasticism. It consisted of white ground, upon which was painted a grid of vertical and horizontal black lines and the three primary colors. He wanted to present the world in "absolute reality" – a world of pure geometry. By using shapes and lines, he wanted to evoke a feeling and sentiment. It is believed that he often used the Golden Ratio in the elaboration of some of his works.

Kazimir Malevich (1878-1935) was a Russian avant-garde artist. Like Mondrian he was a supporter of geometrical abstraction. He worked in a style that he called Suprematism. He created nonobjective compositions of elemental forms floating in white space. He wanted to achieve the higher absolute reality, which he called “the fourth dimension”.

M. C. Escher

M. C. Escher (1898-1972) was a Dutch graphic artist, best known for his mathematically inspired graphics, impossible constructions and tessellations. His works represent the use of mathematics in art in a best possible way.

“Still life and street”, 1937, was his first print of impossible reality. He enjoyed creating images from his own mind rather than visualizing something that already exists. Some of his most famous works are “Drawing hands” (lithograph), where the transition from 2d to 3d is shown; “Sky and water”(woodcut) is the example of transition from one shape to another, in this case from a bird to a fish, so that all the shapes fit into each other like a jigsaw puzzle. Also, getting closer to the center of the plane, the shapes become more flat and amorphous, and their three-dimensionality is increasing as they move towards the edges.

Escher did not have any professional knowledge in math, he understood geometric principles exclusively visually. The most used impossible objects in his works are the Necker cube and the Penrose triangle. He also used polyhedra and geometric distortions. Many of his works also include series of repeated tiles called tessellations, where each objects perfectly fits those next to it, leaving no empty space in between.

“Relativity”, 1953, is his most famous lithograph depicting an impossible construction. It shows a space where the normal laws of gravity don’t apply. Also, the stairs could be used for both climbing and descending from the same direction.

Salvador Dali

Salvador Dali (1904-1989) was a Spanish surrealistic painter who incorporated mathematical elements in several works. The most famous is “Corpus Hypercubus” or “Corpus Christi” which shows the crucifixion of Christ on a hypercube. In “The sacrament of the Last supper” the subjects are set in a huge dodecahedron. This proves that Dali was well introduced with geometry.

Victor Vasarely

Victor Vasarely (1906-1997) was a Hungarian artist best known as the creator of the op-art (optical art) movement. As the word itself says, it is the art created by optical illusions. While looking at those kind of paintings, the viewer gets an impression of movement, flashing, warping etc. The greatest part of those art pieces is made in black and white.

Among Vasarely's most distinguished works, there are "Zebras", the "Gordes / Cristal" series, kinetic art series, "plastic alphabet" series, "the tribute to the hexagon" series. Even though it looks digital because of its geometrical perfection, all of his work is made by hand.

Implementing financial literacy in mathematical class

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Introduction

"Financial literacy is a knowledge and understanding of financial concepts and risks and the skills, motivation and confidence to apply such knowledge and understanding in order to make effective decisions across a range of financial contexts to improve the financial well-being of individuals and society and to enable participation in economic life."

The idea of this project is to create interesting and interactive way of making students more familiar with the terms of economy and finance in both elementary and secondary school, and in this way to motivate them to realize how omnipresent mathematics is in everyday life. Our work is based on PISA 2012 research and test, to which we added our solutions.

Existing evidence shows that adults in both developed and emerging economies who have been exposed to financial education are more likely than others to save and plan for retirement. This evidence indicates that improved levels of financial literacy can lead to positive behavior-change. There is evidence that those with higher financial education are better able to manage their money, participate in the stock market and perform better on their portfolio choice. Moreover, those who have greater financial knowledge are more likely to accumulate higher amounts of wealth.

In addition, financial education is important to economic and financial stability for a number of reasons: consumers can make more informed decisions and demand higher quality services they are less likely to react to market conditions in unpredictable ways, more likely to take appropriate steps to manage the risks, etc.

Financial literacy is considered to be an essential life skill, and it is advised that financial education should start at school. Younger generations are more likely to have to bear more financial risks in adulthood than their parents and because of the changes in the marketplace and social welfare systems they are unlikely to be able to learn from past generations. It is therefore important to provide basis of financial literacy at an early age. In addition, children are often consumers of financial services from a young age. It is not uncommon for them to have accounts with access to online payment facilities or to use mobile phones with various payment options even before they become teenagers, and it is clear that the financial literacy skills would benefit them using such products. It is important for people to be financially literate before they engage in major financial transactions and contracts.

Considering above given facts and the strong connection between mathematics and finance, we believe that implementing basic financial concepts into mathematical classes would benefit both higher motivation in learning mathematics and increase in financial literacy.

Content

The content of financial literacy consists of the areas of knowledge and understanding that one must use in the order to perform a particular financial task. We consider three content areas for financial literacy to be important:

1. money and transactions,
2. risk and reward and
3. planning and managing finances.

For each of these three areas we will introduce theoretical base followed by typical assignment on PISA test and our idea for an interactive class considering this topic. We created three different concepts of mathematical class, one for each student age: 1. 1-4th grade, 2. 5-8th grade and 3. high school.

1 Money and Transactions

This content area includes the awareness of the different forms and purposes of money and handling simple monetary transactions such as everyday payments, spending value of money, bank cards, cheques, bank accounts and currencies. Task in this content area can, for example, ask students to show that they[1]:

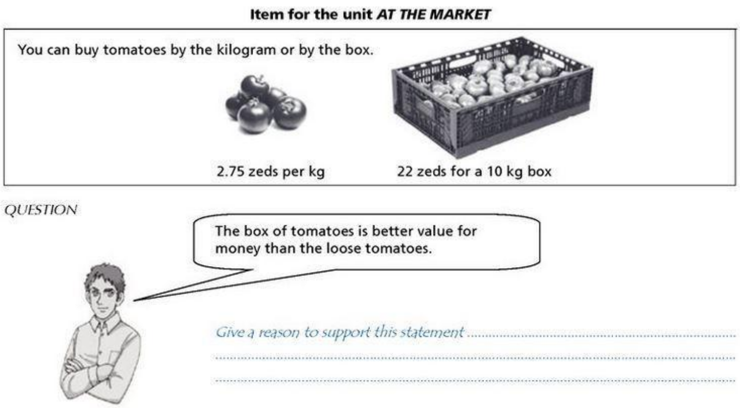
- Are aware of the different forms and purposes of money:
  - Recognize bank notes and coins;
  - Understand that money is used to exchange goods and services;
  - Can identify different ways to pay for items, in person or via Internet;

[1] OECD (2013), "Financial Literacy Framework", in PISA 2012.

- Recognize that there are various ways of receiving money from other people;
- Understand that money can be borrowed or lent, and the reasons of paying or receiving interest.

- Are confident and capable at handling and monitoring transactions:
  - Can use cash, cards, and other payment methods to purchase items;
  - Can use cash machines to withdraw cash or to get an account balance;
  - Can calculate correct change;
  - Can work out which of two consumers items of different sizes would give better value for the money;
  - Can check transactions listed on a bank statement and note any irregularities.

Example from PISA 2012 test:



Our suggestion for an interactive class (1-4th grade):

This class is envisioned to introduce the terms of money and transactions to students at young age through playing a game. Teacher is supposed to bring fake money, pricing lists for many different items and to-do-lists which he/she should distribute to students at the beginning of the class. In addition, he/she should give various roles to them. One half of the class is supposed to be sellers at: flower shop, market (fruit, vegetables, meat, eggs…), bakery, pharmacy, ice-cream stand, etc. The other half is buyer. In the middle of the class these roles should switch. Every student from a group of buyers is given a certain amount of fake money and to-do-list, while every student from group of sellers is given matching pricing list. The game can now start.

The aim of the game is to successfully fulfill to-do-list with the accent on regular exchange of money and goods during each sale. Students which are given the role of buyers should now find the needed items from the list and if there are two sellers of the same item to compare which one has better price. Buying or selling items from the lists will develop calculus skills but also help them understand that money is used to exchange goods and services and make them familiar with the value of it.

2 Risk and Reward

Risk and reward is a key area of financial literacy, incorporating the ability to identify ways of managing, balancing and covering risks and an understanding of the potential for financial gains or losses across a range of financial contexts. There are two types of risk of particular importance in this domain. The first relates to financial losses that an individual cannot bear, such as those caused by catastrophic or repeated costs. The second is the risk inherent in financial products, such as credit agreements with variable interest rates, or investment products.

This content category includes[2]:

- Recognizing that certain financial products (including insurance) and processes (such as saving) can be used to manage and offset various risks (depending on different needs and circumstances):
  - Knowing how to make assess whether insurance may be of benefit.
- Applying knowledge of the ways to manage risk including the benefits of diversification and the dangers of default on payment of bills and credit agreements to decisions about:
  - Limiting the risk to personal capital;
  - Various types of investment and saving vehicles, including formal financial products and insurance products where relevant; and
  - Various forms of credit, including informal and formal credit, unsecured and secured, rotating and fixed term, and those with fixed or variable interest rates.
- Knowing about and managing risks and rewards associated with life events, the economy and other external factors such as the potential impact of:
  - Theft or loss of personal items, job loss, birth or adoption of a child, deteriorating health;
  - Fluctuations in interest rates and exchange rates; and
  - Other market changes.

[2] OECD (2013), "Financial Literacy Framework", in PISA 2012.

- Knowing about the risks and rewards associated with substitutes for financial products; in particular:
  - Saving in cash, or buying property, livestock or gold; and
  - Borrowing from informal lenders.

Example from PISA 2012 test:

Item for the unit *MOTORBIKE INSURANCE*

Last year, Steve's motorbike was insured with the PINSURA insurance company.
The insurance policy covered damage to the motorbike from accidents and theft of the motorbike.

QUESTION

Steve plans to renew his insurance with PINSURA this year, but a number of factors in Steve's life have changed since last year.
How is each of the factors in the table likely to affect the cost of Steve's motorbike insurance this year?

Circle "Increases cost", "Reduces cost" or "Has no effect on cost" for each factor.

Factor	How is the factor likely to affect the cost of Steve's insurance?
Steve replaced his old motorbike with a much more powerful motorbike	Increases cost / Reduces cost / Has no effect on cost
Steve has painted his motorbike a different colour	Increases cost / Reduces cost / Has no effect on cost
Steve was responsible for two road accidents last year	Increases cost / Reduces cost / Has no effect on cost

Our suggestion for an interactive class (5-8th grade):

This class is envisioned to introduce the terms of risk and reward through playing a game. Every student is given a form with three empty boxes. Teacher is telling a story which consists of several parts. After every part he/she stops and gives students few minutes to make a decision between two given possible choices. Every choice is based on whether or not student will risk. We are giving one example of this story:

"It is a lovely Sunday morning. Ben woke up with a big smile on his face. Today his favorite football team is playing a game in his town. Also he and his friends are going to the open cinema tonight. However, he has few important decisions to make. First decision is whether to buy a ticket for the football game online (100 zeds) or in front of the stadium right before the game (120 zeds). He considers this second option because the policy of the stadium is to let fans for free if less than the half of the tickets is sold. What should Ben do?" Now the teacher gives students few minutes to think about this question and fill in the first box. The story continues:

"On his way to the football stadium, Ben wants to buy some snacks. There are two stores in the neighborhood. First store is open 24 hours and the price of Bens favorite snack is 15 zeds. Second store is cheaper, snack is 10 zeds, but Ben is not sure if it is open on Sunday. If he decides to go to this store and it turns out that it is closed, he has to take a taxi (20 zeds) to the first store in order to make it on time for the game. Which store should Ben choose?" Again, teacher gives time to students to make a decision and to fill in the second box before she continues:

"Football game is over and Ben is happy because his team won. He is even happier because tonight he is going to the open cinema with friends (tickets are already bought for 50 zeds each). Unfortunately there is a 50% chance of rain. They can return tickets immediately (30 zeds are refundable) and spend the night at friends' house watching another movie. If they, however, risk and decide to go to the cinema, two scenarios are possible: if it rains movie is canceled and the whole amount of money is refunded but it will be too late to watch a movie at friend's house and they all go home sad, if it doesn't, they will watch the movie as planned. Should they return tickets immediately or not?"

After another short period of time for thinking about this question, teacher reveals to the students the hidden facts of the story (is the cheaper store closed on Sundays, will the stadium be half empty and is it going to rain). Now they know if they made right decisions or not and they can discuss with the teacher what were the right answers if they exist.

3 Planning and Managing Finances

Income and wealth need planning and managing over both the short term and long term. This content area includes[3]:

- Knowledge and ability to monitor income and expenses:
  - Identify various types of income and measures of income (e.g. allowances, salary, commission, benefits, hourly wage and gross and net income); and
  - Draw up a budget to plan regular spending and saving.
- Knowledge and ability to make use of income and other available resources in the short and long terms to enhance financial well-being:
  - Understand how to manipulate various elements of budget, such as identifying priorities if income does not meet planned expenses, or finding options for reducing expenses or increasing income in order to increase levels of savings;
  - Assess the impact of different spending plans and be able to set spending priorities in the short and long term;
  - Plan ahead to pay future expenses: for example, working out how much needs to be saved each month to make a particular purchase;
  - Understand the purposes of accessing credit and the ways in which expenditure can be smoothed over time through borrowing or saving;
  - Understand the idea of building wealth, the impact of compound interest on saving, and the pros and cons of investment products;

[3] OECD (2013), "Financial Literacy Framework", in PISA 2012.

- Understand the benefits of saving for long term goals or anticipated changes in circumstances (such as living independently); and
- Understand how government taxes and benefits impact on planning and managing finances.

Example from PISA 2012 test:

Item for the unit *SPENDING CHOICES*

Claire and her friends are renting a house.
They have all been working for two months.
They do not have any savings.
They are paid monthly and have just received their wages.
They have made this "To do" list.

To do
- Get cable TV
- Pay the rent
- Buy outdoor furniture

QUESTION
Which of the tasks on the list are likely to need prompt attention from Claire and her friends?

Circle "Yes" or "No" for each task.

Task	Is the task likely to need prompt attention?
Get cable TV	Yes / No
Pay the rent	Yes / No
Buy outdoor furniture	Yes / No

Our suggestion for an interactive class (high school)

This class is envisioned to introduce the terms of planning and managing finances to students through playing a game. At the beginning of the class students are divided into three equally numbered groups. Each group is assigned with different material which they use to fulfill the same goal – to calculate how much money they will have after three years' time given that every group had the same amount of money at the beginning. This material consists of two parts where the first part contains theoretical basis of needed financial terms and the second part the actual assignment.

For example, first group is introduced with investing in a bank and buying actions. They are supposed to invest the given amount of money in a bank and after one year time withdraw the money – in this way they need to calculate how much money they will withdraw. After that, they are buying actions with the new sum.

Second group is introduced with the terms of opening a new company and profit. They are supposed to invest the given amount of money into buying machines and materials for the production of the goods they will sell. Furthermore, they need to calculate how much money they will earn each month considering that they need to pay the workers and buy new materials – they are calculating the profit. Finally, third group is introduced with the terms of raising loans and investing in real estate. They are supposed to raise a loan on certain amount of money from a bank and together with the amount of

the original, invest it in a real estate. The task is to calculate how much money they will earn monthly from the rent and how much they need to pay the loan installment.

At the end of the class each group will shortly present theoretical basis of their investment and the amount of money they earned after three years. After presentations they will all together compare the results and decide what the best way of investing money in this particular case is. However, teacher should emphasize that depending on the interest rates, costs of material and machines, costs of real estate and actions these results can drastically change.

Conclusion

As the participants of the Tempus project 2014 ”Visuality and Mathematics” we realized the importance of interesting and well prepared mathematical classes. During our one month visit to Finland we had the opportunity to attend numerous lectures about popularizing mathematics, to see up close the Finnish education system and participate in GeoGebra workshop.

Making children more interested in mathematics through art, nature and computer science is good and successful way. However, we believe that financial literacy is just as equal or even better way, because most certainly every single child will sooner or later have to deal with investing, managing money and savings, bank credits, etc.

Organizing economy based mathematical classes which we introduced in this project, doesn’t require big investments in terms of money and technical support, but it fully depends on willingness of the teacher. This is the reason why we believe that implementing this kind of classes is possible even in undeveloped countries such as our country, Serbia.

Moments from the Short Study Visit in Eger

Ilona Oláhné Téglási

Course coordinator, Eszterházy Károly College, Eger, Hungary

From 15th April to 15th May nine Serbian students came to Eger, to join the „Visuality and Mathematics” course of the Tempus project. Also 24 from the mathematics students of the Eszterházy Károly College took this course, so we had 12 seminars and lectures during the month together. These were the following:

1. Visual communication lesson in secondary school – visit and discussion.
2. Descriptive geometry lesson in secondary school – visit and discussion.
3. Problemsolving in mathematics.
4. Optical illusions – the psychology of seeing.
5. ZomeTool and nanotubes – a useful kit for teaching geomerty.
6. Spatial abilities and their evaluation –tests and tools.
7. Music and mathematics.
8. Visual arts and mathematics from the viewpoint of an artist.
9. Computer graphic softwares.
10. Planning a ZomeTool construction.

11.–12. Building the ZomeTool construction.

The Serbian participants were the following:

Jelena Anđelov	ICT	Marija Radojicić	MISANU
Anželik Anđelov	ICT	Aleksandra Stevanoviś	MISANU
Jovan Petković	ICT	Neda Milić	UNS
Darko Veselinović	ICT	Visnja Stefanović	UNS
Slaviša Radović	MISANU		

The Hungarian students were: Aux László, Bárdi Olívia, Buhala Renáta, Farkas Gábor, Ferencz Zoltán, Gujdi Friderika, Juhász Alma, Katona Tibor, Király Dávid, Kormos Nóra, Kotroczó József, Kovács Balázs, Kovács Péter, Macsinka Gábor, Madarasi Andrea, Molnár Anita, Molnár Bernadett, Osváth Nikolett, Páll Gréta, Pesák Dóra, Telek Zsuzsanna, Szokoli Kinga, Varga Katalin, Virág Csaba.

During the lectures and seminars we got acquianted each other, and could see many different viewpoints of the connection between mathematics and visuality. The main task of the course was to show how visual arts and visualizing problems can help in the development of mathematical competencies, how we can recognize the mathematical structures in artistry, and how we can use it in school. We worked out a project with ZomeTool kit, and at the end of the course we built up a special construction we planned together. The topic was nanotubes, which is a follow up of our previous construction from 2011, a hugh Buckminster fullerene (see: `http://www.youtube.com/watch?v=TEH5XUKB3tc`). The one we created now is "Species of Carbon". It is a 2 meter high „sculpture": the base of it is 2 streaks of graphene plane, on which 3 carbon nanocapsules stand, forming a tetrahedron. Inside the tetrahedron there's a diamond structure, and on the top of the nanocapsules a hugh fullerene molecule. Each capsules are different: one is single-walled, the other is double-walled, and in the third there are 2 smaller fullerene molecules. As we have learned a lot about spatial sight, problemsolving, visuality and tried out different structures from ZomeTool, it was a great job planning and building this „sculpture" – I think, everyone enjoyed it.

Besides the serious work we had several other activities (Serbian and Hungarian students and our professors together). We visited two interesting exhibitions: the "Loss of Sight", which was about how we feel and get orientation when we can't see anything, and the Kepes György Institute with the exhibitions of 3 artist, all of them use many mathematics in their artistry.

As Eger has got many interesting places to visit, and also the sorrounding of the town is rich in sights every day had its own programme: the castle, the Lyceum, Basilica, the Bishop's Park, the Thermal Spa, wine cellars in the Valley of the Beautiful Woman, etc. Our leisure time activities together were the College Days, an excursion to Felsőtárkány (with cooking gulyás outside), and a Farewell Party in the Students' Club.

As you can see from the photos, we had a useful and wonderful month together. It was the "beginning of a beautiful friendship"... We are looking forward to the Summer School where we shall hopefully meet again.

Serbian Students in Eger 2.0 – Report

Ilona Oláhné Téglási

Course coordinator, Eszterházy Károly College, Eger, Hungary

In 2014, from 17 March till 16 April nine Serbian students visited Eger for a Short Study visit. They participated in the course „Visuality in Mathematic Education".

The students from Serbia were:

Anett Arc	BMU
Nina Tomović	BMU
Ljubica Srećković	BMU
Stefan Hačko	UNS
Evelin Kurta	UNS
Miroslava Pušin	UNS
Marina Jokić	UNS
Ružica Radović	MISANU
Jovan Petković	ICT

The programme of the course was a wide range of possibilities how we can make the teaching of Mathematics more motivative and interesting. Our main topic this year was tiling and tessellations, most of the lectures and workshops dealt with this.

The first lecture was about general problemsolving methods, some theoretical background, why visualization is important in teaching Mathematics. Then the students got acquainted with different ways of visualizing mathematical problems: Emőd Kovács introduced the pupils with POV-Ray program, Ilona Téglási and Gábor Geda with the useful GeoGebra, and we also worked with Zometool modeling kit. The students visited a descriptive geomerty lesson in the practical school of the Eszterházy Károly College, where Istvánné Szilvási showed us, how they use Geogebra in real teaching situation. We've heard an interesting lecture from Ferenc

Mátyás, who showed, how tiling can be used to prove a theorem in Number Theory. Tibor Juhász showed us, how algebraic group symmetries appear in Hungarian folk needleworks. Erika Gyöngyösi made a workshop about tetris, pentomino and tangram games, where everyone could try out these games. At the end Ilona Téglási gave a lecture about optical illusions, the perception of seeing – with some examples, how visual artists use such illusions in their artistry. The course also had a goal to organize a small exhibition from the works of the students – they were very creative to use the methods and knowledge they've gained during the course. As an illustration you can see one of the products, made by Anett Arc, Ljubica Sreckovic and Nina Tomovic.

Besides the official programs, the students visited different interesting places of Eger: we discovered the town from above in the tower of Lyceum (the main building of the College), visited the Archiepiscopal baroque Library, the Chapel, the Astronomical Museum, and played in the Magic Tower with interactive physical experiments. We visited the Orthodox Serbian Church, which was very interesting, with its nice wooden and golden altar screen. The students could join our Erasmus students in a trip to Budapest, and another to Hollókő. At the end of the month we had a farewell wine-tasting evening at Zsálya Restaurant.

This year, I think it turned out again after last year's successful course, that those students, who undertake such a challenge to learn abroad, in unknown circumstances, are really open minded, creative and inquisitive. The atmosphere was friendly during the whole course, and after getting the students's report I think they really felt well in Eger.

After all, we didn't say good bye at the end, but: **see you in Belgrad this Summer!**

Summary of the evalution sheets of the EU tempus project “Visuality and Mathematics”

Lilian Wieser

Each of the four participating countries offered another study visit program to several serbian students from four academic institutions. Some of the students visited several countries, so those students got a broader perspective to the whole topic of visuality and mathematics. Each country organized two visits. One in 2013 and another one in 2014.

At the summer school at the Eszterházy Károly College in Eger organized by Ibolya Prokajné Szilágyi and Ilona Oláhné Téglási in 2013 for example there was more focus on the arthistorian part of the whole intersection of all the important disciplines interwoven in the topic of the project. But also the educational aspect left quiet an impression.

> *“Since transferring knowledge of mathematical concepts is always a challenging task, as a lecturer, I am excited about incorporating new ideas into my classes.*
>
> *The experience obtained during this short study program will help me providing lessons with new interactive visualisations which can engage students more deeply. The possibility to visualize parameter changing in real time is adding much value and weight to the lesson, where physical variables are no longer just values to find after plugging meaningless numbers into some formula.”**

With Kristof Fenyvesi at the University of Jyvaskyla in Finnland Students had a closer look to modern physics and the finish school system.

> *"During the short term stay at the University of Jyväskylä we obtained lots of useful information on how teaching mathematics and related subjects can be improved by visuality. Using visuality is easier to attract children and young people to science, because it is usually presented as something very difficult and complex."**

> *"Also, by visiting some schools of Jyväskylä, I took some knowledge about Finish education, that we can bring in our University and improve our educational system."**

In Belgium at the Sant Lucas School of art with Dirk Huylebrouck as the coordinatior, they got to know alternative workshops, to facilitate mathematical thoughts through different medias.

> *"The Zome geometry based on golden section proportions and 2-, 3-, and 5-fold symmetries, was used by NASA for a space station project, Nobel Prize Winner for modelling quasicrystals and so many other mathematicians and research scientists for constructing complex geometric forms. Besides scientific applications, Zometool is also used as a completely new medium for creative art expressions."**

The Austrian part led by Ruth Mateus-Berr let the students try a design method to develop educational tools.

> *"The interdisciplinary work and design thinking methods, done at the workshop, helped me to improve a creative process and team building skills. New approaches at the seminar were a great asset to my knowledge and they inspired me for my further research."**

Also a big influence to the project had the different disciplines the students came from. Most of them are studying computer science or mathematics, but they also came for example from graphic design or electrical engeneering. Due to their studies and interests they had different perspectives on the evaluation sheets they filled after each visit.

> *"I study applied mathematics, but also I attend some subjects about theoretical mathematics. On this project I saw how it is important to connect those two things. If you want to transfer your knowledge to others in a very good way you must have good methods for it."**

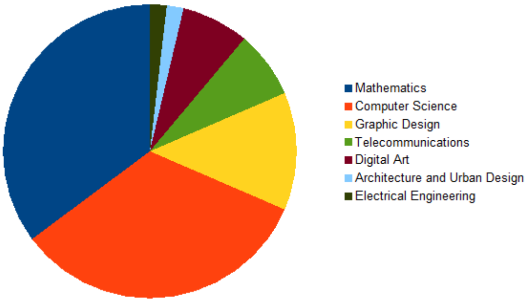


Figure 1: Participating disciplines due to the evaluation sheets

But also the hobbies of the serbian students led to another reflection to the project. One student for example was very thrilled, that he found an academic connection to thoughts he had himself, when he played music.

> *"I played in many orchestras such as school orchestras and also cultural and artistic orchestras. But somehow I was always thinking how mathematics is connected with Arts and that is something which led me to this very interesting VisMath project."**

The same student wrote:

> *"The thought of attending such a project and also to be part of such team for me was like a dream come true."**

In general the students were amazed about this connections they didn't think of before and the facilitation, that not only visuality brings to mathematics, but also the other way around. After reading the evaluation sheets, it seems that this interdisciplinary project were strongly needed by all participants, students and professors. It opened a field, that will lead to useful things for humanity.

> *"The experience and knowledge I got from this project is priceless. I would be very glad if you will in the future realize this project and if I could participate again."**

* All quotations are chosen to display the reaction of the students to the project. They are taken from the evaluation sheets which were filled by them after all visits.