The Closing Conference of the project Visuality & Mathematics

Belgrade, Serbia, September 17–19, 2014

This project has been funded with support from the European Commission.

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Project number: 530394-TEMPUS-HU-TEMPUS-JPHES

Title of the project: Experimental Education of Mathematics through visual arts, sciences and playful activities

Project coordinator: Eszterházy Károly College

Partners: University of Jyväskylä, Belgrade Metropolitan University, University of Novi Sad, Serbian Academy of Sciences and Arts, ICT College of Vocational Studies, Sint-Lucas School of Architecture, University of Applied Arts Vienna

ISBN: 978-615-5297-26-7

Technical editor: Tibor Tómács

Published by
Eszterházy Károly College, Eger, Hungary

Printed by
Kvíz '96 Nyomdaipari és Kereskedelmi Kft., Felsőtárkány, Hungary

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About project

With the cooperation of eight institutions in Austria, Belgium, Finland, Hungary and Serbia the Tempus project Visuality & Mathematics – Experimental Education of Mathematics through Visual Arts, Sciences and Playful Activities supported by the European Union was launched in autumn, 2012. Hungarian, Austrian, Finnish, Belgian and Serbian partners united in mutual understanding to enhace Serbian national education in a specific way. The project „Visual Mathematics" focuses on inter- and trans-disciplinarity: its main goal is to make mathematics a more appealing subject for students through connecting mathematics with arts. The project is about making mathematics more visual and adventurous by illustrating Math with the tools of arts. We specifically wished to give some tools for visual mathematics tuition for Serbian educational institutions, to allow non-formal methodological training of Serbian undergraduate students, to make teachers and students acquainted with teaching and learning methods of visual mathematics at two international summer universities. Training a new generation to accomplish the prerequisites established by a knowledge-based competitive society and economy is a significant goal to reach. Our project aims to achieve this goal by SUPPORTING THE DEVELOPMENT OF TECHNOLOGY AND THE PRAGMATIC EDUCATIONAL METHODS of the educational institutions and their teachers and tutors in Serbia. We also intend to raise students' interest for mathematics and sciences and make these disciplines more appealing to the youth, INVOKING INTER- AND TRANS-DISCIPLINARY INSTRUMENTS.

During the project we are going to expand and MODERNIZE THE TOOL SYSTEM used in the field of mathematics and other sciences in Serbian elementary and secondary schools. First INTERACTIVE TOOLKITS (laptop, projector, "active board") will be purchased for the benefit of each Serbian partner-institution. Then an EDUCATIONAL SMART TOOLKIT will be developed, which will consist of an exercise book and software - both linked to the modern teaching methods. The third part of the educational toolkit is a series of short films that can be used as METHODOLOGICAL HELP by teachers.

We helped in the development of TEACHING METHODOLOGY for Serbian teachers and undergraduate students to use in elementary and secondary mathematics education. Methods were presented to teach mathematics in an exciting way through two Summer Universities, two Experience Workshops, various publications, short-term mobilities for students and through a closing conference.

The WIDER PURPOSE OF THE PROJECT was to make sure that MORE CHILDREN SHOULD CHOOSE MATHEMATICS as their subject of studies. That's the reason why we counted on Serbian students during the project: measured their approach to the present methods of mathematic teaching and during Experience Workshops they had a hands-on experience to new methods and practices.

We believe that our project is a distinctive and creative approach to an existing problem and no similar has been set up in the near past.

We thank the representatives of the 8 partner institutions taking part in the Tempus programme for the hard and constructive work. Many thanks for workers at University of Jyväskylä, Belgrade Metropolitan University, University of Novi Sad, Serbian Academy of Sciences and Arts, ICT College of Vocational Studies, Sint-Lucas School of Architecture, University of Applied Arts Vienna and workers of the facilitator consortium at Eszterházy Károly College.

Conference description

1. PERCEPTION OF SPACE IN PAINTING– Slavik V. Jablan, Ljiljana M. Radović,

Abstract: During the history, perception of space in painting is changed from one- and two-dimensional geometric patterns, that dominate in Paleolithic and Neolithic art, through "hierarchical perspective" and orthogonal axonometry used in Egyptian painting, Byzantine counter-perspective, Renaissance linear perspective, cubistic polycentrism, perceptive perspective, to the non-oriented space of abstract painting. Trying to explain 3D-vision as the reconstruction of a 3D-image from its 2D-projection, that is in general not unique, we will consider different extreme forms of perspective (e.g., anamorphoses), or the formation of ambiguous reconstructions of 2D-projections resulting in visual illusions and impossible figures.

2. DISCOVERING THE LIVING BODY OF MATHEMATICS IN REAL LIFE, Alan Rogerson

Abstract: During the past 50 years the seminal ideas in maths education of problem solving, modelling and integration with other disciplines have finally led to the creation and use of real life themes in mathematics teaching, and thus to a truly humanistic vision of mathematics in society and in the Real World. This paper will look at some innovative international projects that have initiated this paradigm shift in both our conception of mathematics, and how it should be taught. This paper will contrast mathematics as a skeleton collection of rules and axioms with a living body of mathematics which derives from the multitude of ways mathematics is used in Society and in Real Life. Underpinning this analysis is the work of Polya, Kuhn, Lakatos and Wittgenstein.

3. SINISTER MODELS, Dirk Huylebrouck

Abstract: The present paper discusses the importance of the scientific (i. e. mathematical) education of artists, their anthropomorphic preference and their references to nature, yet concludes there is probably no straightforward explanation for preferring one chiral version to another in art. The concept of chirality, i. e. the property of potentially having left- and right-forms is important in art, and more in particular in architecture. For instance, spiral elements render objects chiral with a specific handedness, and this is also the case for dynamic chirality, induced by motion (such as in rotating restaurants or windmills). The issue is relevant not only because of the different aesthetic perceptions of left or right versions of the same object, but also for practical reasons, such as for the design of staircases.

4. DE DIVINO ERRORE, Dirk Huylebrouck

Abstract: ‘De Divina Proportione’ was written by Luca Pacioli and illustrated by Leonardo da Vinci. It was one of the most widely read mathematical books. Unfortunately, a strongly emphasized statement in the book claims six summits of pyramids of the stellated icosidodecahedron lay in one plane. This is not so, and yet even extensively annotated editions of this book never noticed this error. Dutchmen Jos Janssens and Rinus Roelofs did so, 500 years later.

5. PROBLEM SOLVING WITH HANDS-ON AND DIGITAL TOOLS: CONNECTING ORIGAMI AND GEOGEBRA IN MATHEMATICS EDUCATION, Kristóf Fenyvesi, Natalija Budinski, Zsolt Lavicza

Abstract: Our “Visuality & Mathematics — Experiential Education of Mathematics Through the Use of Visual Art, Science, and Playful Activities” Tempus Project started in 2012 with the cooperation of eight European universities and scientific institutions, in order to develop Serbian mathematics education with technological equipment and interactive, experience-centered, and art-related content. The general objectives of this two-year-long project are justified by the findings of the PISA 2012 survey as well, which show that 15-year old Serbian students’ mathematics performance is significantly below the OECD average. For the improvement of the Serbian s students’ mathematical literacy and abilities, what we believe is important is research on new approaches in mathematics education and the increase of experience-centered presentations of cultural, interdisciplinary, and artistic embeddedness of mathematical knowledge, such as the creative applications of mathematics by hands-on models, in digital environments and in real-life problems, in the math class. We are not only working on the development of genuinely new content and methods in mathematics education, but are also collecting all of those estimable practices in experience-centered mathematics education in Serbia, which can be disseminated in the wide circles of Serbian mathematics teachers and can be introduced into the education of teachers as well. In this paper, we highlight efficient practice methods, which are already being applied in a Serbian high school and which connect mathematics education with origami and the open-access GeoGebra dynamic geometry software.

6. VISUAL MATHEMATICS IN EDUCATION, Ljiljana Radović, Slavik Jablan

Abstract: The paper contains description of the course “Visual Mathematics and Design” organized at the Faculty of Information Technologies (Belgrade, Serbia). The course introduces students of graphical design to different areas of visual mathematics: symmetry in art and science, isometric symmetry groups, similarity symmetry, modularity, antisymmetry, colored symmetry, theory of proportions, theory of visual perception, perspective, anamorphoses, visual illusions, ethnomathematics, graph theory, and elements of knot theory. The output of the course is illustrated by representative original student works.

7. MATHEMATICAL MODELING OF ILLUSIONS, Đurđica Takači, Sunčica Zdravković, Sanja Rapajić

Abstact: The mathematical modeling processes of different illusions are analyzed in a view of new technology. Starting from illusions as real problems, all stages and transitions from one stage to another are considered. The cognitive activities following the whole process of mathematical modeling of illusions are analyzed. The packages Mathematica and GeoGebra are used for the geometry constructions of illusions.

8. VISUALIZATION OF FRACTIONAL CALCULUS, Djurdjica Takači, Lila Korenova

Abstract: The visualization and application of definite integral for area problems is considered. The function defined by definite integral the convolution and the composite of integrals are visualized and analyzed. The definitions of fractional integral and derivative are introduced and visualized by using GeoGebra Packages. The presented visualizations were used in teaching calculus for the students, mayor physics, at the Faculty of Science, University of Novi Sad. The students were tested after the presentations and they show good results.

9. MATHEMATICAL WORKSHOPS, LEARNING AND POPULARIZATION OF MATHEMATICS, Tanja Sekulić, Valentina Kostić

Abstract: On the example of mathematical workshop launched for pupils of upper grades of primary school, we discuss the benefits of workshops related to the improvement of teaching methods and students' knowledge and understanding of mathematics. Through illustrative examples from the workshops we have shown different techniques and possibilities for realization of teaching process in the upper grades of primary schools mathematics. Also, the concept of mathematical workshops is described in detail and the effects of the workshops on pupils knowledge of mathematics, the reactions of mathematics teachers and the popularization of mathematics, are analyzed.

10. FRACTAL STRUCTURES IN ARCHITECTURAL HIGH SCHOOL, Milena Jeretin, Jasmina Manojlović, Milena Marić

Abstract: Within this work, it will be presented how the teaching of geometry can be more interesting and more accessible to students in architectural high school by using various modern media. The authors have tried a different approach to motivate students to identify geometric shapes in the world around us and to describe their properties using mathematical language.It also shows the correlation between maths and art. The motive for this approach to teaching math is raising the level of students' motivation as well as the aim to bring them closer to mathematical concepts through visualization.

11. PLATOGROUND-GAME THAT CONNECTS TEACHING MATHEMATICS AND PHYSICAL EDUCATION, Tatjana Stanković, Ljiljana Djuretanović, Nada Ranković

Abstract: Learning environments affect student achievement. Games create enjoyable environment. In this paper we will give an idea how an outside game can connect teaching of mathematics and physical education. We will describe Platoground-game that we have created in order to connect these two subjects and we will give some ideas for future research.

12. DEVELOPING ALGORITHMIC THINKING USING CROCHETING PATTERNS AS EDUCATIONAL TOOL, Milena Životić Ilić, Ivana Đokić, Lilian Wieser,

Abstract: In this paper we are emphasizing the importance of developing algorithmic thinking, for improving problem solving skills. Problem solving competence is important for one of eight key competences defined at EU level - mathematical competence. We will find a relationship between Polya's problem-solving and algorithmic thinking, comparing basic Polya's principles to definition of algorithmic thinking. So, we are considering algorithmic thinking as an important role in high school education for developing mathematical competence. We are using crocheted geometrical shapes and finding a mathematical model to realise it through crochet. We developed algorithms for crocheted models, as a useful educational tool.

13. EXTREME VALUES OF FUNCTION IN GEOGEBRA STYLE, Valentina Kostić, Tanja Sekulić

Abstract: The process of student transitioning from elementary to advanced mathematical thinking in learning of calculus is followed by many difficulties. The role of the teacher is very important in planning, designing and applying adequate methodic solutions by which smooth transitions from lower to higher levels of abstraction, and mathematical thinking, are secured. Methodic solution wich can be efficient in teaching and learning calculus, is visualization, and animation of the basic terms, and processes in combination with symbolic records and definitions. By applying according computer programs, the teacher can realize the teaching process in visual environment. Educational software can link visual and symbolic representations of mathematical objects in dynamic, and interactive environment. This paper presents the possibility of didactic shaping of calculus material using GeoGebra. In dynamic worksheets algebraic and geometric interpretations of the concept of local extreme values of the functions, are connected.

Exhibition description

Exhibition had two parts:
1. "DO YOU LIKE OP-ART?" and
2. "MATHVIS – VISMATH"

Exhibition Space: Mikser House, Karađorđeva 46, Belgrade
Date: Thursday, 18. September 2014, 5 pm

1. "DO YOU LIKE OP-ART?"

In the exhibition "Do You Like Paleolithic Op-art?" in the gallery "Mikser House" (Belgrade) are shown several graphics of large size, 1.5×1.5 m (nine by Slavik Jablan and one by Ljiljana Radovic), the selection of art works of students from Belgrade Metropolitan University inspired by the mathematical structures (symmetrical friezes, patterns, tilings, visual illusions, fractals...) about which they learned from their teachers, S. Jablan and Lj. Radovic in the course "Visual Mathematics" at the Belgrade Metropolitan University. This course, the unique course of that kind in Serbia, and even maybe in this part of Europe is an attempt to establish the new disciplines in art: the Mathematical Art, becoming in the present moment, thanks to the use of computers and new information technology one of the new frontiers in the modern art. The application of Math Art in textile design is represented by the designs of T-shirts with geometrical patterns designed by the BMU students of Textile Design, and geometrical jewelry deigned in the manner of Op art made by Andjelka Simic. Her design is the best example how it is possible to produce wonderful and creative Op-art fashion details for almost zero expenses. The application of new media, dynamic animations produced by BMU students is shown on monitors following the exhibition.

The title of the exhibition "Do You Like Paleolithic Op-art?" shows how it is possible to connect some of the oldest uses of the geometrical art with the Op-art (optical art of the XX century) and modern technology-computer art. The illustration of that are works by Slavik Jablan: mono-thematic graphic works and a hypercube sculpture all based on a single geometrical element: Op-tile that appeared in history 23 000 years B.C. in Mezin (Ukraine) in the reliefs on the bone - bracelet and birds from Mezin. This simple geometric element, a square with the set of parallel diagonal lines can be used for the construction of an infinite collection of geometrical designs, abundantly used in all the history of art: from Neolithic patterns from Vincha, Celtic key patterns, till the works of Victor Vasarely and some other Op-artists. So it is not surprising that the poster for the exhibition remains us to the Vasarely work, and that the title of Lj. Radovic's graphic, based on Koffka cubes is "Homage to Vasarely".

This short comment of the exhibition we can finish by paraphrasing the words "Che is still alive", and say: "Op-art is still alive" by continuing his life in the new kind of art: the Mathematical Art (or simply "MahArt").

2. "MATHVIS – VISMATH"

Feyerabend (2010) argued, that "knowledge needs a plurality of ideas, (...) and that well established theories are never strong enough to terminate the existence of alternative approaches." He considered science as a confused political process, a new experience and argued "against established methods" in science. He calls into question methods of exact and systematic methods and encourages "irrational approaches" as a basis for experimental research. He believes that scientists should use artistic research.

The exhibition MATHVIS – VISMATH, as a collaboration of the Belgrade Metropolitan University and University of Applied Arts Vienna, shows pieces of art of more then 20 artists and scientists, which will be seen on monitors and simultaneously. The installation, classified as a multiple work of different artists and scientists of diverse countries, combining math, computation, molecular-biology, movie, art and design. Professional artists and autodidactic scientists go hand in hand by questioning and discussing relationships and enmities as well as approaches of sciences (mathematics) and the arts and challenge visitors of the exhibition to continue.

Exhibition Space: MIKSER HOUSE, 11000 Belgrade
Opening: Wednesday, 17. September 2014, 7 pm
Duration: 18-21. SEPTEMBER 2014

Artists & Artists Collaboratives

Anjelka Simic, Gerhard Funk, The Imaginary Collective, János Szász Saxon, Julia Monaco, Laszlo Bagi, Luis Filipe Rodrigues, Manuel Holunder Heiss, Milan Licina, Monica Gross, Patrick Karaoulanis, Project Password (Dirk Huylebrouck, Ruth Mateus Berr, Lei Feng, Moje Menhardt, Waltraud Mohoric, Daniel Thomas Moran, Radmila Sazdanovic, Heliane Wiesauer-Reiterer, Gerald Wenzl), Ron Wild, Sama Mara, Stefanie Leinhos, Ulrich Kuehn, Zsuzsa Dardai.

Participating Countries: Austria, Belgium, China, Germany, Hungary, Portugal, Serbia, UK, and USA.

Jury: TEMPUS PROJECT VISMATH research members

MATHVIS – VISMATH takes place, accompanying the research project TEMPUS VISMATH, starting with the Closing Conference of the project at Metropolitan University Belgrade. Objectives of this EU funded TEMPUS CALL is to raise students' interest for mathematics and sciences and make these disciplines more appealing to the youth, invoking inter- and trans-disciplinary instruments and to initiate methodologies of Visual Mathematics, which offer a great possibility for teachers to present mathematics creatively, and in an interesting, appealing way. The project supports the development of technology and the pragmatic educational methods of teachers and tutors in Serbia at elementary, high school, undergraduate, postgraduate and tertiary level.

Full papers

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Do my eyes believe what my ears can hear?

Leslie Burm

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Abstract

Architects create and design spaces. If there is space and a medium, there is sound. Unlike light, sound requires a medium to travel through. No sound without a medium. What is the medium of architects to design spaces and how do architects design and understand space in the context of sound? Within more complex design situations, let's say concerthalls, theatres and auditoria, acousticians are the consultants of architects. The architect will depend on the knowledge and the skills of an acousticians to get to the desired result. But, how do the acousticians design and understand space in the context of sound? How is the ear of an architect/acoustician?

Musicians create and design music. In contrast to the architects they work and perform with sound in space. Musical pieces are, in some way or another, a reflection, an explicification of their understanding of acoustical space. Musicians, and by extension sound artists, use and modulate their instruments to activate and play the space to get to a desired result. The result, their musical composition, could be understood as their comprehension of space. More specific, it makes explicit their comprehension of and the possibilities they hear in the acoustic space in which they are performing. But how is the ear of a musician?

To bridge the space of sound of the architects/acousticians and the musicians one can state the question: What could make an architect a better musician and a musician a better architect? Or, how could an architect learn from a musician and visa versa to design acoustic quality?

1. Why?

Why is a bridge needed? And what could be that bridge? Architects look and see, acousticians calculate and simulate to understand space. We could say that they 'know sound'.

Musicians listen to learn about space. They have sensitive skills to work with sound. We could say that they can 'feel sound'. To work with sound is to work with our ears. We can not depend on our eyes to get an understanding of our ears. For now, I say we process sound with our ears.

Psychoacoustics is the scientific study of sound perception, it deals with the process of hearing, with how we hear and how we receive and process variations in air pressure that reach our ears, which are then transmitted and processed by our brain. To certain extend it defines limits and qualities a sound might need to be processed by the ears. One very clear example is sound pressure level or loudness. If we are exposed to a loud sound over a long period of time, we will damage our ears and our health. Another set of limits is the frequency range of our ears: we can process sounds ranging from 20 Hz to 20 kHz.

Our ears and brain are related to our ability to talk, to produce sounds in the form of speech, of vocal language. Vocal language is composed by sounds produced by air which is pushed from our lungs and modified by our vocal cords in puffs. The produced sound is modified by the space of the mouth and other resonant parts of our body to eventually produce the sound we want. In fact we can say that our whole body is engaged in a vibrating process to produce our individual sound of speech. This whole process is monitored and steered by our brain AND corrected – if needed – through our simultaneous listening. We are in fact all musicians.

Listening, in contrary to hearing, involves an understanding: I make sound to send information to the receiver. Only when the receiver is listening, he will built up an understanding. Listening is an active involvement in the process of communication.

Whether or not a process of listening is initiated depends on the quality of the signal. If a signal, let's say a sound, is too distorted, too obscured and unclear, the receiver will have difficulties in understanding what the sender might want to communicate.

So, if architects want to understand the musical language they will need to learn to listen and need to learn to judge sound quality. Even so, if musicians want to understand spacial configuration, they will need to learn to listen and need to learn to judge acoustic quality.

2. How?

If we narrow the concept of space to a built space, to a system within an enclosed volume. How can we learn about that system and the interaction within that system?

Musicians use their instruments. Architects have the space as their instrument.

How can an architect and a musician learn from each other to get to new knowledge for sound and for space?

Let's consider a concert hall. If we imagine a 'perfect' concert hall, how could it be designed and how does it need to sound like?

Musicians know their instrument. To know ones instrument is to know the qualities of an instrument. This knowing involves qualities like timbre of an instrument. Timbre is to a great extend linked to the physical composition of the instrument.

If we take for instance a violin, it will involve characteristics of materials like the type of wood and its finishing, the specifics of the strings and their connections, the specific resonant capacities of the violin, Once the instrument is activated - in the case of the violin it might be a bow – we can start talking about sound color. At this point we need to extend the system to the way of playing, to the interaction of the bow, operated by the musician, his listening AND the space in which he is performing. If he wants to know how his instrument sounds like in the space while playing, he should have an experience which covers the whole experience of his sound in space.

Let's put this concept in the world of the architect/acoustician. What is the timbre of a space? Parallel to the violin, we can say that it is built up by the materials, their finishing and their connections. But what is the sound color of a space and how can we determine and judge sound color of a space? Within that conceptual frame the architect/acoustician might need to think and act like a musician. He needs to consider his space as a musical instrument, as a system that modulates sound in specific ways.

So, the architect should start playing the space as a musical instrument.

To explore this further I will give an example from music history.

3. Giovanni Gabrieli

Giovanni Gabrieli (ca. 1555–1612) is considered to be one of the driving founders of the polychoral music. The way in which he positioned the choirs in the church gave him the possibilities to start composing with voices that were singing to each other. The composition in space made the sounds of both choirs start to interact. His musical works mirror the transition from late Renaissance to the early Baroque. Gabrieli was well-known because of a characteristic sound in his music, which was associated with St. Mark's Cathedral, Venice, where he made noteworthy contributions in vocal and instrumental music. A commonly encountered term for the composition of the separated choirs is cori spezzati—literally, separated choirs.

The style arose from the architectural peculiarities of the imposing Basilica San Marco di Venezia. Aware of the sound delay caused by the distance between opposing choir lofts, composers began to take advantage of that as a useful special effect. Since it was difficult to get widely separated choirs to sing the same music simultaneously, composers such as Adrian Willaert, the maestro di cappella of St. Mark's in the 1540s, solved the problem by writing antiphonal music where opposing choirs would sing successive, often contrasting phrases of the music; the stereo effect proved to be popular, and soon other composers were imitating the idea, and not only in St. Mark's but in other large cathedrals in Italy. This was a rare but interesting case of the architectural peculiarities of a single building influencing the development of a style which not only became popular all over Europe, but defined, in part, the shift from the Renaissance to the Baroque era [1].

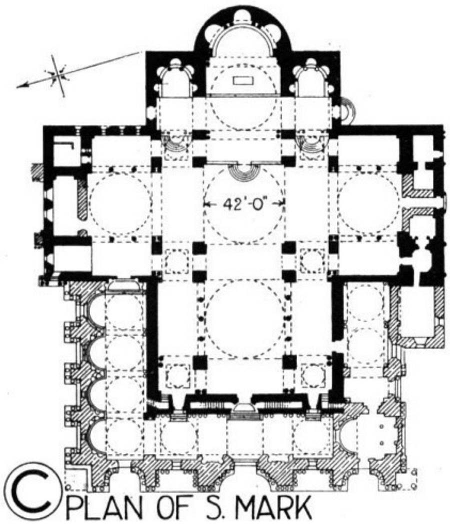


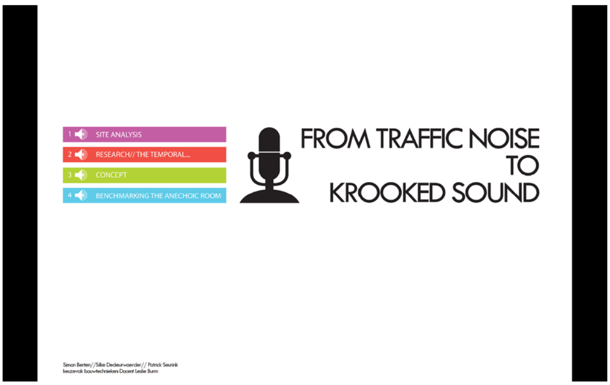
Figure 1: Plan of Saint Mark's Cathedral, Venice

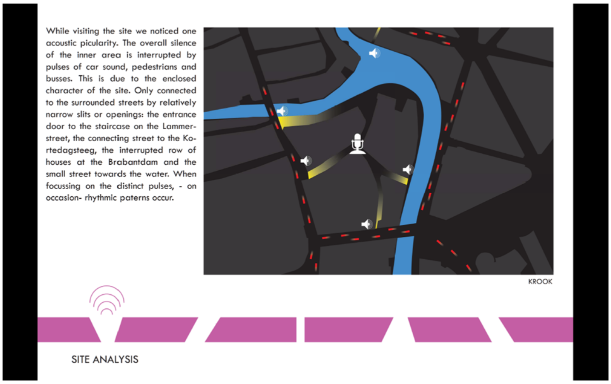
4. Architecture students as musicians

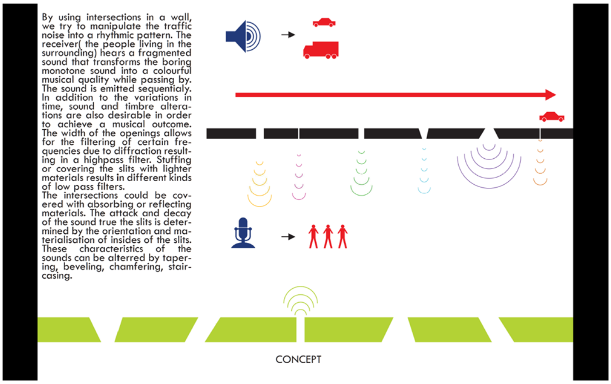
In my teaching I stimulate architecture students to think and to act like musicians, to invite them to listen, as an approach to design spaces or interventions in space. The following example is the work of a team of students that had to design an intervention in an urban context.

As a result from their site analysis, the students started designing a sound barrier. The sound barrier was not imagined as a closed wall, but a wall with perforations. These perforations would modify the continuous sound of traffic into a pulsating rhythmic sound.

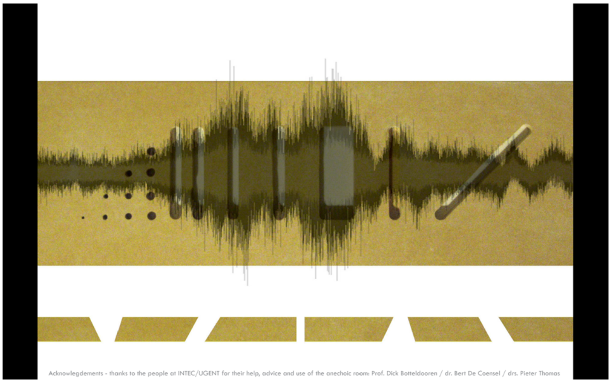
A model of the design was made and tested in an anechoic chamber.




While visiting the site we noticed one acoustic picularity. The overall silence of the inner area is interrupted by pulses of car sound, pedestrians and busses. This is due to the enclosed character of the site. Only connected to the surrounded streets by relatively narrow slits or openings: the entrance door to the staircase on the Lammer-street, the connecting street to the Ko-rtedagsteeg, the interrupted row of houses at the Brabantdam and the small street towards the water. When focussing on the distinct pulses, - on occasion- rhythmic paterns occur.
KROOK
SITE ANALYSIS


By using intersections in a wall, we try to manipulate the traffic noise into a rhythmic pattern. The receiver(the people living in the surrounding) hears a fragmented sound that transforms the boring monotone sound into a colourful musical quality while passing by. The sound is emitted sequentialy. In addition to the variations in time, sound and timbre altera-tions are also desirable in order to achieve a musical outcome. The width of the openings allows for the filtering of certain fre-quencies due to diffraction result-ing in a highpass filter. Stuffing or covering the slits with lighter materials results in different kinds of low pass filters.
The intersections could be cov-ered with absorbing or reflecting materials. The attack and decay of the sound true the slits is deter-mined by the orientation and ma-terialisation of insides of the slits. These characteristics of the sounds can be alterred by taper-ing, beveling, chamfering, stair-casing.
CONCEPT


As a next step we took some MDF test models to the anechoic room at the INTEC department of Ghent University for testing their acoustic performance. Despite the scientific assistance and devices we did not succeed in generating reproducable, scientific test results. The combination of limited time and a complex setup that involves linear motion proved too hard to tackle. None the less, the tests yielded results that indicated that the parameters we were trying to put at work did actually perform quite well. Rythmic patterns could be generated and the spectral changes in the sound depending on the location of the recording were also quite clear. The positioning of the holes was a bit more problematic. Diffraction played a more promininent role than we had estimated, resulting in hardly noticable differences between two consecutive, closely positioned slits. Edge diffraction at the top and sides of the wall is another problem to reconsider for further testing.
As a next step, 3D acoustic computer simulation would be useful for optimising the wall design before taking it back to the lab. To be continued....
BENCHMARKING - THE ANECHOIC ROOM



References

[1] Wikipedia, Venetian polychoral style

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The *Golden Section Puzzle*® in math education

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Abstract

The *Golden Section Puzzle*® (GSP) is a tool-kit developed by the author to be applied for several purposes. It can visualise a series of geometrical information for pupils and, at the same time, it is an enjoyable toy.

1. Description of the Golden Section Puzzle®

The GSP® is a modern 'great grandchild' of the millennia known tangram. The ancient Chinese tangram consists of 7 pieces. A GSP kit consists of 8 shapes of tiles, each available in 8 basic colours. So, the kit consists of $8 \cdot 8 = 64$ tiles.

The 8 tiles can be obtained by cutting a golden rectangle. The proportion of the edges of the rectangle is $1 : \Phi$ (where Φ denotes the so called Golden Section number, 1,618...). Let's divide the golden rectangle into similar smaller golden rectangles in two steps (their edge proportion being $1 : \Phi^{-1} : \Phi^{-2}$), then draw the diagonals of these rectangles. We received the eight unit tiles of the GSP. The lengths of the edges of the individual tiles can be expressed by the powers of the Golden Section number.

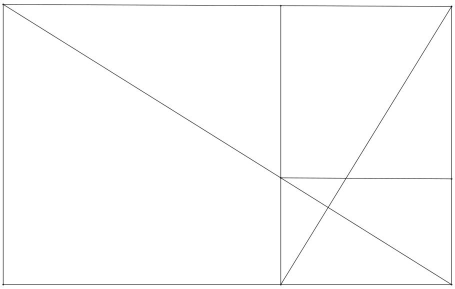

Figure 1: Eight-section of the Golden Rectangle

The 8 colours correspond to the 8 vertices of the so called colour cube (Figure 2): (0,0,0 – black), (255, 0,0 – red), (0,255,0 – green), (0,0,255 – blue), (0,255,255 – cyan), (255, 0, 255 – magenta), (255, 255, 0 – yellow), (255, 255, 255 – white). The back side of each tile is coloured by the complementer colour in front (according to the opposite vertex of the colour cube).

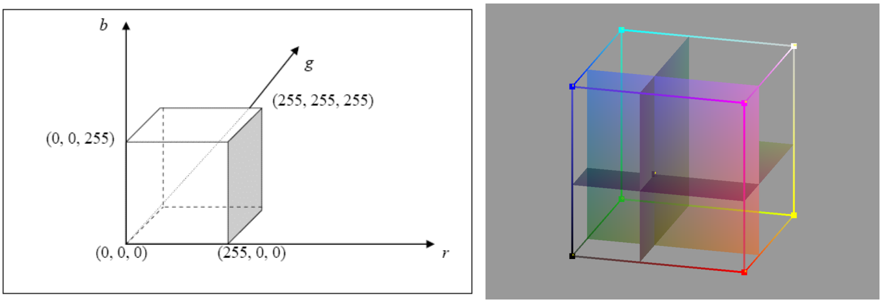

Figure 2: The “colour cube”

2. What is the GSP® good for?

1. Learning the *Fibonacci numbers* and their attributes.
2. Learning the concept of the *Golden Section* and its relation to the Fibonacci numbers.
3. *Measuring* by the help of a ruler designed in units of the powers of the Golden Section number.
4. Learning *similitude*.
5. Learning *proportionality* (in geometry and arts).
6. Comparing different appearances of *symmetries* (mirror-reflection, rotation, similitude, translation, permutation, colour).

7. Studying *theorems of right triangles.*

8. Studying *combinatorics* by tiling with shapes and colours. (e.g., study Euler's problem about the bridges of Königsberg based on the number of edges meeting in a node).

9. Studying the attributes of basic *colours* (symmetries of a non-geometric property).

10. Studying *harmony by* means of *proportions* (in geometry and arts).

11. Studying *harmony by* means of *colours* (in geometry and arts).

12. Studying the phenomenon of *chirality* (applicable in chemistry and biology).

13. Playing *domino* (framing edges and colours into each other).

14. *Tiling*: the millions of possible variations develop the phantasy of pupils.

15. *Constructing shapes* (simlar to tangram) by free variation of the pieces from among the 64 tiles allow to open the dithyrambic phantasy of pupils.

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 25–38

Problem Solving with Hands-on and Digital Tools: Connecting Origami and GeoGebra in Mathematics Education

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Abstract

Our "Visuality & Mathematics — Experiential Education of Mathematics Through the Use of Visual Art, Science, and Playful Activities" Tempus Project started in 2012 with the cooperation of eight European universities and scientific institutions, in order to develop Serbian mathematics education with technological equipment and interactive, experience-centered, and art-related content. The general objectives of this two-year-long project are justified by the findings of the PISA 2012 survey as well, which show that 15-year old Serbian students' mathematics performance is significantly below the OECD average. For the improvement of the Serbian students' mathematical literacy and abilities, what we believe is important is research on new approaches in mathematics education and the increase of experience-centered presentations of cultural, interdisciplinary, and artistic embeddedness of mathematical knowledge, such as the creative applications of mathematics by hands-on models, in digital environments and in real-life problems, in the math class. We are not only working on the development of genuinely new content and methods in mathematics education, but are also collecting all of those estimable practices in experience-centered mathematics education in Serbia, which can be disseminated in the wide circles of Serbian mathematics teachers and can be introduced into the education of teachers as well.

In this paper, we highlight efficient practice methods, which are already being applied in a Serbian high school and which connect mathematics education with origami and the open-access GeoGebra dynamic geometry software.

1. Introduction

Our "Visuality & Mathematics — Experiential Education of Mathematics Through the Use of Visual Art, Science, and Playful Activities" Tempus Project started in 2012 with the cooperation of eight European universities and scientific institutions, in order to develop Serbian mathematics education with technological equipment and interactive, experience-centered, and art-related content (project website: [12]). The general objectives of this two-year-long project are justified by the findings of the PISA 2012 survey as well [18]. PISA 2012 reveals that although Serbia — which scored 449 points — steadily improved in mathematics education from 2003 [18, p. 55], the mathematics performance of 15-year old Serbian students is still statistically significantly below the OECD average. According to PISA's definition of mathematical literacy [18, p. 37-38], Serbian students fall behind the OECD average in their capacity to formulate, employ, and interpret mathematics in various contexts, and also have difficulty recognizing the role that mathematics plays in the world. From the four overarching areas, which the PISA assessment framework for mathematical literacy makes reference to, it is only in quantity that the Serbian students score higher than their overall mathematics proficiency scale. Operations in the other three areas of mathematical literacy, i.e., uncertainity and data, change and relationships [18, p. 101], and space and shape [18, p. 104], cause even more difficulties for them. PISA 2012 measured not only the students' performances, but also examined whether and how their exposure to mathematics content is associated with their performance, and this provides a snapshot of the priorities of Serbian mathematics education. The survey has shown that Serbian students' exposure to word problems is under the OECD average [18, p. 147], as is their exposure to applied mathematics [18, p. 149], while they have significantly more opportunities to learn formal mathematics content during their schooling [18, p. 148]. The examination of Serbian students' engagement, drive, and self-beliefs in connection with mathematics learning shows that the index of their mathematics self-efficacy – the extent to which they believe in their own ability to handle mathematical tasks effectively and overcome difficulties – is also relatively low, while their index of openness to problem solving is high, although the latter is not reflected in their mathematics performance [19, p. 11]. Serbian students' intrinsic motivation to learn mathematics is slightly lower than the average as well, but from the survey results it is also obvious that the educational system is not taking full advantage of their positive attitudes and their openness to problem solving. In Serbia, less than 30% — at most — of students enjoy mathematics [19, p. 69].

The picture provided by PISA 2012 on Serbian students' mathematics education and attitudes is further refined by our own Tempus Attitude Survey 2013 (TAS 2013). We succeeded in identifying a number of features of pedagogical practices applied to the mathematics education of 11–18-year-old Serbian students, which could be recommended to be developed or changed to improve and build more efficiently on students' attitudes towards mathematics and thus support them in achieving better results. We found that the majority of the 2,607 11–18-year-old Serbian students who participated in TAS 2013 do not feel personally addressed to by the content of their math classes: most of them feel math classes are boring and think that the content could be taught in a much more engaging way. The majority of the students agree that they would learn more mathematics if the content of the classes would be more interesting to them. TAS 2013 has also shown that the presentation methods of the mathematics education content in school does not provide an account of the cultural embeddedness of mathematical knowledge and does not connect to the students' real-life world. According to our results, most Serbian mathematics teachers also do not apply all of the methodologies, tools, and equipment for experience-centered mathematics education [9], which could be effectively implemented to support their students' creative and imaginative abilities in the comprehension of complex and difficult mathematical problems and would make mathematics classes more engaging. Although at least a third of the students rarely used computers in mathematics classes, more than half of them never used a computer in their maths class. This demonstrates that not only do students not rely on the support of computer applications in math classes, the teachers also use them only very rarely for the illustration of teaching content (e.g., in the form of PowerPoint presentations). The situation is not significantly better in the case of using hands-on tools, physical models, and other visualization equipment: almost half of the students have never had an opportunity to work with these kinds of physical materials in their math classes. The situation is rather unfavorable in connection to the school presentation of the cultural embeddedness of mathematics. The general mathematics education practice in Serbia almost entirely excludes all accounts of art connections to mathematics (Figure 1).

For improving Serbian students' mathematical literacy and abilities, what we believe is important is research on new, experience-centered, art-related approaches in mathematics education and the increase of presentations of cultural, interdisciplinary, and artistic embeddedness of mathematical knowledge, leading to creative applications of mathematics using hands-on models, the use of digital environments and the incorporation real-life problems into math classes. We are not only working on the development of genuinely new content and methods in mathematics education, but also on collecting all of those good practices in experience-centered mathematics education in Serbia, which can be disseminated in the wide circles of Serbian mathematics teachers and can be introduced in the education of teachers as well. For this purpose, annually we are organizing the European Summer School for Visual Mathematics and Education, which provides the opportunity for practicing mathematics teachers and university students from Serbia to come together and exchange ideas with specialists of experience-centered mathematics education from all over the world.

In 2013 July, the Summer School program took place in Eger, Hungary [13]

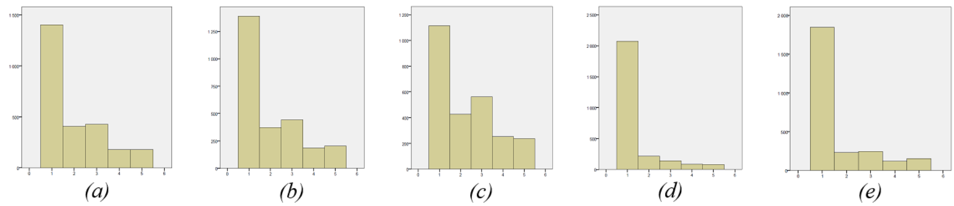

Figure 1: TAS 2013 results. Students rate how often their mathematics teachers (a) use computers; (b) computer-aided presentations, such as PowerPoint; (c) real physical objects or models for visualization; (d) references to artworks, like paintings or sculpture, etc.; (e) or how often they visited art or science museums to support the understanding of mathematical content. The vertical line shows the number of students; the horizontal line: 1 = never; 2 = a few times; 3 = sometimes; 4 = often; 5 = many times.

where the authors of this paper first met. They studied each other's approaches and decided to further develop and publish their method, which connects mathematics education with origami and the open-access GeoGebra dynamic geometry software. The method is already being applied by mathematics teacher Natalija Budinski in her workplace, the Petro Kuzmjak School (Ruski Krstur, Serbia) and in 2014 an Origami-Geogebra Workshop was held at 2nd European Summer School of Visual Mathematics and Education in Belgrade and an origami workshop at the event's Family day by Budinski (Figure 2).

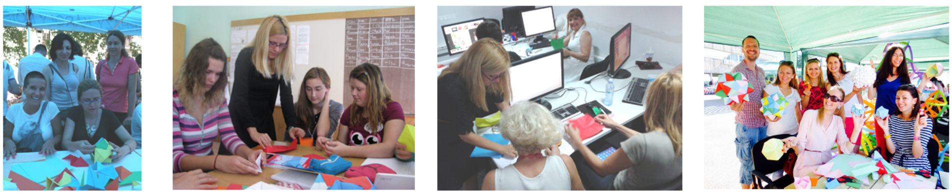

Figure 2: From left to right: Natalija Budinski's origami workshop at the 1s European Summer School of Visual Mathematics and Education in Eger, 2013; Budinski's origami-mathematics class in her school in Ruski Krstur; Origami-Geogebra Workshop at 2nd European Summer School of Visual Mathematics and Education in Belgrade, 2014; Origami workshop at 2nd European Summer School of Visual Mathematics Family Day.

2. The Problem: Thinking Out of the Box

PISA measures not only the extent to which students can reproduce mathematical knowledge, but also how they can apply their knowledge in unfamiliar situations, in real-life contexts, and how well they can use tools, such as rulers, calculators, dynamic geometry software, etc., each of which require a degree of mathematical reasoning: "Strong mathematics performance in PISA is not only related to opportunities to learn formal mathematics, such as solving a quadratic equation, using complex numbers, or calculating the volume of a box, but is also related to opportunities for learning how to apply mathematics (using mathematics in a real-world context)." [18, p. 146] In pursuit of the idea of opportunities for learning different kinds of mathematics in an engaging way, we were looking for a mathematical problem, whose solution requires real-world context application of formal mathematics knowledge, experience-centered approaches, and the genuine ability of "thinking out of the box". The famous Delian problem of doubling the cube we considered an appropriate example for illustrating certain characteristics of our approach, because (a) the mathematical knowledge needed to understand and solve this problem is available to 15–18-year-old Serbian high school students; (b) of its historical origin as well as the fact that famous solution; (c) its origami solution is able to call the students' attention to some unexpected aspects of the mathematical potential of the art of paper-folding; (d) its solution with the involvement of two conic curves can be studied in the GeoGebra dynamic geometry software environment; (e) both the origami and the GeoGebra solutions of the problem requires that the students recognize the epistemological limits of Euclidean geometry, to change perspective and experiment with new approaches.

(a) The Delian problem is based on the mathematical knowledge of calculating the volume of geometric solids, although it is impossible to solve it within the constraints of Euclidean geometry. For the solution, students need some prerequisite mathematical knowledge, which is covered by the Serbian secondary and high school curriculum, such as: basic geometric shapes, the Pythagorean Theorem, similarity of the triangles, simple quadratic and cubic equations, curves like circles, parabolas, hyperbolas, and ellipses in an analytical sense. Students should also be familiar with origami and GeoGebra basics.

(b) There is a plenty of information available on the Delian problem's significance in the history of science and its philological background [21, pp. 82-88], as well as about its mathematical details [4, pp. 122-134]. Teachers can efficiently make use of these resources according to the students' interests and their own demands to illustrate the cultural embeddedness of this mathematical problem. In our class, we were content with the most widely prevalent, to some extent romantic version of the ancient story [2, p. 98], which also contextualizes the otherwise formal mathematical problem of doubling a cube as a real-world application of mathematical knowledge. According to the version we used in our class (cf. T. Sundara Row quotes [10, p. 82, 207] in [20, p. 55]), ancient Athenians were struck by a plague and consulted the oracle at Delos. The oracle advised the Athenians to double the size of Apollo's perfect cube-shaped altar. They constructed the new altar where the sides of the cube were double the length of those of the original altar. But Apollo then made the pestilence worse, as the Athenians, by doubling the side of the original cube, increased its volume not to double that of the original, but falsely, by two cubed, or eight. This narrative might catch students' interest and support their imagination, compelling them to visualize the problem. It also makes possible for the teacher to link the mathematical information with the students' knowledge of Greek culture, and Euclid and Plato, in an interdisciplinary way.

(c) In the 19th century, Friedrich Fröbel, the German educationist and founder of the Kindergarten concept, encouraged the use of paper-folding in education with the aim of conveying mathematical concepts to children [8, pp. 214–225]. The Indian mathematician T. Sundara Row, in his Fröbel-inspired Geometric Exercises of Paper Folding (first published in 1901), already mentioned the Delian problem under the chapter Arithmetic Series and provided Menaechmus, a pupil of Plato's, solution to the problem [20, p. 55]. But it was Margherita Piazzolla Beloch in 1936 who proved that starting with a length L on a piece of paper, she could fold a length that was the cube root of L, and although she might have not recognized it, this way she solved the Delian problem by using origami. Her paper was rediscovered decades later, when the mathematical world started to take origami seriously (see [7, 14]. Implementing origami in mathematics education was systematically researched from the 1950s [15] and it has been found that origami has several pedagogical benefits. Origami allows the students to feel the objects created, rather than just imagine them or see them in pictures. A teacher can blend mathematical vocabulary and content within the steps s/he must go through to teach the folding of a particular model [3]. Additionally, it is well-suited to working with a classroom of 30 or more students, supports community building, encourages cooperative learning, develops planar and spatial reasoning, allows students to create and manipulate basic geometric shapes such as squares, rectangles, and triangles, and in many ways contributes to the students' cognitive development (on several mathematics educational benefits of origami, see: [17]).

(d) The experience-centered process of exploratory introduction to geometry problems and proofs by paper-folding can be successfully supported by using Dynamic Geometry Softwares (DGS) such as the free-access GeoGebra (`www.geogebra.org`) to extend investigations and foster deeper understanding of a proof [5]. GeoGebra is accessible, engaging, encourages students to further explore the geometrical situation and provides opportunities for making and evaluating conjectures of geometrical results.

(e) The experience-centered study of the Delian problem effectively shows what kind of potential and limitation certain mathematical frameworks have, such as Euclidean geometry. Students can make thought experiments with the application of an origami based geometry and they can be introduced to origami axioms, including Geretschlager's [11], who notes that his last procedure (7*) makes origami different from Euclidean geometry. Euclidean constructions are equivalent to origami built from Geretschlager's axioms nr. 1 to 7, but 7* amounts to the solution of a cubic problem, which is not achievable using Euclidean methods. It is the axiom that allows paper-folding methods to solve the classic problems of doubling the cube and trisecting the angle [5, p. 10].

3. Two Solutions to an Unsolvable Problem: An Origami-GeoGebra Mathematics Lesson Plan

Our lesson is planned for 45 minutes. To conduct the lesson, we need at least one computer per every two students, a computer for the teacher, equipped with an LCD projector, and colored paper for the origami experiment. As software support, we need GeoGebra and PowerPoint. Our lesson follows the PISA 2012 mathematics frameworks' structure as it is explained in Figure 3.

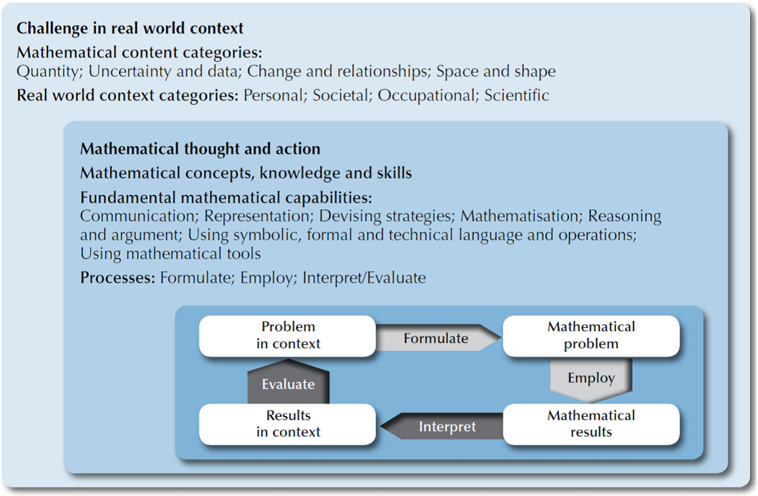

Figure 3: Main features of the PISA 2012 mathematics framework [18, p. 37]

Introduction to the lesson (duration: 5 minutes). In a PowerPoint presentation with several illustrations on ancient Greek cultural artefacts, the teacher provides the basic information on the Delian problem by presenting the advice of the oracle as it is explained in (b) of section 2; and shows visualizations of the mathematical content. In the presentation the teacher underlines with a brief explanation that in the framework of Euclidean geometry the construction of figures and lengths are restricted to the use of only an unmarked straight edge and compass. At the end of the introduction, the teacher starts a dialogue with the students and refreshes their preliminary knowledge on area and volume of geometrical solids.

In search of the solution (duration: 5 minutes). Students are encouraged to share their views and opinions on the Delian problem. The teacher orients the discussion with questions and the students are encouraged to devise different assumptions. After some thinking, students most likely come up with the wrong assumption that *"the size of the altar is x, and if we double it, we obtain the doubled* *volume."* In this case, the teacher reminds the students of the formula of calculating the volume of the cube and they prove together that the assumption is not correct. The formula shows that when we double the cube's volume, the side length of the new cube should be $\sqrt[3]{2}$ times larger. It is important to underline the significance of this assumption and to call attention on the importance of the analysis of the number $\sqrt[3]{2}$, which is not a constructible length in the Euclidean framework [6]. Students might be reminded that, for example, the number $\sqrt[2]{2}$ is a constructible number and during their education they previously learned how to construct it by only using an unmarked straightedge and compass. Students understand that because of the unconstructibility of $\sqrt[3]{2}$, the Delian problem is not solvable in the Euclidean framework. The teacher briefly recounts the most famous attempts to approach and solve this problem from a non-Eucledian framework including Menaechmus, Galois, and the origami solution developed in 1986 by Peter Messer [16].

Solving the unsolvable (duration: 30 minutes). The teacher divides the students into two groups. One group works with origami on the Messer-solution, and the other works with GeoGebra on the Menaechmus-solution of the Delian problem. Groups can be formed by teacher choice or by student preference. Both groups get adequate printed instructions, previously prepared by the teacher. Following that, Origami and GeoGebra students will be matched in pairs and work collaboratively. Origami students create an origami solution of the problem from a squared piece of paper, while GeoGebra students create a dynamic worksheet based on Menaechmus' solution, a sketch of intersecting parabolas.

Origami instructions:

1. Gently fold the squared paper in half;
2. Fold paper as is shown to match the line segments AC and BE;
3. Fold paper in three equal parts;
4. Fold the angle C to match the line AB, and match the point I with line FG;
5. Point C will make the line segments AC and CB in proportion to $1 : \sqrt[3]{2}$;
6. Analyze the proportion! Why is it correct?

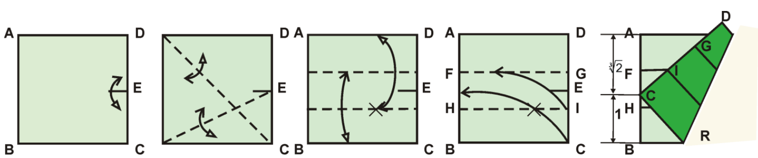

Figure 4: Messer's solution

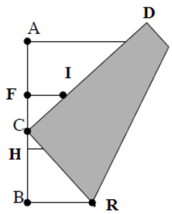

Figure 5: The origami solution

GeoGebra instructions:

1. Open the GeoGebra file;
2. Insert two parabolas: one with the focal length twice of the other. In case you do not know, their analytical formulas: $x^2 = ky$, $y^2 = 2kx$.
3. Use a slider;
4. Observe the intersection, while changing the slider's values;
5. Analyze if there is $\sqrt[3]{2}$. How did you obtain it?

The students work approximately 10 minutes individually on finding the answer to the question. Then each team comes together and for 10 minutes discusses and makes drafts or completes proofs. At this phase it is not required of the students that they prove the solutions; rather, they must make several assumptions and experiment. The teacher is observing their progress and helps if there is a question or the students need support. When the groups obtain some solutions, or maybe even a proof (see Figure 6), the teacher starts an open discussion with the participation of all of the students. Each delegates one student to present the solution origami and the GeoGebra solution. The teacher and students support the presenters and the teacher gently leads the students through the proof. For studying the GeoGebra proof, the teacher may ask questions like: Why do parabolas intersect? How do we formulate that intersection analytically? Do you know how to solve the equation? What about dividing by zero while solving the equation? What is the solution? etc.

The intersections of the parabolas occur at two points. One of them is the origin $A(0,0)$ and the second one is the point B with coordinates $B(\sqrt[3]{2}k, \sqrt[3]{4}k)$. The teacher may ask the student about what is obtained when the value of a slider is changed? If we estimate the slider value on 1, we obtain the solution to the Delian problem as the length of segment AC. The length is $\sqrt[3]{2}$. The possible solution of the student in GeoGebra is shown in Figure 6.

For the study of the origami proof, the teacher may ask questions and may give instructions like: Would you make a sketch? Note the main points. Use the

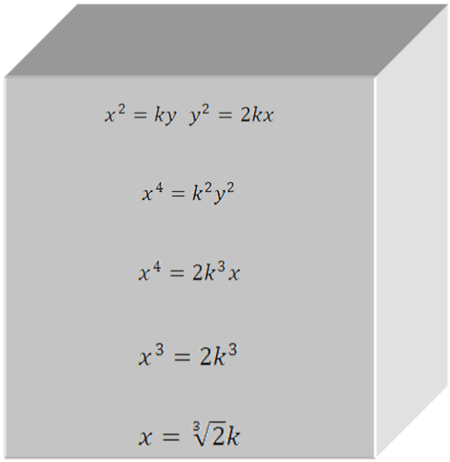

Figure 6: Proof of the problem

Pythagorean Theorem. Are there similar triangles? What does that imply? When you simplify the equation, what is the solution? etc.

Consider the square of paper shown in Figure 5 with the noted points A, B, C, R, F and I. Let us assume $BC = 1$ and let $x = AC$ and $y = BR$. From this it follows $AB = x + 1$ and $CR = 1 + x - y$. If we apply the Pythagorean theorem $CR^2 = BR^2 + BC^2$ and use the notation we get $(1 + x - y)^2 = y^2 + 1^2$. Simplifying the equation we get:

$$y = \frac{x^2 + 2x}{2 + 2x}. \tag{1}$$

Let us consider the triangles ΔIFC and ΔCBR. They are similar. This implies:

$$\frac{BR}{CR} = \frac{FC}{IC}. \tag{2}$$

The segment FC is part of segment AB which satisfies the following equations:

$$AB = AF + FC + CB$$
$$1 + x = \frac{1}{3}(1 + x) + FC + 1$$
$$FC = \frac{2x - 1}{3}$$

If we substitute this in (2) we get:

$$\frac{y}{1 + x - y} = \frac{\frac{2x-1}{3}}{\frac{1+x}{3}}.$$

Simplifying the expression we get:

$$y = \frac{(2x - 1)(x + 1)}{3x} \tag{3}$$

As a result of equating expressions (1) and (3) we get $x^3 + 3x^2 + 2x = 2x^3 + 3x^2 + 2x - 2$ from whence it follows: $x^3 = 2$.

Closing the lesson (duration: 5 minutes). The teacher and students summarize the content of the lesson and the teacher asks the students for their impressions on the content of the lesson and the introduced approaches (Figure 7). As a homework task, the teacher may provide students materials for experimenting with the solution of the two other famous unsolvable problem of Greek mathematics, the trisection of an angle and the squaring a circle.

4. Conclusion and Future Work

Our combined approach connected hands-on activity with computer-based learning. By the combination of different experience-centered approaches, students can improve their skills in reasoning in various contexts. While making origami requires following certain procedures in paper folding, GeoGebra allows the students to create a set of procedures that will lead to the solution. Both approaches illustrate real-world application of mathematical knowledge and bring abstract and difficult geometrical notions closer to the students. The fold diagrams in origami and the interactive visualizations in GeoGebra might give an opportunity to make transitions from visual to formal statements. Due to their visual and interactive nature, both origami and GeoGebra might enable students to see connections in the geometric statements more easily compared to students who receive standard instruction.[1] The origami-based instruction might contribute to students performing higher in achievement tests like PISA as well [1, p. 18].

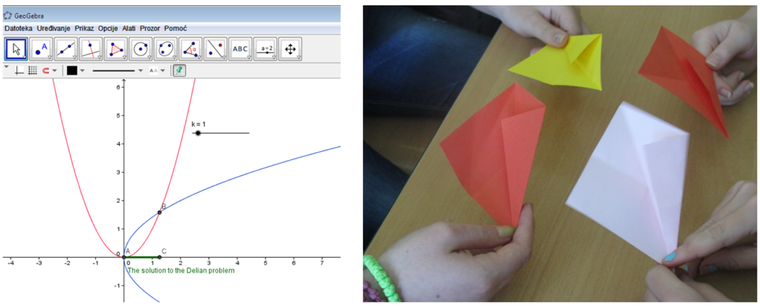

Figure 7: The GeoGebra (left) and the origami (right) solution of the Delian problem

In the framework of the Budinski's Origami-Geogebra Workshop at the 2nd

[1]The GeoMatech Project, started in 2014 by the University of Applied Sciences, Budapest is working on special GeoGebra curricula applied to the Hungarian mathematics education. Numerous results of the project are expected to be implemented in various areas of mathematics education also in other countries. Website: `http://geomatech.hu/`

European Summer School of Visual Mathematics and Education, further opportunities were introduced for combining origami and GeoGebra at the math class: the construction of perpendicular line and the proof of the Pythagorean Theorem. In this paper we can provide only a short summary below of these further opportunities, but our plan is to make a systematic study of the topic in the near future.

Perpendicular line is a basic mathematical notion and it is introduced to Serbian students in the age of 10-12 years. Only few origami folds are needed to construct the perpendicular line and GeoGebra has embedded function for perpendicular line construction. Figure 8 shows constructions of perpendicular line by GeoGebra and Origami.

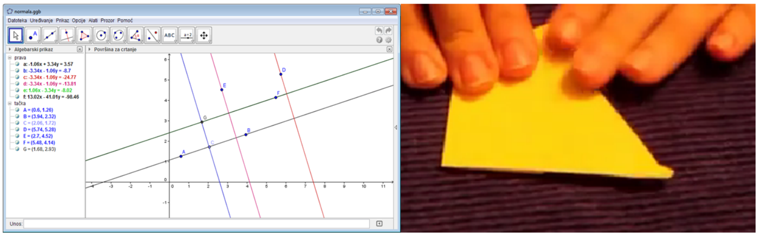

Figure 8: The GeoGebra (left) and the origami (right) construction of perpendicular line

Pythagorean Theorem is a compulsory part of mathematical curriculums all over the world. It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. So far, there are more than three hundred proofs and representations of this theorem. Figure 9 provides the Geogebra and Origami representations of it.

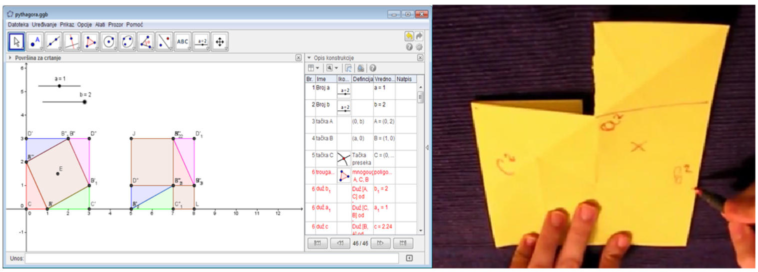

Figure 9: GeoGebra (left) and the origami (right) proof of Pythagorean Theorem

Credits: The authors owe thanks to the members of the "Visuality & Mathematics — Experiential Education of Mathematics Through the Use of Visual Art, Science, and Playful Activities" Tempus Project, especially to Sunčica Zdravković, Ljiljana Radović, Đurđica Takači, Ruth Mateus-Berr, Ibolya Prokaj Szilágyi, Ilona Oláh Téglási, Slavik Jablan, Dirk Huylebrouck, Kálmán Liptai and Raine Koskimaa; to the members of the GeoMaTech Project; to Rainer J. Hanshe, Nóra Somlyódy and to our anonymous reviewers for their valuable support, advice and criticism.

References

[1] Arici, Sevil - Aslan-Tutak, Fatma: The Effect of Origami-based Instruction On Spatial Visualization, Geometry Achievement, and Geometric Reasoning, International Journal of Science and Mathematics Education, December (2013).

[2] Bellos, Alex: Alex's Adventures in Numberland. London: Bloomsbury Publishing (2010).

[3] Boakes, Norma: Origami-Mathematics Lessons: Paper Folding as a Teaching Tool. Mathitudes 1(1) (2008), 1-9.

[4] Burton, David M.: The History of Mathematics: An Introduction. New York: McGraw-Hill (2006).

[5] Coad, Lance: Paper Folding in the Middle School Classroom and Beyond. amt 62 (1) (2006), 6-13.

[6] Courant, R. - Robbins, H.: Constructible Numbers and Number Fields. In: What Is Mathematics? An Elementary Approach to Ideas and Methods, Oxford: Oxford University Press (1996), 127-134.

[7] Demaine, Erik D. - O'Rourke, Joseph: Geometric Folding Algorithms: Linkages, Origami, Polyhedra. Cambridge: Cambridge University Press (2007).

[8] Douglas Wiggin, Kate - Archibald Smith, Nora: Froebel's Occupations. Houghton Mifflin & Co. (1900).

[9] Fenyvesi, Kristóf: The Experience Workshop MathArt Movement: Experience-centered Education of Mathematics through Arts, Sciences and Playful Activities. In: Bridges 2012 Proceedings. Baltimore: Towson UP (2012), 239–246.

[10] Fink, Karl: A Brief History of Mathematics, Chicago – London: The Open Court Publishing Company (1903).

[11] Geretschlager, R.: Euclidean constructions and the geometry of origami. Mathematics Magazine, 68(5) (1995), 357–371.

[12] `http://vismath.ektf.hu/`, as of Jan. 8, 2014.

[13] `http://vismath.ektf.hu/index.php?l=en&m=233`, as of Jan. 8, 2014.

[14] Hull, Thomas: Project Origami: Activities for Exploring Mathematics. Wellesley, Mass: A.K. Peters, 2006.

[15] Johnson, Donovan A.: Paper folding for the mathematics class. National Council of Teachers of Mathematics (1957).

[16] Messer, P. Problem 1054, Crux Mathematicorum, 12(10) (1986), 284-285.

[17] Meyer & Meyer: Teaching Mathematical Thinking through Origami. In: Bridges 1999 Proceedings. Winfield: Southwestern College (1999), 191-204.

[18] PISA 2012 Results: What Students Know and Can Do – Student Performance in Mathematics, Reading and Science. (Vol. I), PISA, OECD Publishing (2013).

[19] PISA 2012 Results: Ready to Learn – Students' Engagement, Drive and Self-Beliefs (Vol. III), PISA, OECD Publishing (2013).

[20] Row, T. Sundara: Geometric Exercises in Paper Folding. Chicago – London: The Open Court Publishing Company (1917).

[21] Zhmud, Leonid: The Origin of the History of Science in Classical Antiquity. Berlin: Walter de Gruyter (2006).

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 39–44

De Divino Errore

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Abstract

'De Divina Proportione' was written by Luca Pacioli and illustrated by Leonardo da Vinci. It was one of the most widely read mathematical books. Unfortunately, a strongly emphasized statement in the book claims six summits of pyramids of the stellated icosidodecahedron lay in one plane. This is not so, and yet even extensively annotated editions of this book never noticed this error. Dutchmen Jos Janssens and Rinus Roelofs did so, 500 years later.

1. Introduction

The book 'De Divina Proportione', or 'On the Divine Ratio', was written by the Franciscan Fra Luca Bartolomeo de Pacioli (1445-1517). His name is sometimes written Paciolo or Paccioli because Italian was not a uniform language in his days, when, moreover, Italy was not a country yet. Labeling Pacioli as a Tuscan, because of his birthplace of Borgo San Sepolcro, may be more correct, but he also studied in Venice and Rome, and spent much of his life in Perugia and Milan. In service of Duke and patron Ludovico Sforza, he would write his masterpiece, in 1497 (although it is more correct to say the work was written between 1496 and 1498, because it contains several parts). It was not his first opus, because in 1494 his 'Summa de arithmetic, geometrica, proportioni et proportionalita' had appeared; the 'Summa' and 'Divina' were not his only books, but surely the most famous ones.

For hundreds of years the books were among the most widely read mathematical bestsellers, their fame being only surpassed by the 'Elements' of Euclid. The 'Summa' was more popular than the 'Divina', though this no longer applies today because we now are especially impressed by the illustrations Leonardo da Vinci made for this book. They were the first truly insightful spatial representations of polyhedra, similar to contemporary 3D computer drawings. Perhaps, this collaboration between Pacioli and Leonardo was glorified in a well-known portrait from

1495 (see Fig. 2). It is usually attributed to Jacopo de' Barbari but this statement is questionable, as is the suggestion Leonardo was indeed the person painted in the background.

Figure 1: About this illustration of Leonardo da Vinci for the Milanese version of the 'De Divina Proportione', Pacioli erroneously wrote that the red and green dots lay in a plane

Figure 2: Portrait (1495) of Luca Pacioli and (?) Leonardo da Vinci

Pacioli's five hundred year old text seems tedious and unreadable to modern standards. Besides, the question arises whether 'The Divina Proportione' was indeed well read and verified, because chemist Jos Janssens (Leiderdorp, The Netherlands) recently happened to notice a curious statement in the book. Of some chapters about the polyhedra Pacioli studied, Janssens did a careful reading, inspired by recent discoveries of errors in Leonardo's geometric drawings (see [1], [2] and [3]). He found a questionable statement, printed in black and white, and thus is cannot be rejected as a matter of interpretation or as an inaccuracy. Janssens corresponded about this statement with the author and with his compatriot Rinus Roelofs (Hengelo), who found out Pacioli's statement was not correct. However, it is difficult to imagine that the error was never noticed, since there are so many publications about this work. However, they are mainly art history books, with a very limited focus on mathematics. Hence the current article about this error – though this paper can also be seen as a call to find out whether the age-old mistake is known or not.

2. An incorrect statement

Because of financial difficulties, 'The Divina Proportione' was only printed in 1509, in Venice, when Paganinus de Paganinus decided to take up the challenge of editing a mathematical book. Thus, it was more than ten years after the publication of the manuscript, of which two versions remain today, one in Geneva and one in Milan. In the latter two, the artwork by Leonardo da Vinci is hand-painted while in the printed version they were replaced by woodcuts. Their accuracy is a subject of debate ([4]) and this is good to know, in order to estimate precision and accuracy of evidence in those times. In any case, in the current paper the discussion is not about a drawing, but a statement that was clearly written down in precise words.

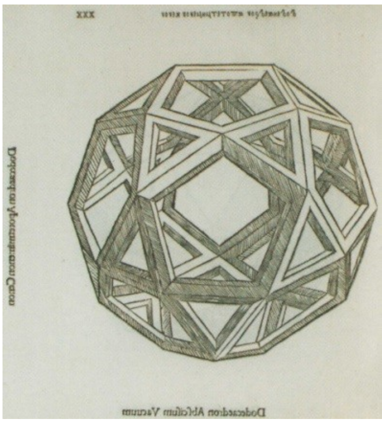

Figure 3: The truncated dodecahedron in the printed version of 'De Divina Proportione'

The statement is about the stellated icosidodecahedron, which is built on a truncated dodecahedron (see Fig. 3), as described in section 'LII' of 'The Divina Proportione' (the text was translated based on the French translation, see [4], and checked in the German version):

> *And the body that is created in this way, is composed by the flat truncated dodecahedron, inside, which shows itself to the mind only through the imagination, and by 32 pyramids, of which 12 are pentagonal, all of equal height, and of which the 20 others are triangular, all of equal height. The bases of the pyramids are the faces of the aforementioned dodecahedron and they mutually correspond, that is to say, the triangles to the triangular pyramids and the pentagons to the pentagonal pyramids. Projected onto a plane, this body will always rest on 6 tops of pyramids, one of them being a pentagonal pyramid, the other five triangular. When this body is seen in the air, it seems at first sight absurd that these vertices satisfy this property, but something like this, noble Duke, is of such great abstractness and deep science that I know that who understands me will not deny it. As for the dimensions of this body, they are obtained by the very subtle practice of primarily algebra or almucabala, which is known to few people, and is well demonstrated by us in our work, with methods allowing understanding them easily.*

Figure 4: Fra Luca Pacioli donating his book to Duke Ludovico Sforza

3. Proof of a wrong theorem?

Addressing the 'noble Duke ' in such a mathematical text was not unusual in those times. After all, Ludovico Sforza was the patron who had paid Pacioli (see Fig. 4). The self-flattering wording about the use of 'algebra and almucabala' seems outdated too, though the author right when he pretends checking geometric properties through calculation was little known in his time – after all, Descartes was just born. Now it happened Jos Janssens did what Pacioli said; he looked at the stellated icosidodecahedron. It seemed at first glance that the six points mentioned by Pacioli (the five red and one green on Fig. 1) indeed lie in one plane. Janssens Pacioli wondered if this had been verified on a concrete physical model, or whether an 'algebra and almucabala' calculation had actually been performed.

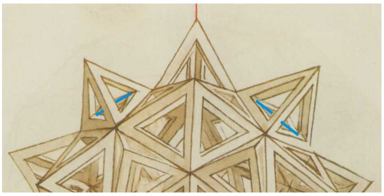

Figure 5: The same drawing as in Fig. 1, but from the Geneva version: the blue lines illustrate an error in the drawing

Janssens immediately doubted about the geometric verification on a model, because the drawing of such a model showed a lack of precision which would not have occurred when the sketch would have been made from an existing model: some lines at the top of the figure are broken, while they should have been straight (see the blue lines on fig. 5). Because the trigonometric calculations seemed rather unattractive, Janssens asked Rinus Roelofs to check it out. Roelofs would not really calculate the statement either and do the algebraic work indirectly, using the modern 3D software Rhinosceros. And perhaps the lack of such software also explains why something was not observed during five hundred years, but now noticed by Janssens and Roelofs: the claim is not true. The green dot clearly comes out of the plane defined by the five red dots (see Fig. 6).

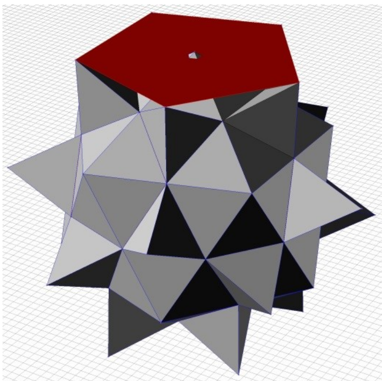

Figure 6: Roelofs' negative answer to Janssens' question about Pacioli's theorem

Did Pacioli actually prove his 'theorem' about the six coplanar points, as he announced with so much commotion to Duke Sforza? A real model he probably did not have, otherwise he would have found out the polyhedron wobbles when standing on the top of a five-sided pyramid. Why no one ever noticed an error in such a popular book may be easier to explain: the calculations are quite complicated (though not impossible) and long-winding and perhaps no one took time and effort to actually execute them. Besides, Roelofs also let the calculations to a computer. Or maybe someone noticed the error, but it got lost in the numerous historic and artistic considerations that have glorified the work of Pacioli and Leonardo - rightfully so.

References

[1] Huylebrouck, Dirk (2011). *'Lost in Triangulation: Leonardo da Vinci's Mathematical Slip-Up'*, `http://www.scientificamerican.com/article.cfm?id=davinci-mathematical-slip-up`.

[2] Huylebrouck, Dirk (2012). *'Lost in Enumeration: Leonardo da Vinci's Slip-Ups in Arithmetic and Mechanics'*. The Mathematical Intelligencer; DOI: 10.1007/s00283-012-9326-8.

[3] Huylebrouck, Dirk (2013). *'Lost in Edition: did Leonardo da Vinci slip up?'*. To appear in Leonardo, 2014.

[4] Luca Pacioli (1991). *'Divine Proportion'*. Librairie du Compagnonnage, 1980.

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 45–64

Sinister models

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Abstract

In a previous paper (see [1]) the author already examined the concept of chirality, i. e. the property of potentially having left- and right-forms, and its importance in art, and more in particular in architecture. For instance, spiral elements render objects chiral with a specific handedness, and this is also the case for dynamic chirality, induced by motion (such as in rotating restaurants or windmills). The issue is relevant not only because of the different aesthetic perceptions of left or right versions of the same object, but also for practical reasons, such as for the design of staircases. The present paper discusses the importance of the scientific (i. e. mathematical) education of artists, their anthropomorphic preference and their references to nature, yet concludes there is probably no straightforward explanation for preferring one chiral version to another in art.

1. Introduction

The word sinister would stem from the Latin word 'sinestra', meaning 'left-hand', but through times it got the connotation of 'evil-hand', and so it now also means 'inauspicious', 'wrong', 'perverse' or 'dishonest'. In heraldry, the word 'sinister' simply meant 'left of the bearer', as opposed to 'dexter' and there is an easy way to distinguish left- and right-handed people: just ask them to cross the arms (acknowledgment for sharing this information to Rinus Roelofs, The Netherlands); see figure 1.

Sty Another test for experiencing how important the left and right issue is for the body consists in turning a foot and a hand in opposite directions, that is, one clockwise and the other counterclockwise. It is virtually impossible; see figure 2.

In order to draw some attention to this 'visual mathematics' topic of left-right symmetry, I organized the first official 400 meters run on track. This happened during the 2011 Diamond League Memorial Van Damme in Brussels. The 45000 spectators liked the first '400m clockwise' quiet a lot since they spontaneously

Figure 1: One of these statues is left-handed, the other right-handed; find out which one

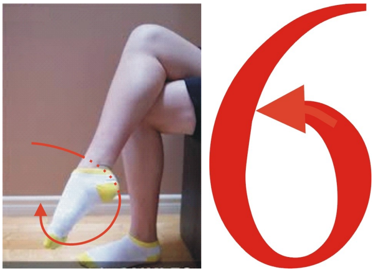

Figure 2: Another left-right test: turn a foot clockwise while describing the symbol for 6 counterclockwise

began clockwise Mexican waves during the meeting. The race itself got a surprising result too: the youngest runner won. The faster and more experienced runners had problems ending the race in a curve and the clockwise sense messed up their usual race plans.

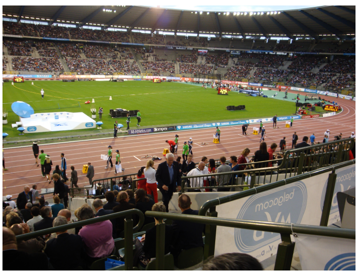

Figure 3: During the 2011 Diamond League meeting in Brussels spectators were surprised to see the runners starting in the opposite direction

In 2013 I repeated the experience in a track cycling event. During the Ghent Six Days a special race was organized with four cyclists who tried to set the new world record 500m time trial. Sponsors were happily surprised because for once racers followed the same sense as their names and slogans painted on the track, while normally they follow the opposite sense; see figure 4.

Figure 4: Cyclists normally follow the sense opposite to the direction of the sponsor messages

The riders were scientifically followed by a sport science team from the University of Leuven. Prof. Werner Helsen and his colleague Ann Lavrysen measured the heart beat and the stress factors. Not surprisingly, the riders had quite a hard time to get used to the opposite sense as a cycling track is quite steep and almost vertical in the bends. One of them simply could not really go full speed as he was too frightened. Yet the 20 years-old Robin Venneman was more audacious and crowned himself on 2013 November 20 as official world champion 500 meter clockwise time trial on track; see figure 5.

Figure 5: The participants and the new world record holder, Robin Venneman

The event also created unexpected attention from bicycle producers: why are the gears always on the right side? This seems more practical as most bicycle repairers are right handed, but for mountain bikes and in cyclo-cross situations where the bicycle has to be lifted and carried on the shoulder, it does create a problem for left handed riders who get the chain and the gear in the back; see figure 6.

We will return to these sporty matters of which the most media worthy matters were mentioned here in the introduction as to catch the attention of the reader, but

Figure 6: The gear of a bicycle is always on the right side, and this causes problems

first we will focus some more serious art and architectural matters, and some mathematical justifications, since after all this paper is addressed visual mathematics and art readers.

2. Importance for urbanism

One aspect that immediately strikes any observer in a city is the difference created by the mere fact that the traffic uses the left or right lane. To get an idea it has more surprising consequences than commonly expected, look at the drawing of an imaginary symmetric bus given in figure 7. There is no reason why the left or right side of the drawing should be the front or the back of the bus. When asked in which direction the bus would move, adults can't tell, but little children age 6 to 11 will immediately get the answer. In countries where cars drive on the right, the left side is the one shown in the illustration. Indeed, the drawing shows no door, so the entrance is at the other side, and thus when moving the bus will go to the left.

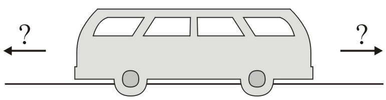

Figure 7: An "achiral bus", at least, when it is not known if its country of origin drives on the right or on the left lane

The reason why some countries drive right and others left, doesn't really seem known for sure. Some sources note in a distant past swordsmen held their weapon in their right hand, simply because the majority of the people are right-handed. Thus, would be able to defend themselves quickly against robbers jumping out of the bushes along the road. Others argue that for exactly the same reason they walk on the left side on a road, in order to defend themselves against opponents coming from the other direction. Therefore, it would follow that in forested areas people would walk or drive on the right side, while in open plains and deserts they would walk on the left.

Figure 8: From Macau to China traffic switches from left to right by an ingenious cloverleaf as shown on "Google Earth"; it has implications for the construction of parking garages

French authors often turn to Napoleon who would have been left-handed and thus would have used "the other side" of the road. Or else, they say, Napoleon may have wanted to confuse the English adversary. Though it may indeed have been Napoleon who forced most people on the European continent to drive right, the explanation only pushes the question further back in time: why then did the British walk on the left side? American writers point to 'covered wagons', or 'prairie schooners', the icons of the American Old West. Often, the driver sat on one of the horses, and there were two horses, he chose the left so that his right hand would be in the centre and his whip would thus reach other horses easily. However, it does not explain why then the driver of a traditional stage coach would sit in the middle. Also, why then would horseback riders cross the weapon to the left as shown in images of jousting knights who fought with the left shoulder towards each other; see figure 9. However, in the so-called "South-Italian style", the opposite was the case.

Figure 9: Jousting knights rode right

Other modes of transportation totally mess up all logic: boats cross each other on the right side, even in the Channel between the left British and right French. On the continent however, boats have to look carefully what the signs along the waterways impose about sailing left or right. The reason is that on some rivers with a strong water flow boats prefer to sail upstream at the outside of the curve and thus they sometimes change side; see figure 10. This change of direction is not unusual, and thus traffic signs need to warn for that danger. ‘Ghost sailors’ don’t exist, in contrast to ‘ghost drivers’.

Figure 10: On the same river, the Rhine, boats sails on the left (left image) and on the right (right image)

As trains were constructed by British engineers, their tracks follow British rule almost all over the world: one usually gets out of a train on the left. A bonus is trains in the Channel tunnel do not have to cross. Yet, even here there are exceptions: in Russia trains run right, except for a single track somewhere in the Moscow area, as it was English built. Note the situation is simpler in the air: everyone flies on the right, even in British airspace. Still, an airplane pilot sits on the left, a helicopter pilot on the right; see figure 11.

Figure 11: In airplane the pilot sits left, in a helicopter he sits right

Of course, one could imagine architects would keep in mind the above considerations when planning a building along a road where the traffic drives on the left or on the right, or, more generally, in more general urbanistic questions. However, no plan is known to the author where, say, an English plan was symmetrically adapted to, say, a French situation.

3. Running clockwise or counter clockwise?

"Ben Hur" was right: the movie showed Roman chariots racing clockwise. Some say the reason was the shortest turn was left, as Romans drove left, but this is not sure. So little is known the cartoon series “Asterix at the Olympic Games”, which also exist in a movie adaptation, shows chariots turning right, though the series even exists in Latin and is rather well documented; see figure 12. It remains surprising the ruins of so many Roman “via” or stadiums do not provide a clear-cut answer to the question about the “Roman chirality”. There seems to be but one indication, about a track leading to a quarry near Swindon, England: in 1998, archaeologists noticed the deep grooves, going away from the quarry, were on the left side.

Figure 12: In “Asterix and the Olympic Games”, Obélix runs clockwise

The confusion is understandable: as mentioned in the introduction, even today the word sinister still sometimes has an “evil” connotation, and thus it seems unlikely a chariot would reel off "to the dark side". Admittedly, it had one compelling advantage: when walking or driving counter clockwise in the arena, Caesar or his representative could be saluted easily, as Romans stretched the right arm to do so. This brings us back to the sport topic mentioned in the introduction. All logic seems to be missing, as the counter clock sense of the Roman chariots did not survive systematically. Horse races are not always held counter clockwise, even in the same country. In Australia, for example, the races in Melbourne run counter clockwise, while in Sydney they are clockwise; see figure 13. Actually, the orientation of the hair on the front of the horse head seems to be an indication for its footedness, but this could not be confirmed.

The modern version of horse races, the Formula 1, enjoys a similar confusion and they are even totally unrelated to the direction of the traffic in the organising country. Cars drive right in Bahrain, China, Spain, Monaco, Canada, Germany, Hungary, Belgium, and Italy and their Grand Prix goes clockwise (at least in 2010). However, Turkey, Korea, Brazil and Abu Dhabi drive right too, but their races run counter clockwise. In Singapore, the Grand Prix is counter clockwise too, but public transport goes left. In Australia, Malaysia and Great Britain they drive left too, but Formula 1 follows a clockwise sense. Admittedly, the most elegant solution is the Japanese, where they also drive left, but the racing track has an eight shape so that counter and counter clockwise alternate; see figure 14. By the way, this is also the case for the test track of the famous British TV-show Top Gear.

In absence of this logic, one can only guess what the reasons are for the architects

Figure 13: In some races horses show their left side to the spectators, in other their right side

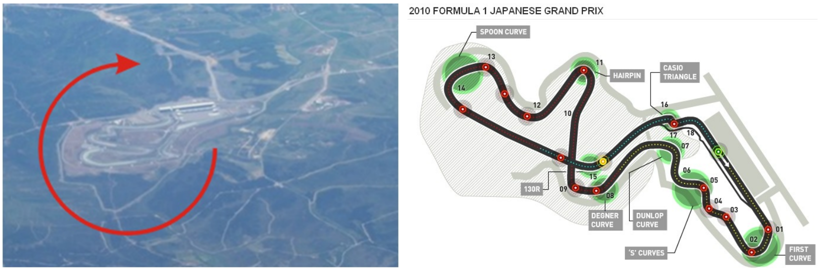

Figure 14: The Grand Prix in Turkey follows the opposite sense of the traffic in the country, most probably because this way the race can better take advantage of the geography (left); in Japan however, the eight shape of the track annihilates all chiral sport advantages (right)

designing the tracks to go clock- or counter clockwise. Some telephone calls to the Belgian organisation for the design of the cyclo-cross learned the organisers only follow the geography of the terrain. A steep start in an uphill curve for instance creates an extra difficulty, while a long straight track downhill advantages high spectacular speeds.

Though in mechanical sports the left-right issue may be overruled by the technical aspects, in physical sports it seems not to be overlooked as lightly as it always has been in sport. Yet every woman and man has a leg she or he prefers to start with, a "stronger" leg, which is not necessarily the right leg, just as not every person is right-handed. Soldiers for example learn how difficult it is to walk straight in a desert with no roads or indications. Right-footed people will walk into a large circle, counter clockwise, often without realizing they do so. And perhaps this is one of the most obvious explanations for the left turn and driving.

It implies left-footed skaters for instance are disadvantaged, and especially the short track ice skaters. They know very well they suffer because of their asymmetrical sport as many have severe back injuries. Their sportswear even has a special "patch" on one side to avoid frictions. It is noteworthy that in every picture of a short track start, every rider has her or his left foot first. This contrasts with athletics, where some start left, some right. In the 100m sprint, there is no problem, because every athlete can choose his preferred foot, but what about the 200 or 400m? Athletes invariably grumble about the disadvantage of the inside lane, and so it is curious no athlete ever complained she or he lost because of his left-footedness. In some long endurance athletic races in a stadium, the sense is sometimes changed, to avoid main in the joint pains.

Women and men practice their sport separately because of different physical constitutions, but right-and left-footed do their sport together, though left-footed athletes are discriminated. In horse races the advantage of a right footed horse is well known and here the solution is simple for a jockey with a left footed horse driving on a counter clockwise track: change the horse. But runners don't have that solution; see figure 15.

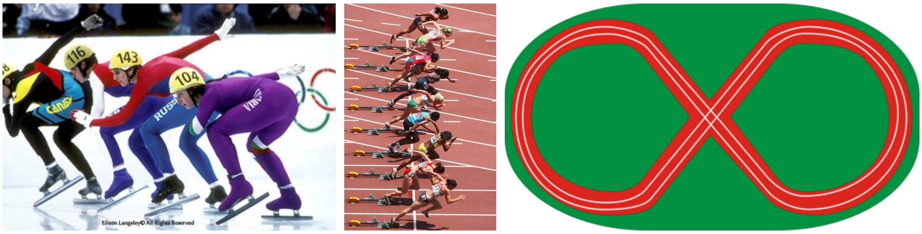

Figure 15: Short track skaters know right footed skaters have an advantage as they all start with their right foot behind; runners do not care though an 8-shape track seems fairer

4. Twisting or twisted buildings

In more recent times, sometimes the athletes or horseback riders don't run themselves anymore but a building does the work for them. So one would expect that as carousels or merry-go-round mimic the chirality sport or horse race track of the city or country where they are located. So, logically, a galloper should turn counter clockwise in Melbourne, and clockwise in Sydney. However, pony fairground rides always go clockwise, so that children can climb on the pony, by swinging their right leg over the horse, at least, when the interested children stand on the outer border of the attraction. There are fair attractions where pony rides first have to go to the centre of the mill, and climb on the horse from the center of the attraction. In that case, they run counter clockwise; see figure 16.

This confusion has implications for modern fair attractions: the rule that in Europe clockwise would dominate while in America counter clockwise is contradicted very often. Today, one finds various configurations which different rotation senses in the same amusement park. It is surprising, because in a bus or on the train, many people insist to sit direction the car or train will drive, since they otherwise

Figure 16: In the Belgian amusement park Bobbejaanland the "Cog Mill" runs clockwise, but a dolphin attraction counter clockwise. And nobody ever complained about "chirality sickness"

suffer from travel sickness, but Luna parks are not aware of complaints because of "chirality sickness".

Some modern architectural realisations seem like very tall carousels, such as the revolving restaurants located on upper stories of hotels or skyscrapers. The tower restaurants have revolving rates varying between 20 minutes to one hour. This speed seems to be chosen as to offer the client a complete 360° view during one meal (luckily, there are no revolving fast food restaurants). Otherwise, a 24h movement, following the direction of the Sun would have seemed more appropriate.

It is surprising most revolving restaurants turn clockwise. The Ginza Sky Lounge in Tokyo seems to be the rather unique exception though the web site of the Macton Corporation, specialized in the building of revolving restaurants, affirms it has a "unique drive design assuring smooth, vibration-free rotation while accommodating inevitable movement in the structure ... allowing the operator to control both turntable speed and direction (clockwise vs. counter-clockwise movement)". It is unknown however why thus different operators around the world have a predilection for a clockwise "chirality".

Architect David Fisher pushed the idea of a revolving floor further and came up a complete rotating building. His so-called 'dynamic towers' are skyscrapers changing in shape as each floor rotates around a central axis were said to be realized in Moscow and Dubai by the end of 2010 (!). As for now only some spectacular movies can be admired (`http://www.youtube.com/watch?v=4muhc_QUGcI`); see figure 17.

Maybe this is for the best: an architectural aphorism says "the only kinetic architecture that stands the test of time is a fountain". So maybe it is better just to simulate movement, as some architects have done. An Internet search shows left and right twisted building seem to be equally abundant; see figures 18, 19 and 20.

Little information can be found to explain the sense of the torsion in architecture; it does not seem to be of any concern. In contrast to what was said above, we are tempted to make a call for "architects to wake up", and maybe the links below can provide the first suggestions to a more conscious use of chirality in architecture.

Figure 17: In its edition of January 25 2011, The New York Times reported David Fisher's dynamic tower might come to New York

Figure 18: Some left handed buildings

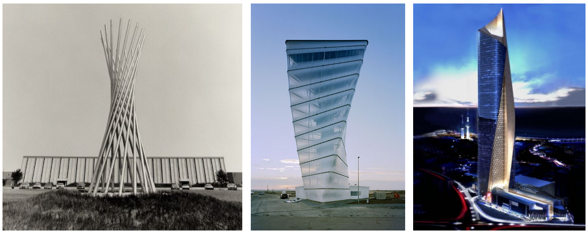

Figure 19: And some right handed ones

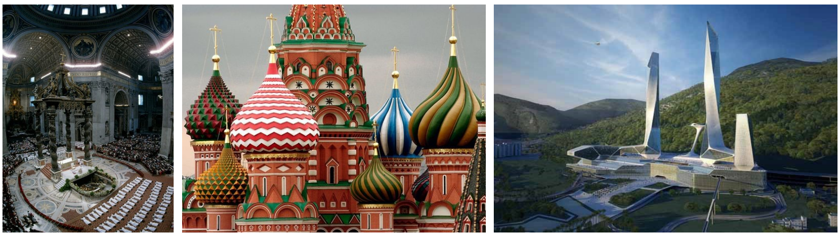

Figure 20: And finally some left- and right handed buildings

5. A possible explanation: scientific education

Artist, architects and engineers have a more or less important background in mathematics, physics, and sometimes in chemistry. In fact, it was this way I came to the topic of chirality, after a lecture by Prof. David Avnir, of the Hebrew University of Jerusalem (he won the Israel Prize for Chemistry in 2011). He stressed, for instance, how left-right symmetry can change a medication into a poison; see figure 21.

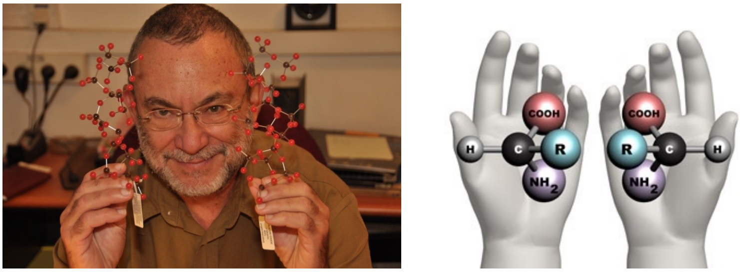

Figure 21: David Avnir and symmetric versions of a molecule

Through high school education, one gets acquainted to the right-hand rule, a common mnemonic for a notational convention for a coordinate system in 3 dimensions. When the thumb of the right hand is along the x-axis, the index along the y-axis, and the z-axis along the middle finger, the coordinate system (x, y, z) is called 'right' handed; see figure 22.

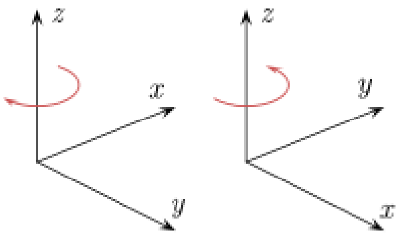

Figure 22: A left coordinate system (x, y, z) and a right coordinate system

This right handed coordinate system is the one that is used almost all the time in the mathematics and physics education for architects, especially in vector calculus, which is important for structural analysis. When corkscrew lies along the positive x-axis and turns to counter clockwise toward the positive y-axis, it will move in the direction of the z-axis. The direction in which a screw turns can be an inspiration for architects, as this sense of torsion is so generally used; see figure 23.

Note it is not as easy as it seems to teach this topic. The vector or cross product of two vector $\mathbf{a}$ and $\mathbf{b}$ has different notations, such $\mathbf{a} \times \mathbf{b}$ or $\mathbf{a} \otimes \mathbf{b}$ or $\mathbf{a} \wedge \mathbf{b}$ and is defined by $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin(\mathbf{a}, \mathbf{b})\mathbf{u}$, where $\mathbf{u}$ is defined as a unit vector perpendicular to $\mathbf{a}$ and to $\mathbf{b}$ such that $(\mathbf{a}, \mathbf{b}, \mathbf{u})$ is right handed. A common proof for $\mathbf{b}\times\mathbf{a} = -\mathbf{a}\times\mathbf{b}$

Figure 23: A screw (left) could have been the inspiration for the handedness of big screw like attraction park rides (right)

states that $\mathbf{b} \times \mathbf{a} = |\mathbf{a}||\mathbf{b}|\sin(\mathbf{b}, \mathbf{a})\mathbf{v}$ and $\sin(\mathbf{b}, \mathbf{a}) = \sin(-(\mathbf{a}, \mathbf{b})) = -\sin(\mathbf{b}, \mathbf{a})$. However, this proof omits the fact that $\mathbf{u} = -\mathbf{v}$ and thus following this proof $\mathbf{b} \times \mathbf{a}$ would be equal to $\mathbf{a} \times \mathbf{b}$ instead of being its opposite. A correct definition of the angle between the two components avoids this confusion; see figure 24.

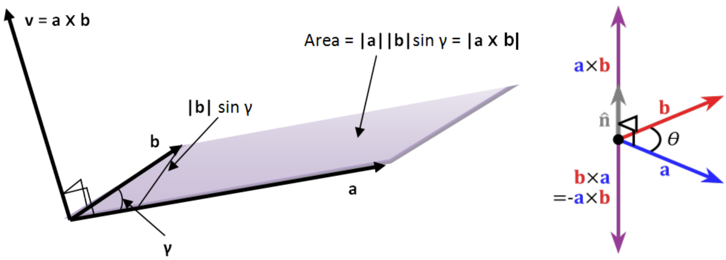

Figure 24: The definition of the vector product uses the angle from the first vector to the second, and thus this angle is always between 0 and 180°, and never negative

Anti-commutativity as in a vector product is even more important in higher mathematics. A so-called Lie algebra is vector space over a field F with operation [,], called the Lie bracket, for which [x , y] = - [y , x]. Names of famous mathematicians, such as Sophus Lie and Hermann Weyl are associated to the study of this anti-symmetry.

In elementary mathematics education, the phenomenon of dyscalculia refers to a specific developmental disorder, like dyslexia, but for learning or comprehending arithmetic, such as learning how to manipulate numbers. Yet even children without dyscalculia can have troubles when learning numbers, for instance when they use different languages as their native tongue or in their education. 27 is pronounced as ‘twenty seven’ or ’20 + 7’ in English, but becomes ‘sieben und zwanzig’ or ‘7 + 20’ in German. Also, while writing a multiplication, the numbers are written from the right to the left and not from left to right as usually. Our numbers mix left and right, because of historical reasons and traditions, but that complicates teaching them.

6. Another explanation: anthropomorphism

Sometimes, artists or architects do reveal their reason for their choice for one handedness above another. In Malmo, Sweden, the name Calatrava gave to his building says it all 'Twisting Torso'. The Skyscraper was inspired by a sculpture by Calatrava. It shows a human body turning clockwise. Of course, this only pushes the explanation further back: why then did Calatrava let the torso turn to its right side? See figure 25.

Figure 25: The building Twisting Torso (left) and the sketch on which it was based (right)

This reminds a related question: stirring tea or coffee. Right hand will mostly stir counter clockwise. In fact, there is a belief everyone must do so: in the movie version of Kipling's Jungle Book, Mowgli even learns the etiquette rule to do so, regardless the question if he would be left or right handed. Superstition has its say too: stirring clockwise would bring bad luck or wake up the devil. Some cookbooks explicitly say preparing dough for a cake or bread should be done by rotating it clockwise. Probably, for a right handed person, rotating counter clockwise goes easier.

It can have a consequence when making architectural sketches: a right handed person will more easily and more naturally draw a circle counter clockwise. On many sketches one can actually see that: the end of a hand drawn circle often is lighter, while the beginning, where the pen is placed on the paper is firmer. So, a right-handed architect will draw his circles in the first sketches of his designs counter clockwise. This might change in the near future, as architects tend to create their first ideas immediately on computer. Yet, it would be interesting to know if the architects of the left and right handed buildings given above, probably still designed in the hand sketching era, would have been themselves right or left handed.

7. Nature?

More in general, maybe chirality matters were not only influenced by human nature, but by nature in general. This is rather well documented for some architects, such as Victor Horta or Frank Gehry. The Belgian architect Horta had a large collection of pictures of all kinds of plants. In his housing and office complex "Der Neue Zollhof" in Düsseldorf, Germany, Gehry said he realised a dream of making the curved facade with the sinuous lines of square windows look like the spotted back of a deer. That would be why some buildings lean to the left, others to the right (see [2] and figure 26).

Figure 26: Striking similarity in plants and Horta's architecture (left and middle), and Gehry's "Der Neue Zollhof" in Düsseldorf, Germany

Since inspiration from nature does indeed seem substantiated, it would have an important consequence: most building should then turn in the same direction. Indeed, 92% of the vines for instance turn clockwise (implying a sensational tip for wine makers: produce a special left-handed wine from those 8% opposite rotating grapes). 94% of spiral shells run in the same direction, and who has never, architect or not, admired the beauty of shells, thus impregnating her or his visual memory with a dominant rotation. Note chirality raises a question for nature too: some plants have left and right 'version', and thus the question arises what happens when crossing these plants; see figure 27.

Figure 27: What happens when crossing these plants?

On a still larger scale, it is not unthinkable the situation of a building project on earth would have an influence on its chirality. Indeed, on the northern and southern hemisphere the prevailing winds differ. Without exception, northern cyclones rotate counter clockwise, southern clockwise. It is related to the so-called Coriolis force; see figure 28. At a smaller scale it has hardly any influence: the myth water changes it direction when flowing out of a bathtub while crossing the equator only proves the dexterity of the tourist guide, motivated by the financial incentive of watching tourists. However, the location in one hemisphere or another does imply the orientation towards the wind differs, and so this should have had some consequence in urbanism. In Belgium, for instance, at the sea side, farms the part of the roof in the South-western direction almost reaches the ground, so that the prevailing strong South-western winds can easily slide over the house. At the opposite side the roof does not go so low, and there is the entrance, sheltered from the winds by the farm itself.

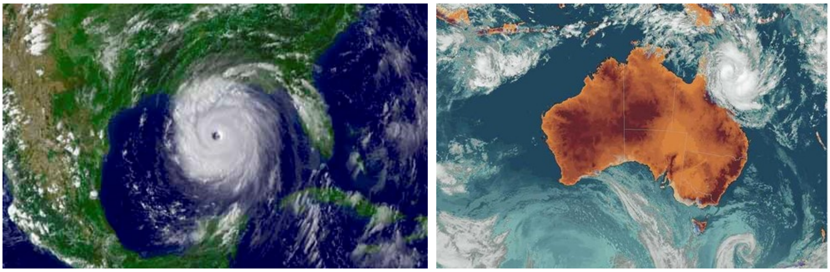

Figure 28: Tornados turn clockwise over the USA(left), but counter clockwise over Australia

On an even larger scale, it is surprising spiral galaxies are not evenly distributed in left and right turning versions. An thus, if there is a God, what he left or right handed? And if there is no God, why is there this dominance for one chiral version?

8. Indifference

As a matter of conclusion, we stress the most important consideration about chirality seems to be ... the lack of any. Fuller domes, named after Richard Buckminster Fuller, who popularised them, can be constructed starting from whatever polyhedron by subdividing more and more polygons in triangles such that the new vertex (on the normal from the center of the polygon to the surface) lays on the sphere. Usually, an icosahedron is taken as initial polyhedron but the Fuller procedures works just as well for any other polyhedron; see figure 29.

That, it surprising the two kinds of snub dodecahedra were never used as initial polyhedra. There is a left turning snub dodecahedra and a right turning one, its enantiomer. In architecture this notion has never been noticed, left aside it would have been exploited for a practical or a creative purpose; see figure 30.

Similarly, the two versions of a snub cube or of different tessellations have never been exploited in any design or ornament; see figures 31 and 32.

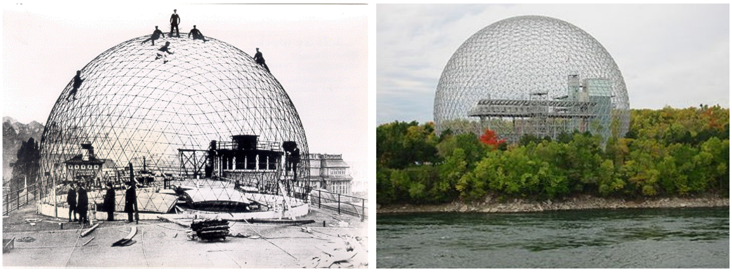

Figure 29: The first "Fuller" dome ever, the geodesic dome planetarium in Jena, Germany, constructed by W. Bauersfeld in 1926 (left) and Fuller's "Montreal Biosphère" (1967, right)

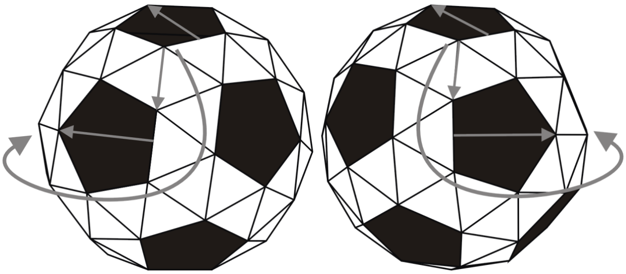

Figure 30: A left and a right snub dodecahedron

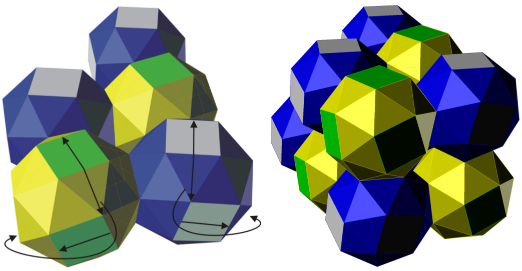

Figure 31: Snub cubes come by 2 to fill space

9. Example 1: staircases

In medieval castles, spiral staircases were designed so that a right-handed defender could more easily stop an attacker coming from below. Placing his right foot on a lower step, the defender could fully swing his sword down to the attacker. Thus, spiral staircases in old castles run clockwise when going up – in most cases, there were exceptions. A funny legend pretends the members of the Kerr clan were mainly left-handed so their headquarters, the Ferniehirst castle in Scotland, has a counter clock wise climbing staircase for that "genetic" reason. Today, this chiral

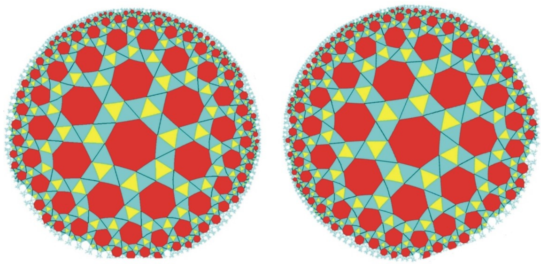

Figure 32: Two hyperbolic tessalations, by Radmila Sazdanovic (Serbia/USA)

aspect of a stair is a tourist attraction; see figure 33.

Figure 33: A medieval spiral staircase moved up in clockwise sense (left) while in the opposite direction for left handed knights (right)

Today, this question about how spiral stairs should turn seems to be forgotten by modern architects. However, it is easier to go up the stairs on the toes, and to go down on the heels. So, since going up is harder than going down (in mountain countries cars going up have priority), ascenders should have the advantage to use a handrail. Since the majority of the people are right handed, even today spiral stairs should ascend clockwise so that smaller part, used for going up, is at the side of the banister. If also lets people cross on the stairs left shoulder to left shoulder. Unless the stairs are not large, in which case ascending counter clockwise seems appropriate, so that the ascending person can hold the handrail on the wall; see figure 34.

10. Example 2: windmills

A special case where the explanation seems known and acceptable is the case of windmills. English scientist A. Rowan, who works in The Netherlands, was surprised when seeing all those Dutch traditional wind mills turning in the same counter clockwise direction. His explanation is that trees twist slightly while growing, ‘following the sun’, just as young sunflower plants (note full grown sunflowers

Figure 34: People going up on this stair (using the smaller part, which is easier when climbing on toes) have to hold the ramp with their left arm; did the architect think about this?

do not turn, despite what a common story tells). Thus, when used as a beam for the axis of a windmill, traditional wind mill builders made them turn in one direction, the counter clockwise one; see figure 35.

Figure 35: Traditional wind mills turn in the direction imposed by the trees used as their rotation axis

Figure 36: An erroneous windmill from Hungary and another wrong drawing, from Serbia

However, there are trees growing in the opposite direction. Also, modern wind turbines rotate clockwise, as a consequence of some strange agreement made in the seventies in Denmark among wind turbine producers. Thus, some older models, such as in Ireland, turn otherwise, but the modern general tendency is clockwise.

Nevertheless, it suggested the two pictures of windmills given in figure 36 are most probably wrong.

References

[1] Avnir, D. and Huylebrouck, D., (2013). *On Left and Right: Chirality in Architecture.* Nexus Network Journal, 15 (1), 171-182.

[2] Karel Vollers, *Twist and Build: Creating Non-orthogonal Architecture*, Uitgeverij 010, The Netherlands 2001, 288 pp

[3] Web site about stairs: `http://en.wikipedia.org/wiki/Stairway`.

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 65–73

Developing Algorithmic Thinking Using Crocheting Patterns as Educational Tool

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Abstract

In this paper we are emphasizing the importance of developing algorithmic thinking, for improving problem solving skills. Problem solving competence is important for one of eight key competences defined at EU level - mathematical competence. We will find a relationship between Polya's problem-solving and algorithmic thinking, comparing basic Polya's principles to definition of algorithmic thinking. So, we are considering algorithmic thinking as an important role in high school education for developing mathematical competence. We are using crocheted geometrical shapes and finding a mathematical model to realize it through crochet. We developed algorithms for crocheted models, as a useful educational tool.

1. Introduction

Eight key competences have been defined at EU level. Those competences represent a combination of knowledge, skills and attitudes that are necessary for "personal fulfilment and development; active citizenship; social inclusion; and employment" [1]:

- communication in mother tongue
- communication in foreign languages
- mathematical competence and basic competences in science and technology
- digital competence

- learning to learn
- social and civic competences
- sense of initiative and entrepreneurship
- cultural awareness and expression.

"Mathematical competence is ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations, with the emphasis being placed on process, activity and knowledge." [2]

According to a given mathematical competence we are analyzing Georg Polya's principles of solving problems. In everyday life there are problems that have to be solved. Solving a problem means, to find an adequate procedure for its solution. Thinking about daily procedures is helping each person to develop basic algorithmic skills.

2. "How to solve it", George Polya

The father of problem-solving, George Polya, gave specific advices to teachers, how they should teach mathematics. According to his principles, the main role of teaching mathematics is to develop thinking, clarity and persistence. The most important part of thinking, developed in mathematics, is solving problems. [3]

In his book "How to solve it", that has been published in 1945, Polya identifies four basic principles of solving problems:

1. Understand the Problem
2. Devise a plan
3. Carry out the Plan
4. Look back. [4]

These principles indicate that problem-solving process consist of following steps: [5]

- Identifying the problem
- Understanding the problem
- Representing the problem
- Solving the problem
- Communicating the results

3. Algorithmic Thinking and Algorithm

Gerald Futschek gave a definition of algorithmic thinking as a special problem solving competence, which consists of several abilities:

- Analyze given problem
- Specify problems precisely
- Find the basic action that are adequate to given problems
- Construct correct algorithms to given problems using basic actions
- Think about all possible special and normal cases of a problem
- Evaluate algorithms
- Improve the efficiency of algorithms. [6]

Beside these abilities, algorithmic thinking is influenced by many other human cognitive factors as abstract and logical thinking, thinking in structures, creativity and problem solving competence.

Let us compare Futschek's definition of algorithmic thinking with Polya's problem-solving process:

- Analyze given problems ⇔ Identifying the problem
- Specify problem precisely ⇔ Understanding the problem and Representing the problem
- Construct correct algorithms to given problems using basic actions and Think about all possible special and normal cases of a problem ⇔ Solving the problem.

This comparison represents real connection between algorithmic thinking and problem-solving process in mathematics.

Algorithmic thinking contributes to the understanding of problem solving and because of that has pedagogical value, as Donald Knuth said:"... a person that is taught how to deal with algorithms: how to construct them, manipulate them, understand them, analyzed them has a general-purpose mental tool which will be a definite aid to his understanding of other subjects. The reason for this may be understood in the following way: It has often been said that a person does not really understand something until he teaches it to someone else. Actually a person does not really understand something until he can teach it to computer, i.e. express it as an algorithm." [7]

Algorithm is a precisely-defined sequence of rules telling how to produce specified output information from given input information in a finite number of steps, so "... the attempt to formalize things as algorithms leads to a much deeper understanding than if we simply try to comprehend things in the traditional way" [7]

4. Problem-solving, Algorithmic Thinking and Crocheting

Polya's Second Principle of problem solving, Devise a plan, says that there are many ways to solve a problem. It is important to find a strategy for a solution. It can be done by solving many problems. Choosing a strategy is very easy, Polya says. And he gave a partial list of strategies that could be used for solving problems [8]:

- guess and check
- make and orderly list
- eliminate possibilities
- use symmetry
- consider special cases
- use direct reasoning
- solve an equation
- look for a pattern
- draw a picture
- solve a simpler problem
- use a model
- work backward
- use a formula
- be ingenious.

Each crocheted geometrical shape has a specific rule that is used to build it.

Doing crocheting we could start from patterns and build a model (look for a pattern, solve a simpler problem) or we can try to decompose into the pattern a model that is already made (use model, work backward). In both cases we are drawing picture, using symmetry, considering all possible cases, using direct reasoning and finding a formula that describes the process of crocheting a pattern or a model.

The main goal is to construct an algorithm based on a mathematical model of crocheting a specific pattern or a specific model.

Mathematical Foundations of Crocheting

The material used for doing crocheting is a yarn, a three dimensional line. With this line, the crocheter is building meshes, which are the basic elements of crocheting.

One can build with a crochet needle the following types of meshes:

1. the chain stitch **O**

2. a single crochet **X**

3. a double crochet ŧ

After the double crochet you can also build a triple crochet, a quadruple crochet and so on, but those can be seen as modifications of a double crochet. These symbols are internationally used to draw crocheting patterns.

A main property of crocheting is also notable in the term of the chain stitch. Logically, by using a yarn, we are obliged to build one mesh after the other. We are building chains of meshes. Or ROWS. After the first Row, we can do a flip stitch to turn the work around and to build another row above the row before. If we do that several times, we will get several rows.

One single mesh is as high as wide. So actually it is not far away to see it as a Cartesian coordinate system, dimension NxN, where each mesh is a crossing point of both sets. If we would be extremely correct, we should see it, because of the three dimensionality of the crocheted object, as a coordinate system dimension NxNx1 - because the surface is as thick as one mesh.

Our measurement in crocheting does not necessarily have to be in centimetres or millimetres. Because of that we are able to define a new measure - 1 mesh.

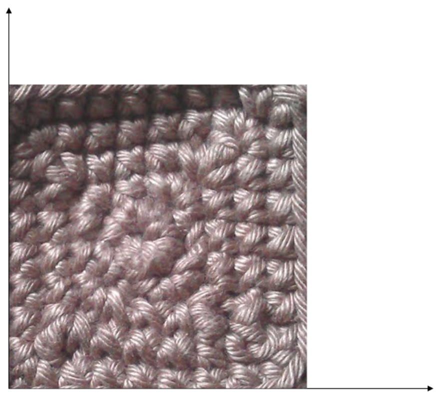

Figure 1: Cartesian Coordinate System, $N \times N$

5. Examples of Algorithms

We were curious: is it possible to crochet a perfect circle, triangle, square? And, if it is: is there an algorithm that will describe building regular 2D geometrical shape, or one can rely on intuitive approach?

5.1. Crocheting a Circle

Let us consider a perimeter of circle given with formula $P = 2\pi r$, where r is radius of a circle. Since we can not have a half of a mesh or a third of a mesh, there is a need to take approximation of number π as $2\pi \approx 6.28 \approx 6$.

Assume that $r = 1$ mesh, then $P = 6 \cdot 1$ meshes $= 6$ meshes. And, more further, let it be $r = 2$ meshes, then we have $P = 6 \cdot 2 = 12$ meshes. Then, if $r = 3$ meshes, $P = 6 \cdot 3 = 18$ meshes.

So, we have now following results:

$$r = 1 \Rightarrow P = 6 \text{ meshes}$$
$$r = 2 \Rightarrow P = 12 \text{ meshes}$$
$$r = 3 \Rightarrow P = 18 \text{ meshes.}$$

The picture below shows the idea of crocheting circle.

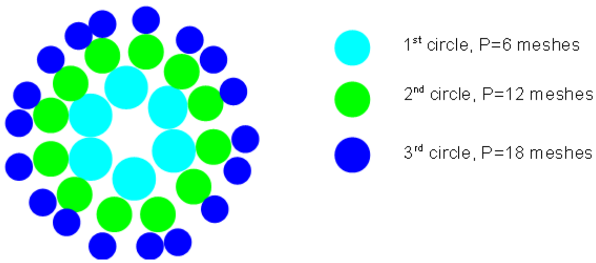

Figure 2: Crocheting a Circle

Observing the results of perimeter we can conclude that those numbers are elements of arithmetic progression with a common difference $d = 6$ and initial term $a_1 = 6$.

Based on previous we have following algorithm:

Step 0: build a chain of 3 or 4 meshes

Step 1: build a circle using the chain from Step 0

Step 2: make a circle with 6 meshes (initial term of arithmetic progression)

Step 3: build a circle with $6 + 6 = 6 + 1 \cdot 6 = 12$ meshes, by stitching two meshes in every $6/6 =$ 1st mesh of previous circle

Step 4: build a circle with $6 + 12 = 6 + 2 \cdot 6 = 18$ meshes by stitching two times in every $12/6 =$ 2nd mesh of a previous circle (and one stitch in all other meshes)

Step 5: $6 + 18 = 6 + 3 \cdot 6 = 24$ meshes, by stitching two times in every $18/6 =$ 3rd mesh of every previous circle (and one time in all other meshes)

Step N: $6 + (n-1)6$ meshes by stitching two times in every $[(n-1)6]/6 = (n-1)$th mesh (and one time in all other meshes).

Since we have to think about all special cases there is a question: what if we want to make initial circle with random number of meshes? In this case we have to modify the given algorithm. The modification consists of modification Step 2 and Step 3.

Step 2: build a circle with K meshes

Step 3: IF K mod 6 = = 0 THEN DO Step4
ELSE
(find the smallest number M that is greater then K) AND (M mod 6 == 0)
AND (make a circle with M meshes) // M is initial term of arithmetic progression

Figure 3: Crocheted Circle

5.2. Crocheting a Square

Let us think of a perimeter of a square. Since mash can approximate a unit square, we are counting unit squares in the perimeter. Let a be a length of a side of a square.

$a = 1$ there is 1 mash in a "perimeter"
$a = 2$ there are 4 mashes in a "perimeter"
$a = 3$ there are 8 mashes in a "perimeter"
$a = 4$ there are 12 mashes in a "perimeter"
...
$a = n$ there are $4(n - 1)$ mashes in "perimeter"

In each new square, we need to have 8 new meshes. According to this we are able to construct an algorithm for crocheting a perfect square based on arithmetic progression (initial term of arithmetic progression $a_i = 4$, common difference $d = 4$):

Step 0: build a chain of 2 or 3 mashes

Step 1: build a circle using the chain from Step 0

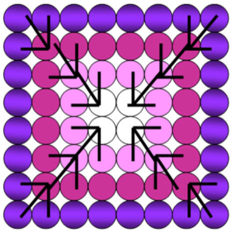

Figure 4: Crocheting a Square

Step 2: make a square with $a = 2$, with 4 mashes (initial term of arithmetic progression)

Step 3: make a square with $a = 4$, with $4 + 8 = 12$ meshes, by making 3 mashes in every $4/4 = 1$st mash of a previous circle

Step 4: make a square with $a = 6$, with $12 + 8 = 20$ mashes, by making 3 mashes in every $12/4 = 3$nd mash of a previous circle

Step N: make a square with $a = n$, with $4(n\breve{}1)$ mashes, by making 3 mashes in every $4(n - 3)/4 = (n - 3)$th mash of a previous circle

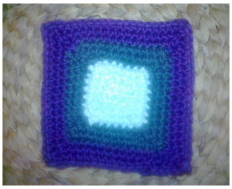

Figure 5: Crocheted Square

6. Conclusion

To develop mathematical competence, one of eight key competences that have been defined at EU level, we compared Polya's principles of problem-solving to definition algorithmic thinking.

Developing algorithmic thinking each person could develop basic mathematical competence.

The main goal is to construct an algorithm based on mathematical model of crocheting a specific pattern or a specific model.

In our own experience, students should work on

- making a clear idea of a model
- visualisation of a model

- calculation of some function (in our example perimeter for a sequence of circles/squares)
- ability to make a drawing of a model
- skill of crocheting, so she/he could make experiments
- think and rethink, going back, if experiment fail
- make an algorithm, i.e. put a solution in a reusable form, from beginning to the end

This is an example how students could be encouraged to develop their mental processes which will lead them to succeed in problem solving. Combination of thinking, visualizing, drawing and handcraft make this sort of teaching usable for students with different cognitive learning styles - visual, auditory, and kinaesthetic/tactile.

That is why we are considering crocheting patterns as a good educational tool for developing algorithmic thinking and using it in a teaching mathematics.

References

[1] Developing Key Competences for School in Europe, `http://eacea.ec.europa.eu/education/eurydice/documents/thematic_reports/145EN.pdf`

[2] `http://europa.eu/legislation_summaries/education_training_youth/lifelong_learning/c11090_en.htm`

[3] `http://cmc-math.org/members/infinity/polya.html`

[4] `http://www.math.wichita.edu/history/men/polya.html`

[5] L. Zsakó, P. Szlávi, ICT Competences: Algorithmic Thinking, Acta Didactica Napocensia, Volume 5, November 2, pp. 50. 2012.

[6] G. Futschek, Algorithmic Thinking: The Key for Understanding Computer Science. In Lecture Notes in Computer Science 4226, Springer, pp. 159 - 168. 2006.

[7] D. E. Knuth ,Computer Science and Relation to Mathematics, American Mathematical Monthly, 81, pp. 323-343. 1974.

[8] `http://www.math.wichita.edu/history/men/polya.html`

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 75–82

Perception of Space in Painting

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Abstract

During the history, perception of space in painting is changed from one- and two-dimensional geometric patterns, that dominate in Paleolithic and Neolithic art, through "hierarchical perspective" and orthogonal axonometry used in Egyptian painting, Byzantine counter-perspective, Renaissance linear perspective, cubistic polycentrism, perceptive perspective, to the non-orientable space of abstract painting. Trying to explain 3D-vision as the reconstruction of 3D-image from its 2D-projection, that is in general not unique, we will consider different extreme forms of perspective (e.g., anamorphoses), or the formation of ambiguous reconstructions of 2D-projections resulting in visual illusions and impossible figures.

Keywords: perception, space, perspective, visual illusions, impossible figures

1. Preliminary notes

In this paper we will try to give brief view on perception of spaces throughout history of art and panting as well as mathematics and design. During the history, perception of space in painting is changed from one- and two-dimensional geometric patterns that dominate in Paleolithic and Neolithic art, through "heirarhical perspective" and orthogonal axonometry used in Egyptian painting, Byzantine counter-perspective, Renaissance linear perspective, cubistic polycentrism, perceptive perspective, to the non-orientable space of abstract painting. Trying to explain 3D-vision as the reconstruction of 3D-image from its 2D-projection, we will consider different extreme forms of perspective or the formation of ambiguous reconstructions of 2D-projections resulting in visual illusions and impossible figures. We will start with idea of symmetry which is in behind all different perceptions of spaces.

2. The Idea of Symmetry

The laws of nature and the objects of human creation are representations of symmetry. The word symmetry originates from Greek science and corresponds to the term "common measure" ($\zeta\iota\mu$ = „common“, $\mu\varepsilon\tau\rho\omega\nu$ = „measure“), directly pointing to one of the two most important problems of the Greek mathematics– the question about the commensurability of two line segments. The other important area of study for Greek mathematics was the theory of regular geometrical figures: regular polygons, polyhedra (Platonic bodies), uniform (Archimedean) polyhedra, inextricably connected to symmetry. The other meaning of the word symmetry originates from Greek philosophy and aesthetics and it is connected to a spectrum of philosophic-aesthetic terms: harmony, proportionality, balance, well-behaved form, etc. Throughout history, the universality of symmetry was reduced to its simplest form– bilateral or mirror-symmetry. In the written form, the word simetria appears for the first time in the Latin text „Tratato I“ by Francesco di Giorgio Martini. At that time, the term „symmetry“ had already lost its universality and was related only to architectural structures and their bilateral symmetry. One of the main Renaissance theoreticians of architecture, Vitruvius, and other Renaissance architects often used this word in their writings.

We find symmetry almost everywhere in nature. Certainly, any symmetry can be followed by a breaking of symmetry, dissymmetry, and some deviation from perfect symmetry which results in variety of forms Symmetry in art reflect symmetry in nature. Since Paleolithic times, the oldest period of human civilization, symmetry has played an important role. After representing a single motif, a deer's head, the Paleolithic artist continued to repeat this figure by reducing and stylizing it, with the result being a frieze based on translation (parallel motion). In the same way, the series of figures in an Egyptian fresco perfectly illustrates translational symmetry (on the left in Figure 1). Throughout history symmetry has occurred in very different forms as artists tried constructing different symmetric artworks such as rose-windows from the Chartres cathedral, or the Op-art works by Victor Vasarely based on squares(on the right in Figure 1) .

Figure 1: Symmetry in arts

Rennaissance scientists and artists were fascinated with regular polyhedra, i.e., ideal geometrical bodies. We all know the famous Leonardo da Vinci's drawing „Vitruvian man“– the composition of a human figure according to ideal proportions. From ancient times, beginning with Egyptian culture, the theory of proportions was applied to sculpture and architecture, i.e., buildings and sculptures were proportioned according to a canon. To an even greater extent, detailed and precise canons were used in ancient Greece, and in the Renaissance Leonardo tried to use these canons to inscribe the human body in the circle and square, archetypal forms representing Heaven and Earth.

3. Paleolithic and Neolithic ornaments

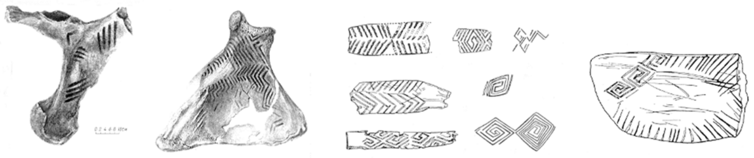
The first examples are symmetric ornamental patterns and friezes that Paleolithic man drew on the walls of caves or engraved on bone, giving testimony to his ability to recognize, record, and create symmetry. A handprint was probably the oldest symbol in the history of mankind, the first attempt of a man to leave evidence of himself.

Figure 2: A prehistorical art

A prehistorical artist simplified the drawing of a herd of deer by stylizing it and, reducing it to a repetition of pairs of horns, and obtained a symmetrical pattern called a frieze or bordure – a decorative motif based on translational repetition (Figure 2).

We have found that the oldest examples of ornamentation in Paleothic art were from Mezin (Ukraine) dated to 23 000 B.C. Note that 23 000 years is a time period ten times longer than the complete written history of mankind. At first glance, the ornament on the right side of Fig. 3a appears to not be significant, it is an ordinary set of parallel lines. On the right side of Fig. 3b this pattern is transformed into a set of parallel zig-zag lines– an ornament with a symmetry group of type pmg, generated by an axis of reflection perpendicular to another axis of glide reflection (Fig. 3b). Let's see how the creative process for the design of this ornament may have developed. Imagine a modern engineer who begins a construction project. At first he makes a rough sketch, and then he begins to work more seriously to solve the problem. The next series of ornaments from Mezin is more advanced. The previously mentioned sets of parallel lines are arranged in friezes and meander patterns (Fig. 3c, d).

In Figure 4a we see the final result, the masterpiece of Paleolithic art – the Birds of Mezin decorated by meander ornamentation. The man of prehistory has applied the symmetry constructions that he learned, and he has preserved them for posterity. On the mammoth bone, modeled in the form of a bird, he engraved



(a) (b) (c) (d)

Figure 3: Basic patterns from Mezin

the meander pattern which represents the oldest example of a rectilinear spiral in the form of meander ornamentation.

(a) (b)

Figure 4: (a) Bird of Mezin; (b) developed bracelet

The next artifact is an engraved bracelet from the same excavation site (Fig. 4b). If we look at this bracelet in developed form, we notice that there is a continuous transition from one ornament to another via a third ornament: on the left corner, reminiscent of the famous print „Metamorphoses" by M.C.

Escher. You can see the meander ornamentation, then the set of parallel zigzag lines used as a symbol of water, and again the continuous transition to another meander ornamentation. [2,3].

The ornaments on Fig. 5 look very different one from another. Among them are black-white and colored ornaments, and at first glance, it appears that there is no unifying principle. Their common property is that they all consist of a single element (module). Notice the small black-white square in the middle. It consists of a set of parallel diagonal black and white lines (strips). If this square is used as the basic motif, then all of these ornaments can be constructed from it. We call this method of construction the principle of modularity. Our goal is to construct all ornaments or structures by using the smallest number of basic elements (modules) and to obtain, by their recombination, as many different ornaments (structures) as possible. This module, a square or rectangle with a set of parallel diagonal black and white strips, we will call an Op-tile; it is the basis of Mezin meander patterns (Fig. 4).

The whole of Neolithic art is characterized by the use of spiral ornaments, dating from the period from 6 000 to 2 000 B.C., where the most important ornaments are those from the Tripolian culture, coming from the excavation sites of Butmir,

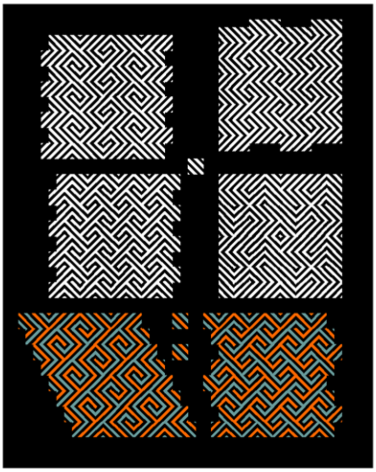


Figure 5: Modular key-patterns

Lepenski vir, and from other cultures such as the Aegean cultures. The Neolithic period is the time when ornamental art (especially the construction of „black-white“ patterns) greatly flourished and most of the ceramic ornaments originate from Neolithic times.

Figure 6 shows a series of ornaments from Titsa culture (Hungary) and Vincha (Serbia), dating to 3 000- 4 000 B.C. Notice that ornaments from Vinca (Fig. 6b) are all based on meanders, continuing the tradition of Paleolithic ornaments from Mezin and Scheila Cladovei They are painted on ceramic and can be found in similar Neolithic settlements. By observing the numerous examples of Neolithic „black-white“ ornaments, with the black part („figure“) congruent to the white part („ground“), we conclude that all of them originated from basketry, matting, plaiting, weaving, or textiles and then were copied to the stronger media of stone, bone and ceramic. Many of these ornaments are obtained from interlaced patterns. Antisymmetry is the symmetry of positive and negative, light and shadow, black and white, „over-under“. Therefore, antisymmetry can be used for so-called dimensional transition.

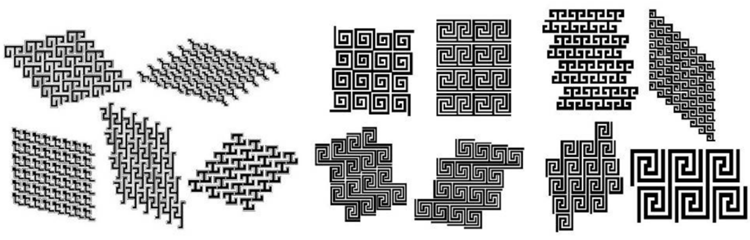


Figure 6: Neolithic ornaments from Titsa and Vincha culture

The best textile patterns were copied to ceramic vessels which requires great skill since the surface of the ceramic vessels are curved. We can find similar examples all over the world (e.g., in Neolithic Lapita ceramics from Fiji (Fig. 7a), or Anasazi ceramics, (Fig. 7b).

(a) (b)

Figure 7: (a) Lapita ceramics (Fiji); Anasazi ceramics

4. The development of perspective

Now, we will put attention on Ancient civilizations. The first is Egyptian civilization and how did they represented the space. They have been using so coaled low of frontality, i.e. orthogonal representation where all the shapes are on 90 degrees. They didn't know about perspective, but they used "hierarchical perspective" the most important persons were the biggest, as figures of pharaohs, while the figures of slaves are very small. Also, they used "the principle of superposition: the figures on the bottom are the closest one and so on. Minor scenes at the bottom of a painted image are shown at a far smaller scale than the main figures higher up. They also used to put the plane of projection down in frontal plane, and did have canons for ideal proportions.

In art of Assyria and Babylonia, "hierarchical perspective" is also used. This period of arts is famous on relief with time dimension: they represent the series of persons in few different levels.The art of Ancient Greek is very famous. During the geometric period, people and animals depicted geometrically in a dark glossy color, while the remaining vessel is covered by strict zones of meanders, crooked lines, circles, swastikas, in the same graphical concept. They learned to use local perspective in form of local dilatation, but there is no some general principle.

In the middle age we can find the inverse perspective in the Byzantine art. Inverse perspective, also called reverse perspective, inverted perspective or Byzantine perspective, is a convention of perspective drawing where the further the objects are, the larger they are drawn. The lines diverge against the horizon, rather than converge as in linear perspective. Technically, the vanishing points are placed outside the painting with the illusion that they are "in front of" the painting. The name Byzantine perspective comes from the use of this perspective in Byzantine and Russian Orthodox icons; it is also found in East Asian art, and was sometimes used in Cubism and other movements of modern art.

The first attempt of linear perspective can be found in the period of early Renaissance. The earliest surviving use of linear perspective in art is attributed to Donato di Niccolò and Masaccio. Finaly, we came to Leon Battista Alberti (1404-1474).

Figure 8: Battista Alberti's set for perspective drawing

He was humanist scholar, natural scientist, mathematician, cryptographer and architect. Alberti was the first who put the theory of perspective into writing, in his treatise on painting, Della pittura (1435). Alberti described how an artist could get a correct view of a scene by observing it through a thin veil, or velo. The idea is that we can get a correct image of some object seen through such a veil or a window by tracing the outline of the object on the window glass.

Here we have to mention gret Leonardo da Vinci (1452-1519). Leonardo describes another kind of perspective, which we now called atmospheric perspective, which anticipate the doctrines of impressionism. Distant objects appear smaller, less distinct, paler, and bluer.

From the moment when painters solved the construction problems, accepted the exact rules of the linear perspective, and become able to consequently represent more sophisticated 3D objects, sceneries of cities or mass-scenes, they started experimentation and search for the new possibilities how to use the extreme forms of perspective: unusual, non-conventional viewing points or angles. For Example, in Dali's panting Christ of Saint John of the cross, 1951, dominates the traditional motif of the cross, emphasized by using such limiting, extreme perspective, and the combination of two perspective views in the same painting: the simultaneous use of two centers of perspective. Cubism also used a new way to represent space object from several different point of view on the same picture in the same time, so instead monocentism as in linear perspective, now we have polycentrism.

5. Impossible figures

The basic building block of impossible objects is the Kofka cube: a regular hexagon divided into three congruent rhombs. All three sides of the Kofka cube are identical, so we cannot tell from which of three equally possible points of view it is being viewed, whether it is convex or concave, or even if it represents a 3D-object, or is it a regular hexagon consisting of three rhombuses, which, acted upon by plane isometries, results in a rhombic tessellation.

Impossible figures are figures that contradict our sense of visual 3-dimensional perception. But, are impossible figure possible? At first glance the two cubes on the left side of the slide represent an impossible object. However, you can take an ordinary cube, add to it a part in the form of an open book, join it to the cube, and obtain a real 3D-object which gives an impossible figure in retinal projection. The Penrose tribar can also be modelled by a real 3D-object.

Figure 9: Impossible figures

In the process of visual perception our eye and brain makes a choice and accepts the simplest solution even if it contradicts our perception of 3D-objects and represents an impossible object.

References

[1] Barrett C., Op art, Studio Vista, London, 1970

[2] Jablan S., Symmetry, Ornament and Modularity, World Scientific, New Jersey, London, Singapore, Hong Kong, 2002.

[3] Jablan S., Modularity in Science and Art, Visual Mathematics, 4, 1, 2002 (http://www.mi.sanu.ac.rs/vismath/jablan/cover.htm).

[4] Radovic Lj. and Jablan S, Antisymmetry and Modularity in Ornamental Art, Visual Mathematics, 3, 2, 2001 (reprint from "Bridges" Proceed., 2001, pp. 55-66),http://www.mi.sanu.ac.rs/vismath/radovic/index.html)

[5] Sarhangi R, Jablan S. and Sazdanovic R, Modularity in Medieval Persian Mosaics: Textual, Empirical, Analytical, and Theoretical Considerations, Visual Mathematics, 7, 1, 2005 (reprint from "Bridges" Proceedings, 2004, pp. 281-292),(http://www.mi.sanu.ac.rs/vismath/sarhangi/index.html)

[6] Collection of the Vasarely Museum, Pécs.

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Belgrade, Serbia, September 17–19, 2014. pp. 83–90

Fractal structures in architectural high school

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Abstract

Within this work, it will be presented how the teaching of geometry can be more interesting and more accessible to students in architectural high school by using various modern media. The authors have tried a different approach to motivate students to identify geometric shapes in the world around us and to describe their properties using mathematical language. It also shows the correlation between maths and art. The motive for this approach to teaching math is raising the level of students' motivation as well as the aim to bring them closer to mathematical concepts through visualization.

Keywords: teaching mathematics, subject of arts, computer application.

1. Introduction

A large number of students in our school are interested in studying architecture. The curriculum for mathematics in architectural high school and the Faculty of Architecture are quite different. We wanted to introduce the students to certain concepts which are studied at the faculty on an intuitive level. We were highly motivated because we believe that introducing students to some of the mathematical concepts at an intuitive level will be of great help in mastering tasks easier in the future.

We decided to use the term fractal because it is a very interesting mathematical concept that is an integral part of the course of Mathematics at the Faculty of Architecture and fractal structures are present everywhere around us. They can be seen in the architectural buildings, art, nature, genealogies and our body.

We tried to present this topic in each class.

While preparing for the work in the classroom, we became familiar with the works of other colleagues on this topic. We tried to adjust this complex mathematical concept to high school students, but also to monitor the content done at the university. Experience of colleagues that we found on the Internet was of great help.

Overview of activities by grade:

2. The first grade

2.1. On Maths lesson

The moment in which we made a cut and introduced the term fractal was after teaching unit Similarity. We thought this was a good time to talk to students about self-repeating of shapes, figures and bodies. Teaching units we previously dealt with in detail were Isometric transformation and Homothety. Also, at the end of these topics, the students had a knowledge test.

Slide presentations were of great help. A lot of our presentations were based on the use of these sites. We used a rich knowledge Data base of Creative school (`http://www.kreativnaskola.rs`). We presented the visual beauties of this field of mathematics by displaying images and films. It was very convenient to use a ready material for demonstrating some of the concepts and figures, as well as characteristics, exploring the definitions, theorems.

The students collaborated more with each other, but they were also more active in the student-teacher communication. They willingly participated in analyzing the presented images, films, self-repeating details.

All displayed presentations, photos, movies, students are able to find on our website, which was created just for this theme and re-examine them individually. Web page address is `http://www.alas.matf.bg.ac.rs/~mm97045/homotetija/`.

2.2. Homework

We insisted on visualization and identification of fractal shapes in the world around us. Furthermore, self-repeating of shapes was one of the themes on which we put the emphasis. The students had to make fractal out of paper for their homework. Some of them did this on maths lessons and some independently at home. In addition to paper visualization, the children were given the task to identify in their surroundings a fragment of fractal geometry and to photograph it. They had to forward the pictures to the teachers by e-mail. Apart from this type of homework, the students also had a regular homework within they solved the problems from the exercise book. Our opinion is that only a combination of traditional and contemporary teaching approach can provide with the best results.

2.3. Fractal paper design book

While working on the creating the fractals out of paper, a new idea emerged that we want to elaborate, and it is making a paper collection of fractals which would be nicely painted and ornamented and which would be used as a mean of introducing new students to this term. We called this picture book A FRACTAL PAPER DESIGN BOOK(FRAKTALNICA). The students were highly motivated to participate in the making of paper collection of fractals. The most suitable paper models were put into a collection.

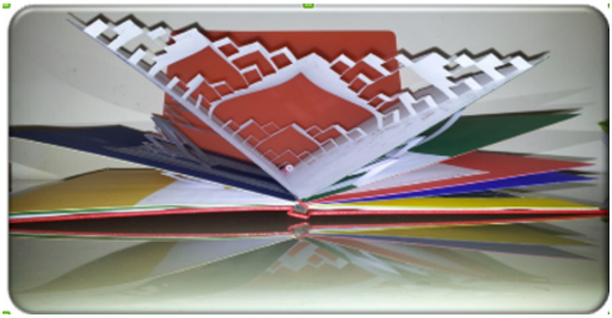

Figure 1: A fractal paper design book

A fractal paper design book can be seen on the blog `milenajeretin.wordpress.com`.

2.4. Assessing knowledge

At the end of this teaching process, the students were tested. This part of the teaching process was implemented by maths teachers Milena Jeretin i Milena Maric with students in classes A14 and A15.

3. The second grade

Considering the fact that the students were very interested, we wanted to expand their knowledge and apply it. Although the topic Similarity is not in the curriculum for mathematics in the second grade, we decided, within one maths lesson, to revise gained knowledge about the concept of Fractal, amend it and prepare the students for its application in the subject of Art, which is intensively studied in the second grade by our students.

3.1. On Maths lesson

Within the maths lesson we revised the concept of self–similarity and fractals and complemented their gained knowledge with where we can observe fractal structures in our body and the world around us using presentation and films. During that lesson the students drew several simple examples of fractals: Koch curve, Koch snowflake, Sierpinski triangle, Pythagoras tree. Due to the fact that our genealogy (genealogical tree) has a fractal structure, we connected the fractals with genealogical tree. Students were given for homework to do their genealogy in electronic form, on paper or in the form of 3D models. In addition, they had the opportunity to re-watch all the content from math class on the blog `milenajeretin.wordpress.com` and for their homework they had to individually complete the test of fractals at the same blog. They were divided into groups with a final project for homework, at the agreed time, to do a poster, film or presentation on fractals.

Figure 2: Genealogical tree

3.2. On Art lesson

On Art lesson, the students linked the fractals to the rhythm and gradation and made drawings which illustrated the application. Using the examples from the world around us, the students were acquainted with the types of rhythm and trained for their own observation. Using the examples from the history of art, they recognized the significance and understood the application of mathematics in the works of art, applying the knowledge gained in the math lesson. They were introduced to the symbolism of numbers and the way of making stained glass and ornaments in medieval art, after which they started making their own stained glass. At the end of the lesson they received the instructions for the homework assignment in which they used gained knowledge of optical art to create rhythm using gradation in composition. On `www.pinterest.com` the students put the photos of their work and their analysis. The aim for the students was to enrich personal vocabulary with artistic terms as well as to adapt to using different techniques, tools, media and methods in problem solving.

In order to facilitate the monitoring for students and encourage their interest, we established a link on the Internet on the blog milenajeretin.wordpress.com, site `http://manojlovicjasmina.wix.com/ats-crtanje` and pinterest `www.pinterest.com`.

Final group work which the students had to do for their homework was presented on the following lesson which was mutual for Math and Art, after which we organized an exhibition.

Figure 3: Exhibition

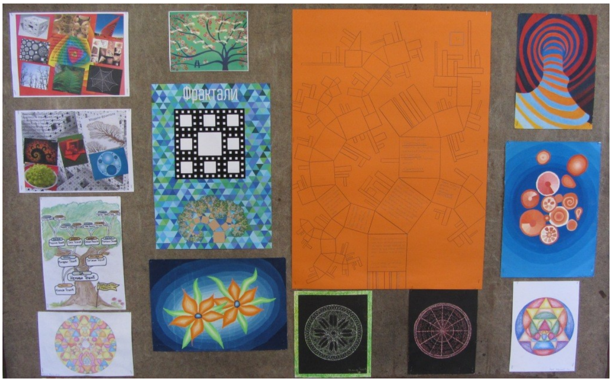

Figure 4: Exhibition

This part of teaching process was implemented by Milena Jeretin, maths teacher and Jasmina Manojlović, art teacher with the students in classes A24 and A25.

Our activities in the next two lessons are theoretically devised and are yet to be implemented. The goal is to shape the four-year story that one generation has learned, told and made about the topic of fractals. We also want the future generations to improve this idea with us.(see [19] Fraktali i ritam)

4. The third grade

At the beginning of the school year we visited the Ethnographic Museum and realized the workshop “The search for the fractal structures in the cultural heritage”in the cooperation with Tijana Čolak-Antić. Each group of students should look for designs on the exposed clothes in the Museum. They became familiar with the cultural heritage by Tijana Čolak-Antić representive of the Museum. Students observed a fractal structure in the exposed objects.

Figure 5: Pirot rug

They drew a fractal design on the paper or used applets on the site http://csdt.rpi.edu/african/African_Fractals.

At the end of the school year, when the students have learned about sequences, we plan to organize a maths lesson where they will see a presentation of Geometrical sequence and row, by Kata Jovanović, Gorica Acketa, Ratka Čorda from the site http://www.kreativnaskola.rs and organize jewellery workshop and pendants on the topic of fractals.

This part of the teaching process will be implemented by Milena Jeretin, maths teacher and Jasmina Manojlović, art teacher with the students.

5. The fourth grade

In the fourth grade one maths lesson is planned to renew and amend with new content the topic of Fractals, and then in the subject Composing shapes to make 3d models of fractal structure, which has already been done in the previous year in the experimental group. Students were acquainted with the application of fractal structures in architecture throughout history and creation of buildings of exceptional beauty. They are encouraged through discussion to find and analyze by themselves the application of rhythm by gradation as well as to establish correlation with mathematics. After that an artistic problem was established and the students approached the realization of special model using gained knowledge and materials they chose by themselves. On `www.pinterest.com` they put the photos of their works and performed analysis using gained knowledge.

The aim of these lessons was for students to understand and adopt the methods that exist in the field of synthesis in art, maths and architecture on the level of more complex artistic research, within the spatial model.

This was implemented in the experimental group of students by Milena Jeretin, maths teacher and Jasmina Manojlović, composing shapes teacher.

6. Conclusion

Mathematics is a subject that most students consider to be extremely difficult. The nice thing about mathematics is that it is a science that can be found in almost all spheres. Tendency to bring complex mathematical concepts closer to the student through the connection with other subjects and visualization should become the trend and goal for the maths teachers.

Our previous experience shows that this approach is correlated with students' motivation to work.

That's why our new challenge is to connect the concept of fractals with some other professional teaching subjects and improve Fractal paper design book with new art techniques, as well as the formulation of didactic content on Fractals through all four years.

References

[1] Matematika za I razred srednje škole; P. Miličić, V. Stojanović, Z. Kadelburg, B. Boričić; Zavod za udžbenike i nastavna sredstva, Beograd 2004 godine

[2] Matematika, Zbirka zadataka i testova za i razred gimnazija i tehničkih škola; Ž. Ivanović, S. Ognjanović; Krug, Beograd 2012 godine

[3] Matiš, baš svuda!; K. Dal i S. Nordkvist; Propolis Plus d.o.o. za Srbiju 2011 godine

[4] Geometrijski niz i red (ppt prezentacija); K. Jovanović, G. Acketa, R. Čorda, `www.kreativnaskola.rs`

[5] Izometrijske transformacije (ppt prezentacija); V. Poljarac, `www.kreativnaskola.rs`

[6] Eksperimentalno određivanje fraktalne dimenzije, H. Eklić, math.e Broj 4, veljača 2005 godine

[7] Mandelbrotov skup, Š. Šuljić, math.e Broj 5, lipanj 2005 godine

[8] Тајни код, Priya Hemenway, Загреб, 2009.

[9] Moderna umetnost, Đulio Karlo Argan, 1770-1970-2000, Clio, Beograd, 2005.

[10] Uvod u likovne umetnosti , Dr. Pavle Vasić, Univerzitet umetosti u Beogradu, 1988.

[11] Istorija umetnosti, Marija Karla Prete, Alfonso de Đoris, MK Panonia, Novi Sad, 2004.

[12] Kako nacrtati ono što vidite,Rudi de Rejna,Narodna knjiga, Neven, Beograd, 2005.

[13] Teorija forме, Kosta Bogdanović, Bojana Burić, Zavod za udžbenike i nastavna sredstva Beograd , 2004.

[14] Principi komponovanja, Marina Skrinjik Ćorić http://www.slideshare.net/razrednamarina/ss-31535223

[15] KURS: Matematika u arhitekturi 1, Prof.Dr Mirjana Petruševski http://elearning.rcub.bg.ac.rs/moodle/course/view.php?id=22

[16] http://eskola.hfd.hr/mini_projekt/mp7/fraktali_2.htm

[17] www.geogebra.org

[18] www.kreativnaskola.rs

[19] http://www.youtube.com/watch?v=OEp7YK6WEXE

[20] http://www.youtube.com/watch?v=8ZuvvYiLPOQ

[21] http://www.youtube.com/watch?v=dZM45mfJQ40

[22] http://www.youtube.com/watch?v=XwWyTts06tU

[23] http://www.youtube.com/watch?v=CMWETzKf7wk.

[24] http://ahyco.uniri.hr/seminari2007/fraktali/fraktali_opcenito.html

[25] http://mathworld.wolfram.com/SierpinskiCarpet.html

[26] http://csdt.rpi.edu/african/African_Fractals

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 91–96

Extreme Values of Function in GeoGebra Style

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Abstract

The process of student transitioning from elementary to advanced mathematical thinking in learning of calculus is followed by many difficulties. The role of the teacher is very important in planning, designing and applying adequate methodic solutions by which smooth transitions from lower to higher levels of abstraction, and mathematical thinking, are secured. Methodic solution wich can be efficient in teaching and learning calculus, is visualization, and animation of the basic terms, and processes in combination with symbolic records and definitions. By applying according computer programs, the teacher can realize the teaching process in visual environment. Educational software can link visual and symbolic representations of mathematical objects in dynamic, and interactive environment. This paper presents the possibility of didactic shaping of calculus material using GeoGebra. In dynamic worksheets algebraic and geometric interpretations of the concept of local extreme values of the functions, are connected.

Keywords: extreme values of function, GeoGebra, cognitive-visual approach

1. Introduction

Until the sixties mathematical education was based on a formal definition of concepts and predominantly deductive methods. The cognitive psychologists findings on the biological basis of cognition, about the work of the brain hemispheres, and the study of how people perceive, learn, remember and think, had an impact on mathematical education. Since then, efforts have been made to apply the cognitive theory in the classroom so that mathematical education can be improved.

Tall and Vinner stated that one of the key reasons for failure in mathematical education is the difference "between the mathematical concepts as formally defined and the cognitive processes by which they are conceived" [1]. In their work two ways of adopting mathematical concepts are stated: the concept definition and the concept image. The concept definition is a form of words and symbols used to specify that concept. The term concept image is defined as "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes". During the formation of the concept, the relationship between the concept definition and the concept image should be reciprocal [1].

Educational research suggests that in teaching practice we should find the optimal relationship between the concept definition and concept image, and should strive to the balanced relationship between logical-analytical and visual-creative thinking. Visualizing concepts that are being introduced or processed in combination with symbolic entries and definitions contribute to the efficiency of the educational process.

Realization of the cognitive-visual approach means that we apply graphics, schemes, tables, concept maps, presentations, interactive learning materials, applets, animations etc. in teaching process. Many computer applications can provide an environment that allows teachers to display mathematical concepts and processes, and students to explore, analyze and draw conclusions.

2. Student difficulties in learning calculus

Functions first derivative is one of the basic topics of calculus with which students of high school first encounter in fourth grade. Students have difficulties accepting the geometrical interpretation of the first derivative itself. Even bigger problems occur when the basic theorems , which link the functions first derivative with monotonicity and the extreme values of the functions, are to be applied. Difficulties occur because students, besides the great knowledge of basic math, must form a new way of thought, which is called "Advanced mathematical thinking" in literature (thinking which is based on formal definitions, axioms and theorems, with which, logical deduction is applied).

The role of the teacher in the process of transitioning students from elementary to advanced thinking is very important. The results of pedagogic researches give certain guidelines to teachers, in order to make that transition as simple as possible for students. When adopting new terms, it's better to go from specific and simpler to more abstract and complex. The existing representations which students have about certain terms need to be connected and on that basis new terms should be built. Students easier, and better adopt new terms of calculus when in teaching, concepts of image and definition, are combined. This approach allows students to make connections between symbolic and visual representations, and thus increase their cognitive abilities.

3. Cognitive-visual approach to teaching calculus by using GeoGebra

GeoGebra is a dynamic mathematical software where mathematical objects are shown in two ways: algebraic and graphic (geometric). Users can simultaneously use a computer algebra system and an interactive geometric system. This tool extends the concepts of dynamic geometry to the fields of algebra and mathematical analysis.

Function first derivative, definite integral and other concepts of calculus are simply presented in GeoGebra, both algebraically and graphically. It is a powerful tool for visualization and animation, so that students can be shown various positions and relationships of given objects. Therefore, GeoGebra can display, in addition to examples of the concepts, examples of mathematical processes through dynamic action. GeoGebra is used in teaching and learning of calculus as a cognitive tool for explaining, exploring and modeling of mathematical concepts and processes. GeoGebra environment encourages students‘ projects in mathematics, experimental and guided discovery learning.

4. Interactive Examples in Teaching and Learning Calculus with GeoGebra

In this paper we are presenting examples of how Geogebra is used in classrooms to explain and explore concept of local extreme values of functions. Dynamic worksheets which we are presenting, the author has prepared and used in classroom with students of the fourth grade of high school. The basic idea while preparing GeoGebra dynamic worksheets is connecting algebraic and geometrical interpretation of the concept of local extreme values of functions.

The problem which is given to the students is as it follows: link each of four given functions with its derivative function. Functions are given with their graphs, and their derivative functions are given analytically, namely with formula (figure 1).

Before solving the problem, the following is to be repeated with students: the notion of the critical number (critical point) of the function, the notion of the local maximum and minimum of the function, necessary and sufficient conditions for local extreme points and procedure for determining local extremes of the function. Students on the basis of the graph of each function, get essential data of critical numbers and local extremes of the function. As the derivative functions are given by formula, by conducting the corresponding algebraic procedure, students first determine the critical numbers, and then the points of local extremes. For the students to link the function with its derivative function, they first need to connect the facts that they have learned by graphical and algebraic solving of the problem.

Students can check if they solved the problem correctly (check your solution), as well solving the problem again ("reset" button). In case that students haven't

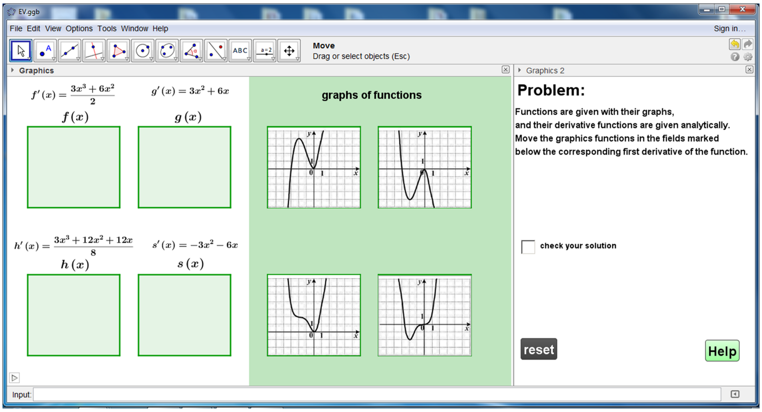

Figure 1: Problem and home menu

solved the assignment correctly, or if during solving they have certain difficulties and concerns, they can use additional explanations by clicking the "help" button.

The assistance that students can receive is organized in multiple levels, and is shown by checking the corresponding boxes or clicking on the button (figure 2).

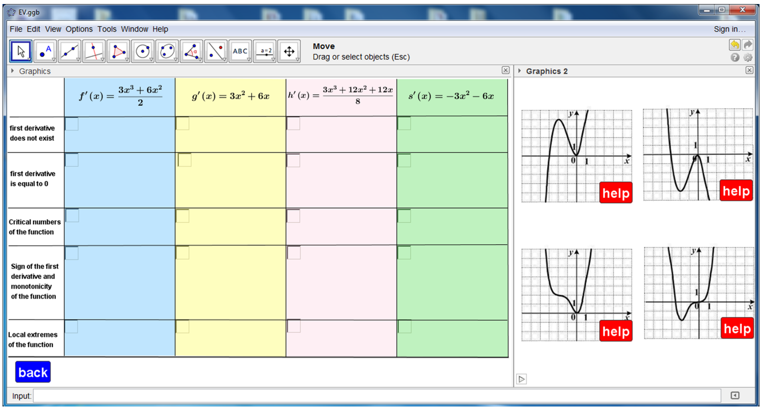

Figure 2: Assistance in solving problems

The levels of help for derivative functions are (figure 3):

1. Real numbers for which the first derivative of the function is not defined.

2. Zeroes of the first derivative.

3. Critical numbers of the function.

4. Sign of the first derivative and monotonicity of the function.

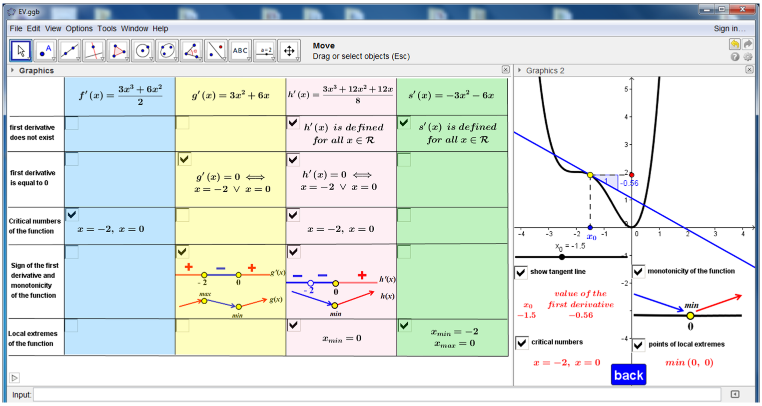

Figure 3: Levels of help for derivative functions and graphs of the functions

5. Local extremes of the function.

Assistance with graphs of the functions is given in following levels (figure 3):

1. Tangents of the graph of the function and value of the first derivative - Students can move the slider and observe the movement of the point on the function curve and the tangent line which is constructed at that point. The slope of the tangent line is geometrically represented by the corresponding right-angled triangle. Also moving the slider, students can follow the changes in the value of the functions first derivative.

2. Critical numbers of the function.

3. Monotonicity of the function.

4. Points of local extremes.

According to their needs, students can choose any type of help, and with independent research they can come to certain conclusions and knowledge. After using selected levels of assistance, by clicking the "back" button, students have the possibility to return to solving the problem, and checking the solution.

Impressions and findings of students in a class where they used GeoGebra worksheets, point to their satisfaction because they had the option to solve the problem independently, and to create the way by which they get to the final solution, by themselves. Solving the problem students experienced as a game with images, and they were very motivated to solve this GeoGebra puzzle.

5. Conclusions

One way to overcome difficulties students faces in learning calculus is for the teacher to shape the learning environment which connects symbolic and visual representations of terms, concepts and processes of calculus. Because of the dual representation of the mathematical objects, algebraic and geometric, GeoGebra is a powerful cognitive-visual tool for teaching and learning calculus.

Visual and dynamic interaction between the user and GeoGebra environment helps students to form their visual understanding and thinking. Also, students can easier apply and link their visual understandings with formal-symbolic language of calculus. In that way their understanding of the basic concepts and connections between them, is fast and proper. GeoGebra allows teacher to encourage and guide students to independently research, discover, and through their own experience obtain new knowledge. Didactically designed contents in GeoGebra allows students to independently forge their own way by which they obtain new knowledge.

References

[1] D. Tall, S. Vinner, Concept image and concept definition in mathematics, with special reference to limits and continuity, Educational Studies in Mathematics, Vol 12, pp. 151–169. 1981.

[2] S. Vinner, Concept definition, concept image and the notion of function, The International Journal of Mathematical Education in Science and Technology, Vol 14, pp. 293–305. 1983.

[3] D.Tall, Biological brain, mathematical mind and computational computers, Proceedings of the 5th Asian Technology Conference in Mathematics, Chiang Mai, Thailand, pp. 3-20. 2000.

[4] R. J. Sternberg, Kognitivna psihologija, Jastrebsko, Naklada Slap, pp. 212-248. 2005.

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 97–112

Empathy in Teacher Education – Generating Motivational Attitudes Collaborations between arts & mathematics

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Abstract

This document presents a reflection on two workshops given at the Summer University 2013 in Eger (Hungary) and 2014 in Belgrade (Serbia). Theory and methods of Design Thinking were applied to use a design method to develop new insights to problems with the subject mathematic, empathize with target groups and find innovative individual solutions for them. Recognizing a problem of somebody, understanding the problem challenges to define it and develop customized individual solutions. Objective was to sensitise math teachers for troubles with the subject and co-develop ideas for innovative approaches.

Keywords: visuality, mathematics, empathy, interdisciplinarity, applied design thinking

1. Introduction

The idea of the Summer University was to inspire teachers of mathematics with art and design, to achieve theories, methods and tools, in order to develop new teaching approaches. Students in school might experience mathematical problems more tangible and visible in future times with their new inputs. Therefore in this case a method, called Applied Design Thinking was presented and applied.

1.1. Austrian and Serbian Educational System

Opposite to the Austrian School system, Serbian teachers have to engage in life-long-learning and – as experienced during the Summer University in Eger and Belgrade during the Tempus project, funded by EU – they are eager and highly engaged even to participate at international teachers training with English as conference language. It also needs to be emphasized that their monthly income depends on collected credit-points for improving competencies. In Austria only primary teachers were called upon to make current teachers training, but level of secondary 1 & 2 was not obliged yet to do anything at all by law. Further on it is important to stress that in Austria teachers colleges (PH) have a unique monopole for teacher further training and are funded by the ministry of education on opposite to Universities, who know the current state of research and even though staff of Universities are primarily used at teachers colleges for this. There is no such a summer school for teachers in their holiday time, where they could engage and exchange experiences in Austria.

1.2. Benefit of Summer schools

Summer Universities of that range haven't yet been implemented in Europe so far. The Tempus EU project demonstrates a high range of excellence of Serbian teachers and the necessity of such "Train the Teacher-Universities" in summertime with interdisciplinary approach in order to enhance creative and innovative classroom lectures. It must be stressed, too, that the social aspect, the gathering of teachers within breaks or after the conference, triggered by discussions about lectures and workshops, are regarded as highly important. Teachers united and started from one year to the other to develop group-work and exchange. Due to Tempus EU, teachers did not only participate at conferences, held lectures and workshops but even prepared English translations of their talks and prepared several classroom-lectures which need to be explored.

2. Motivation of teachers and scholars – Empathizing

Standards in educational reforms, poverty and an increasing diversity of the school population force teachers to engage and understand motivation of students. But also ministries and responsible governmental institutions as employers are challenged to understand and support motivation of teachers.

2.1. Teachers

What can governments do to foster optimum motivation of teachers? At the INSEA 35th Triennal conference in Melbourne 2014, McBrien (2014) described that elementary and secondary education in the U.S.A. "has increasingly been homogenized and diminished by a business-style high-stakes and accountability model. Teachers, tyrannized by fear of losing their jobs, forsake creativity for more time to drill children in standardized test-taking procedures, and research indicates a reduction of class hours for social studies, art, music, and physical education as a result (Smith & Kovacs, 2011, 201-225; Spohn, 2008)". Further Hong (2012, 417-440) emphasized, that "nearly 50% of new teachers leave the profession after just five years". Even though governments should have heard about constructivist approach, they seem to be untouched of motivational knowledge. The constructivist thinker and Viennese Heinz von Foerster (one of the founders of cybernetics) explained that human beings do not behave like machines (Riegler 2010). Von Foerster's famous distinction between trivial (input-output) and non-trivial machines (input – not predictable output) is a starting point to recognize the complexity of contemporary belief in measurement of quality and standardization of education, especially within the fields of art. Von Foerster (1993) describes a trivial machine, which is a machine whose operations is not influenced by previous operations and can be compared to a "well defined problem" (i.e. single, guaranteed solution) in cognitive psychology (Schraw et al. 1995, 2006). It is analytically determinable, independent from previous operations, and thus predictable. For non-trivial machines, however, this is no longer true as non-linear equations define various solutions and cannot be foreseen, human being is a non-trivial machine, if compared (e.g. Riegler 2010; Von Förster 1993, 134-151). Von Foerster already criticized the disinterest of governmental policy in the question of "how students learn" and "what they need", suggested cynically to think about a "mathematical organ" to explain mathematical abilities and considered bad performance of the American school system (A Nation at Risk 1983) results from: "overemphasis of sport, overstrained and underpaid teachers, design of teaching contents by local committees, politisation of schoolbooks, etc." (Von Foerster 1993, 126f). 1990 he already addressed the problems of motivation, which still exist and haven't been changed yet in the USA and many other countries.

Standards reduce autonomy to teachers, they do not get the feeling of individual initiative and entrepreneurial thinking. Part-centered standards would allow them more individual design of classroom activities and individual promoting of student's skills. No obligatory for continuing education in teachers training does not enhance better teaching qualities on the current state of research. Quality of continuing education must be ensured by research and provided by research-governed Universities, as international state of the art. Point-evaluation might encourage teachers as well as participation at Summer-Universities, which enhance international and intercultural communication, exchange of experience and knowledge. For international participation at such Universities, points need to be at least doubled as expression of appreciation and estimation and encourage the feeling of relatedness and being competent. Competitions for development of tools-for-teachers or other items should be enhanced and European calls for such co-operations and initiatives promoted. According to the Society of Human Resource Management, appreciation is at least as important as salary, works as an "emotional salary" (Lengyel-Sigl 2014, K12; Krishnamoorthy 2014).

2.2. Scholars

What can teacher do to foster optimum motivation of scholars? Alderman (2008, ix) considers a crucial element as essential: teachers should understand why a student may not try and understand his or her motivation. Alderman (2008, 7-8) describes a social-cognitive motivational example in his book by two students with similar beliefs about their ability to learn math and how contrasting instructional environment influences motivation and behaviour: Both believed that either you have an ability to learn math or not and further that if you do not have the ability, there is nothing you can change or do about it. One of them was put to a "low math group" and he spent even less and less time on math, confirming his belief about his math ability. The other one was put into a challenging math group and improved her math scores, spent more time with math, became proud and started to change beliefs about her abilities.

Objective of two two-hours-workshops at the Summer School at Belgrade Metropolitan University was to sensitize teachers of mathematics from primary to secondary school for the individual approaches of students to the discipline, the importance of empathy and interdisciplinary collaboration and the application of design methods in math class.

Diverse and customized learning is needed in the classrooms of the 21st century because 1st: today's classrooms are typified by academic diversity, students with identified and non identified learning problems, students who underachieve because of various reasons, scholars from diverse cultures, economic backgrounds and 2nd scholars of varying interests and modes of learning. Tomlinson et al. (2003, 1) describe that there are various indications that "most teachers make few proactive modifications based on learner variance". They consider that differentiated or academically responsive instruction is needed in schools of the 21st century because "it provides support in theory and research for differentiating instruction based on a model of addressing student readiness, interest, and learning profile for a broad range of learners in mixed-ability classroom settings". Vygotsky (1978) believed that an individual learns in his or her "zone of proximal development". Further on the workshop aimed to address pervasive teacher beliefs like artists cannot be mathematical talented and the other way round. If the teacher is supposed to push the child into her or his "zone" she or he needs to engage with the child. Teachers very often are totally in love with their subject and have difficulties in empathizing with others.

Interdisciplinary collaboration might improve translations and understanding, even curiosity for each other. One method presented first at the Summer University in Eger and further on in Belgrade is Design Thinking, developed by designers themselves and propagated by design-consultancies (like f.e. IDEO). Applications of Design Thinking are believed to encourage pupils to act interdisciplinary and in teams with special rules and procedures. Main element is the practice of empathizing, prototyping various solutions for the others and iterative feedback culture. Since the 1970 the approach of development of "things" changed. First this appeared in the IT sector, where programmers have realized the fact that they often do not meet users' needs. Since some years honest meant participatory methods have reached the world of designers. Participatory approaches in politics are still not transparent yet. In these workshops a design method analyzed interest of the target-group (child in school) and suggested teachers to think "out of the box" and apply artistic strategies in order to develop innovative attitudes and approaches in teaching.

3. Interest, Motivation, Trends and Challenges

3.1. Interest & Motivation

Mathematics is considered as one of the highest esteemed disciplines of today's world. "Interest in mathematics could be regarded a predictor for mathematics achievement" (Heinze et al 2005, 212), and still we face problems of motivation and interest for the subject. The also found that there was a significant correlation between the scales fear or failure and fear of mathematics classroom as well as a negative correlation between interest in mathematics and boredom (2005, 216). We have to face, too, the circumstances that female students from countries like the Netherlands and Norway at the TIMSS testing showed poor interest in these subjects and countries like Russian Federation, Armenia and Iran showed high estimation for these disciplines, regardless of gender (comp. TIMSS Advanced 2008; TIMSS Advanced 2015, 6). The last years many interpretation of motivation with psychological functions have been described. Two current need-theories (Self-determination theory and Self-worth theory) contribute to the understanding of motivation more extensively. Deci & Ryan (1985, 2004) believe that human beings have basic needs (Deci & Ryan 2004, 9): Besides health and satisfaction (Maslow 1954) they need autonomy (f.e. Deci 1975), which means to decide and initiate, rule by own conduct, social relatedness (Baumeister & Leary 1995; Baard et al 2004, 246), which means respect and trust for collaboration and a feeling of competence (f.e. Skinner 1995), which means meeting challenges well. Further on, motivation depends on the importance of the social nature in schools (Weiner 1990) and the role of the environment (Bandura 1986).

3.2. Trends: STEAM

Art and Design have been the key subjects in schools the last centuries and empowered with new innovative ideas f.e. outputs of the TEMPUS EU project: Visuality & Mathematics. Temporarily governments and school policy are business oriented and aim for highest success in STEM subjects. Some countries even reduce their art classes in school to favour of STEM subjects. The latest trend, which could be observed, was to include arts to STEM: STEAM. Studies on STEM education focusing on science, technology, engineering and mathematics first began in the 1990's. In 2006, Yakman in the USA came up with another concept, STEAM, with arts included to STEM. STEAM education should be considered an education to help students have comprehensive viewpoints for this education would teach all those subjects such as science, technology, engineering, arts and mathematics by associating one with the other. 2014, Boo-Yun described at the INSEA conference in Melbourne, that the Ministry of Education, Science and Technology in Korea has announced a promotional system with STEAM education being applied to model schools. "STEAM education is replaced with other terms as 'convergent education', 'convergent education to grow highly potential brains', 'creative convergent education', 'science-art convergent education' and others" (Boo-Yun 2014). At the same conference, McBrien criticized the educational shift in the United States, where "elementary and secondary education has increasingly been homogenized and diminished by a business-style high-stakes and accountability model. Teachers, tyrannized by fear of losing their jobs, forsake creativity for more time to drill children in standardized test-taking procedures, and research indicates a reduction of class hours for social studies, art, music, and physical education as a result (Smith & Kovacs, 2011; Spohn, 2008, 3-12). Nearly 50% of new teachers leave the profession after just five years (Hong, 2012)". 'Race to the Top' inspires governments to support STEM, which emphasizes science, technology, engineering, and math over the humanities, arts and social sciences. McBrien believes as well as the authors that arts are important as separate disciplines of study. Arts should act as a means to explore other subjects, "through the arts". In Austria, the chamber of industry considers even to create a school "through design", where design plays a key role of the curriculum, where other subjects are built around. Funding agencies such as the U.S. National Science Foundation (NSF) and European Science Foundation are increasingly setting research agendas to support synthesis (Gutmann, 2011; Simon & Graybill, 2010; Klein 2010).

4. Methods

4.1. Metropolitan University Summer University 2014, Workshop descriptions

In the first workshop 16 (2 men, 14 women) teachers participated as well as two students of design, in the second workshop 14 (2 men and 12 women) math-teachers participated, as well as one language teacher and another. It must be assumed that out of 27 interviewed pupils, 20 women were interviewed and only 7 man. Why this was done cannot be analyzed but further research must be taken.

4.2. Applied Design Thinking

Applied Design Thinking (Mateus-Berr 2013, 73-116) is a method, borrowed from a general design-process. This applied method can be described as a specific workshop, containing explicit rules and tasks. It focuses on empathising, iterating, prototyping, testing in iterative circles and with clear defined team-roles. Online open-source tools from Stanford University were developed further. This workshop encourages empathising in pairs and designing solutions for each other. These are discussed, feedback is given, various solutions again created, a.s.o. The participants have to listen carefully to each other and understand the others' needs.

It is believed that empathic approach improves understanding of individual points of view on a subject and its problems and facilitates customized teaching attitudes. Especially teachers of mathematics due to their subject appear very fond of one solely and short solution. Comparing to subjects as design education, problem-solving tasks demand various solution findings and curiosity for new creative ideas. Action research was applied to involve various target groups and make teacher aware of different problems of students, not for a serious meant empirical study, but outcomes are interesting, too.

After a short introduction of theory and methodology of Design Thinking, teachers of the first workshop were sent to the field to ask anybody about his or her problems with mathematics within a timeframe of fifteen minutes. Further on they were challenged to discuss the outcomes in teams and following try to develop solutions for the interviewed pupils in pairs within fifteen minutes. Before starting the group-work they were asked to define the problem of the interviewed person and describe the profile of the interviewee. This appeared extremely difficult. It was discussed that there was very little information about the problem and that they would have had to dig deeper in similar future projects. Anyhow the lecturer asked to describe a problem and debated a designerly approach to the task, stressing a creative brainstorming. After all participants in doubt with solutions could present their cases and debated various possibilities of solutions, the teachers worked in pairs on solutions for about twenty minutes.

Due to the target –group at the second workshop, which mainly has not joined the lecture and workshop of Design Thinking at the Summer University in Eger, the lecturer considered that more had to be said about the objectives and the methodology. Methods of active listening was experienced on order to make the teachers aware that even a "small problem" might not be understood correctly, if people do not intensively engage with each other.

4.3. Teachers Beliefs

When the task for the interview was explained, some teachers were convinced that they would know the problem already: Bad experience in math would refer to bad teachers. As the teachers present were highly engaged, they would not consider themselves as bad teachers, but "the others". But who are "the others" and why are they bad teachers?

Prejudices and beliefs still manifest disciplinary attitudes: During breaks some teachers tried to convince the lecturer of one beautiful formula, not trying to understand and translate the meaning to a non-mathematician. As it was not translated to the lecturer, whose field is more situated within the arts and design, a discussion break of, how “beautiful” can be defined. This topic appears a little similar problematic as the item of “truth” in philosophy. As Heinz von Foerster & Pörksen (2006) claimed: “The inventor of truth is a liar” and translation did not take place, as DeWachter (1976, 53) underlines the need of translation into the language of the others: “Until there is willingness to change one’s mind and translate conviction into a language the other will fully appreciate, no interdisciplinary communication has taken place.”

Another participant asked the lecturer if she was good in maths at school or not. By explaining her experiences she was interrupted with the message: “I thought you would have been bad in school”. After she continued with her story of experience that she had a crucial test to be admitted to the Matura she had one special lesson in mathematics with a teacher who would ask her what she was interested in. After explaining her interest to listen to music, he explained that she could calculate the way of a turntable needle with integral, differential functions. She was so much inspired by this idea, that she passed with distinction and wrote one of the best grades in math.

Teachers apply learning strategies they know from themselves but very often do not cope with individual needs and stick to preconceptions (Reitzer 2014, 134).

4.4. Workshop descriptions

WORKSHOP 1

The profiles were defined as follows: Thirteen profiles have been described; people between 10 and 65 were interviewed. After describing the profile and defining the problem, solutions were carried out as concepts with the objective to attract the distinctive target groups for mathematic or improve the running system.

Profile 1

Vanja, 25, female, manager at Metropolitan University described that she did not like the maths teacher from her secondary school.

For Profile 1 it was suggested to include more pedagogic knowledge into teaching, work in groups and send her to a math seminar where she could make different experiences with individualities of math teachers, but especially the idea arose that her personal experience is a matter of fact in many schools and team-teaching with constructive exchange of feedback and willingness to improve should become state of the art.

Profile 2

35 old man, concierge desk at the University described that when he was young maths did not appear useful, usable to him.

As for Profile 2 the relation of math to the real world has been missing, it was suggested to let him do a task with a chocolate he should divide between a certain amounts of pupils, in order each of them would get an equal part.

Profile 3

Woman, about 27 years old considered mathematic as boring because of the professor that was too strict and demanding and she felt it too difficult to understand and not useful to talk about quantitative methods.

For Profile 3 it was considered that teachers should be more creative with tasks and should empathize that not everybody regards math as interesting and easy and further that it would need more open-minded teachers.

Profile 4

Woman about 24 years old, student believed math as too boring in school because the acquisition of knowledge in school was not useful to resolve everyday problems.

Profile 5

A teenage-girl from secondary school has a brother who is a mathematician. Due to family interest for the field of mathematics she refused to learn it in school. It is supposed that she likes to go out, ride bike and do sports in general.

Profile 6

Woman about 30 years does not like math, has children

For Profile 4-6 no solutions were found.

Profile 7

A little girl about 10 years old, who likes mathematic because she has not learned difficult things yet.

For Profile 7 it was thought about how the motivation keeps her interested in the subject. Therefore it was suggested to involve games in and after school, symmetry workshops and combinations of arts and math (f.e. do your portrait duplicate one side and look if your face is symmetrical ...)

Profile 8

Woman about 25 years old who finished secondary art school, who never liked math classes and blames it to the bad organization of her math teacher in primary school who - as she believed – would pack too many things to learn in a session. Therefore she changed to an art school in order to have less math lessons.

Profile 8 was analyzed as fond of art, for this purpose the teacher should use more topics from this area (f.e. if the lesson is about symmetry, the teacher could handle the tasks by introducing symmetric pieces of art). As this woman was considered about too much input in one math lesson, the contents of one lesson should be spread to more hours. Further on it was considered as helpful if math lessons were organized as workshops (f.e. learning about Pythagoras, students could draw his findings by three fractals).

Profile 9

A woman about 30 years old was interviewed, who is a lawyer and does not like math at all. She did not remember any lesson at all but after a while due to a discussion she concerned extremely complicated the field of exponential functions in secondary school.

The participant who interviewed Profile 9, suggested an interesting task, namely the situation of a man, asking for investment advisory, to whom it could be explained with the chess field and the formula: $1 + 2 + 2^2 + 23, \ldots .2^{64-1} = 2^{63-1}$. The credit rate would increase according to an exponential function. Further on it could be interesting to ask questions at court in an algorithmic order if the answers are "yes" and "no". This participant appeared extremely interested on this target group because her sister was lawyer, too and she does not know fractions at all, but would be needed by dividing properties or in percent counting.

Profile 10

Woman about 20-30 year old, works at a marketing office.

For Profile 10 it was suggested to design a marketing campaign for mathematics. As this is her profession it could be easily analyzed what she would have needed in math classes to be more interested and further on, let her solve the problem independently.

Profile 11

A woman, waiter described that she hated math, calculations, but geometry was even worse.

For Profile 11 possible solutions were suggested as some kind of active workshop, role-games (to pretend that she is selling things and has to calculate), to do some things with her hands (cutting models, origami, and some new methods) in order to bring into classroom activities some more real-life activities. And further on she should be encouraged with tasks for evaluation, where more solutions are possible.

Profile 12

A woman of 65 years, retired few months ago. She is a remarried widow who has two grown up daughters. She has been math teacher in Belgrade for many years and also holds a magister in mathematical science. She was described with blue eyes and very pretty. She has been wearing a sailor-type dress with many colours.

Profile 12 could not be completely analyzed because she has been fond of mathematics every since, but her interviewee designed paper-dolls with different dresses including math patterns on it and sounds related to mathematic.

Profile 13

Man, about 55 years, football player. He did not like calculation and geometry in school. He could not find any sense in math lessons and if he did something wrong, he was punished by his teacher. Finally he explained that actually math can be found everywhere.

For another Profile, which could not be identified (13?), it was considered to design a game related to strategies of game theory connected with football as well as to give him tools of rhyme order (numbers, metrum) for inventing own poems, f.e. Petrarcas Sonnet.

WORKSHOP 2

The profiles were defined as follows:

Profile 1

Woman 20-25 years, staff-member of the University, dislikes math and cannot define the problem properly. She described two problems: 1st that the teachers did not put enough effort and 2nd that math is not her favourite subject. She had no idea how she ever could like math, but decided that she likes geometry more than other math elements.

Though she had good teachers in school, she rejected to like math. She is not consistent with her opinion. Suggestion of the interviewee would be that she should teach herself mathematics. It is believed that the best way of learning is to teach.

Profile 2

Man, approx. 60 years, who was working as an accounter. He told the interviewer that his secondary education followed circles. He had problems with math because he did not understand numbers, but he liked geometry. Younger students helped him a lot and he preferred to study in groups, with somebody else. He believed that scholars of today were in better conditions and that technology forces people to learn math. His problem can be defined in the fact that his lower education was described as bad and that he had existential financial problems. The suggested solution was to teach him numbers via geometry and technology and to offer him to learn in groups.

Profile 3

Man, about 70 years, who likes to play chess but did not like mathematics in school because his teacher was too strict and he considered himself as too lazy. The suggested solution for this man would be to play chess with the computer when his friends were absent and to explain him the combinatory basis of mathematics within chess.

Profile 4

The interviewer met a young female person of about 24 years. She did not like math because it was too boring to her, but she liked geometry because of the drawings. She believes that the problem lies in how math is taught. The solution could be to show her how to use cloth in creating cloth and patterns to design things for her collections. This task could also used at fashion shows at catwalks (in schools?) to show different formations: circle, triangle, etc.

Profile 5

Woman, 45 years old, worker. The interviewer defined her mathematical thinking as not developed as she did not understand what teachers would try to explain her and therefore she did not have any motivation to engage with the subject. A solution was suggested to offer her individualized lessons.

Profile 6

Woman, round 23, who studied tourism and has been working in a wellness center. She was described not to be very open for discussion, nor to discuss her poor interest in math classes. She believed that it is fine, some people appreciate a subject and others don't. She called her math teacher a "Hitler" in his way of teaching and giving grades. So her experiences with math were boring and frustrating. But she communicated about her interest in geography and Serbian language. In Geography they had competitions in groups, where they would even meet after school and she described how she enjoyed it.

Her problem was defined as being afraid of math, afraid of the teacher and believing that math was not her case. As solution it was suggested to apply teaching in groups, using a more informal approach, dealing with terminology and tasks which just sometimes would concentrate in math and not to look at it as the "one and only" subject.

Profile 7

Woman, 37, hairdresser. She described her problem with difficulties in multiplications, even counting 7x8 and 6x9 etc. The interviewer confirms that she might not have been good in additional tasks, missed some steps and that she had no clear understanding about the relationship between "t" and "x". When asked how she would solve the problem she had a simple solution, by using a calculator all the times. Solutions for her were suggested as using social games as "Monopoly" or "Jamb" or a puzzle, which would include multiple pieces. If she would get into troubles she could be encouraged to improve her additional skills and practice multiplying until it works in the game, but everyday practice would be the base. Her colleges could check what she learned if they asked her every day and several times the same example (f.e. 3x7, 5x8, 6x9 . . .). Also several computer games were suggested to help her to improve her math skills.

Profile 8

Male ("War-child"), born in 1942. He finished faculty of engineering but had problems in secondary school with math. To his opinion the teachers were bad and not competent enough. They were engineers and not mathematicians. They had big difficulties to make content visible. But in primary school he did not have problems with the subject, even loved math as it was taught more vividly and picturesquely (and this could be considered as one of the solutions of his problem).

For Profile 9-14 no solutions were found.

5. Lessons learnt

Although these workshops haven't been planned to be analyzed empirically, the time was too short and the task to superficial, it is possible to track that teachers experienced new approaches in engaging with motivation of students. Empathizing with diverse people, trying to understand their problems and obvious disinterest in the subject, they developed possible strategies for motivational empowerment. Individual interests and learning strategies are highly underestimated. Needs should be identified and divers classroom-lessons created and applied. Practical oriented students might need hands-ons and applied real-life situations, theoretical orientated students would need theoretical background knowledge, cooperative learning and diversity of methods also proved good performance with students, but are practiced seldom (Reitzer 2014, 134). At the first workshop in Eger, teachers did not appreciate that much a design strategy, but preferred a recipe book for math lessons. In the second workshop at Belgrade, therefore more explanations have been done. Applied design thinking strategies were applied in order to rethink teachers' attitudes and students' needs. If the combination of art and mathematics will change the interest of the pupils remains unclear. Mathematicians believe that math will open literacy to other subjects, artists take up the same for their discipline. Besides the "conflict of the faculties" it is considered to be clear that each discipline has its own language and culture and needs rather to collaborate then to annex, as manner of respect. Fischer (1984, 227) claims the use of "open mathematics", which means discussions, argumentations rather the teaching of complicate theories. This theory might reach the "real lives" easier than complicated "boring" "closed mathematics". Mathematics has been visualized by geometry. As art visualizes, inherits and addresses diverse sensory organs, Fischer considers art as essential communication tool and "visual literacy" as important competence. Arnheim (1997) described the creation of innovations by means of images, because words do not reach sufficient the expression. As mathematics is visualization of abstract circumstances, it should be applied to illustrate the visual evidence of mathematics and make them available for other areas, but arts used for mathematics just triggers associations and have nothing to do with the real discipline (comp. Fischer 1984, 243). It is believed that interdisciplinary approach of diverse subjects might relate clearer to mathematical problem solving and better understanding of students' needs. And through theses simple case study with teachers it was proved that for teachers problem definition and empathy is difficult and individual approach not "state of the art".

References

[1] Alderman, M. Kay (1999, 2003, 2008) Motivation for Achievement: Possibilities for Teaching and Learning, London: Routledge Taylor & Francis

[2] Arnheim, Rudolf (1954, 1974, 1997) Art and Visual Perception. A Psychology of the Creative Eye. Berkely, Los Angeles, London: University of California Press

[3] Baard, P.P.; Deci, E.L., Ryan, R.M. (2004) Intrinsic Need Satisfaction. A Motivational Basis of Performance and Well-being in two Work Settings. http://www.selfdeterminationtheory.org/SDT/documents/2004_BaardDeciRyan.pdf (3.5.2014).

[4] Bandura, A., Cervone, D. (1986) Differential engagement of self-reactive influences in cognitive motivation. Organizational Behavior and Human Decision Processes, 38(1), 92-113

[5] Baumeister, R. & Leary, M.R. (1995) The need to belong. Desire for interpersonal attachements as fundamental human motivation. Psychological Bulletin 117, 497-529.

[6] Boo-Yun, Lee (2014) Study on Art Education-Focused STEAM Education Program http://www.insea2014.com/d/program/InSEA2014_Abstracts_Oral_Presentations.pdf p.216

[7] Brian, Martin (1988) Mathematics and social interests. http://www.bmartin.cc/pubs/88search.html. Published in Search, Vol. 19, No. 4, July-August 1988, pp. 209-214

[8] Deci, E.L. (1975) Intrinsic Motivation. New Yor: N.Y. Plenum. Contemporary Educational Psychology 25, 54–67 (2000) doi:10.1006/ceps.1999.1020, available online at http://www.idealibrary. com. http://mmrg.pbworks.com/f/Ryan,+Deci+00.pdf (1.5.2013).

[9] Deci, E., Ryan, R.M. (2004) Handbook of Self-Determination Reserach. Rochester: University of Rochester Press.

[10] DeWachter, M. (1976). Interdisciplinary Team Work. Journal of Medical Ethics, 2. http://www. ncbi.nlm.nih.gov/pmc/articles/PMC2495117/(11.9. 2011).

[11] Fischer, Roland (1984) Offene Mathematik und Visualisierung. In: mathematica didactica 7, S. 139-160.

[12] Foerster, H.v. (1993) Wissen und Gewissen, in J. Schmidt (Hrsg.) Versuch einer Brücke Frankfurt am Main: Suhrkamp.

[13] Von Foerster, Heinz, Pörksen, Bernhard (2006) Wahrheit ist die Erfindung eines Lügners: Gespräche für Skeptiker. Heidelberg: Carl-Auer-Systeme Verlag

[14] Heinze, Aiso, Reiss, Kristina, Rudolph, Franziska (2005) Mathematics achievement and interest in mathematics from a differential perspective. In: ZDM 2005 Vol. 37 (3) http://subs.emis.de/journals/ZDM/zdm053a11.pdf

[15] Hong, Ji. Y. (2012) Why do some beginning teachers leave the school, and others stay? Understanding teacher resilience through psychological lenses. DOI:10.1080/13540602.2012.696044 p. 417-440

[16] Karaman, Abdullah, Cendel (2008) Conceptualizing international field experiences in teacher education with a systems approach: the interplay between worldviews and social episodes. Thesis (Ph.D.) University of WisconsinMadison, 2008.

[17] Krishnamoorthy, Shankar (2014) The Positive Power of Appreciation in: HR Magazine, Vol. 59 No.7 `http://www.shrm.org/publications/hrmagazine/editorialcontent/2014/0714/pages/0714-mgmttools.aspx`

[18] Lengyel-Sigl, Sabine (2014) Wie funktioniert Wertschätzung? Wichtigstes emotionales Gehalt auszahlen. In: Der Standard. Bildung & Karriere K12 26./27. Juli 2014

[19] Maslow, A (1954) Motivation and Personality, Harper & Row, New York

[20] Mateus-Berr, Ruth (2013) Applied Design Thinking LAB and the Creative Empowering of Interdisciplinary Teams. 2013 In: Springer Encyclopedia on Creativity, Invention, Innovation and Entrepreneurship (CI2E). Eds: Elias G. Carayannis, Igor N. Dubina, Norbert Seel, David F. J. Campbell, Dimitri Udiszuni. New York: Springer

[21] Mateus-Berr, Ruth (2014) Art and Design as Social Fabric. In: ARIS (Eds. Bast, Gerald, Carayannis, E; Campbell, D.), Springer, New York

[22] Mateus-Berr, Ruth (2014) Push-Button-Artists and Antinomy Caused by Constructivist Approach in: Conference Proceedings Columbia University, New York

[23] McBrien, Jody (2014) Learning In, With, and Through the Arts:Partnerships for Arts-Infused Education (PAInT), p.247 `http://www.insea2014.com/d/program/InSEA2014_Abstracts_Oral_Presentations.pdf`

[24] Patton, M. Q. (2002). Qualitative research and evaluation methods. Thousand Oaks, CA: Sage.

[25] Riegler, Alexander (2010)http://www.univie.ac.at/constructivism/HvF.htm (18.5.2013)

[26] Reitzer, Christine (2014) Erfolgreich Lehren: Ermutigen, Motivieren, Begeistern. New York: Springer

[27] Schraw, G., Dunkle, M.E., Bendixen, L.D. (2006) Cognitive processes in well-defined and ill-defined problem solving. DOI: 10.1002/acp.2350090605

[28] Skinner, E.A. (1995) Perceived control, motivation and coping. Thousand Oaks. CA: Sage.

[29] Smith, J.M., & Kovacs, P.E. (2011). The impact of standards-based reform on teachers: The case of No Child Left Behind. Teachers and Teaching: Theory and Practice, 17(2), pp. 201-225. DOI: 10.1080/13540602.2011.539802

[30] Spohn, Teacher Perspectives on No Child Left Behind and Arts Education: A Case Study DOI:10.3200/AEPR.109.4.3-12, p. 3-12

[31] TIMSS Advanced 2008. Trends in International Mathematics and Science Study. http://files.eric.ed.gov/fulltext/ED494176.pdf

[32] TIMSS Advanced 2015. International assessment of advanced Mathematics and physics. `http://timssandpirls.bc.edu/timss2015-advanced/downloads/T2015_TIMSSAdvanced.pdf`

[33] Tomlinson, Carol Ann, Brighton, Catherine, Holly, Hertberg, Holly, Callahan, Carolyn M., Moon, Tonya R., Brimijoin, Kay, Conover, ,Lynda A., Reynolds, Timothy (2003) Differentiating Instruction in Response to Student Readiness, Interest, and Learning Profile in Academically Diverse Classrooms: A Review of Literature, Journal for the Education of the Gifted. Vol. 27, No. 2/3, 2003, pp. 119–145. Copyright © 2003 The Association for the Gifted, Reston, VA 20191-1589.

[34] Vygotsky, L. (1978). Mind in society. Cambridge, MA: Harvard University Press.

[35] Weiner, B. (1990). History of motivational research in education. Journal of Educational Psychology, 82(4), p. 616-622.

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 113–120

Galilean geometry of the plane

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1. Principle of Galilean relativity

Principles of relativity address the problem of how events that occur in one place or state of motion are observed from another. And if events occurring in one place or state of motion look different from those in another, how should one determine the laws of motion?

Galileo approached this problem with a thought experiment which imagined observations of motion made inside a ship by people who could not see outside.

He showed that the people isolated inside a uniformly moving ship would be unable to determine by measurements made inside it whether they were moving!

> ... have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still.
>
> – *Galileo, Dialogue Concerning the Two Chief World Systems* [Ga1632]

Galileo's thought experiment showed that a man who is below decks on a ship cannot tell whether the ship is docked or is moving uniformly through the water at constant velocity. He may observe water dripping from a bottle, fish swimming in a tank, butterflies flying, etc. Their behavior will be just the same, whether the ship is moving or not.

Galileo's thought experiment led him to the following principle.

Principle of Galilean relativity. *The laws of motion are independent of reference location, time, orientation or state of uniform translation at constant velocity. Hence, these laws are invariant (i.e., they do not change their forms) under Galilean transformations.*

2. Galilean transformations of the plane

Observe the transformation $g\colon R^2 \to R^2$ given by:

$$g(x,y) = (x+a, y+bx+c), \tag{1}$$

where $a, b, c \in R$.

Lemma 1. *Transformation $g\colon R^2 \to R^2$, given with formula* (1) *is affine transformation.*

Proof. Let's write transformation (1) in this way:

$$x' = x + a,$$
$$y' = y + bx + c,$$

where $a_{11} = 1$, $a_{12} = 0$, $a_{21} = b$, $a_{22} = 1$, $q_1 = a$, $q_2 = c$. We see that linear part of this transformation is

$$\begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix},$$

and translation vector is

$$\begin{pmatrix} a \\ c \end{pmatrix}.$$

So, transformation (1) is affine transformation where matrix

$$A = \begin{pmatrix} 1 & 0 & a \\ b & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

is transformation matrix. □

Equation (1) is called Galilean transformation of the plane (further in the text denoted as $g = [a, b, c]$). Equation (1) can be written in matrix form:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ c \end{pmatrix}.$$

These transformations make a transitive group of transformations called **Galilean group of transformations G**. Geometric invariants of this group is a geometry that is called **Galilean geometry**. Observe two subgroups of the group G:

$$G_1 : \{g_1 = [a, b, c] : a, c = 0, b \in R\},$$
$$G_2 : \{g_2 = [a, b, c] : a, c \in R, b = 0\},$$

and their transformations:

$$g_1(x, y) = (x, y + bx)$$

$$g_2(x, y) = (x + a, y + c).$$

Transformation from group G_1 is affine transformation called shear, and transformation from the group G_2 affine transformation called translation. Notice that, every Galilean transformation can be written as:

$$\begin{pmatrix} 1 & 0 & a \\ b & 1 & c \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

that is, elements of the G_1 and G_2 are generators on Galilean group of transformations.

3. Invariants of Galilean group of transformations

3.1. Lines

There are two types of lines, ordinary and special lines. Ordinary lines are presented with:

$$y = kx + n,$$

spacial lines are lines parallel to ordinate, so their equation is:

$$x = p.$$

Lemma 2. *Galilean transformations map lines to lines.*

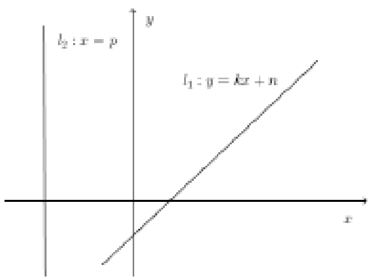

Figure 1: Special and ordinary line

Proof. If point A is a point on line $l\colon y = kx+n$, then A has coordinates $(x, kx+n)$. Image of the point A under Galilean transformation $g = [a, b, c]$ is:

$$g(x, kx + n) = (x + a, kx + n + bx + c).$$

If we put $x' = x + a$, then

$$g(x, kx + n) = (x', (k + b)x' + n + c - ak - ab).$$

If we have line $l\colon x = p$, then its image under transformation g is line

$$l'\colon x' = p + a.$$

□

Since parallelism is preserved under affine transformation it is also preserved under Galilean transformations too.

3.2. Distance

If $A_1(x_1, y_1)$ and $A_2(x_2, y_2)$ are two different points in Galilean plane. Then distance between these two points is given by formula:

$$d_{A_1A_2} = x_2 - x_1.$$

Notice that value in the formula can be positive, negative or zero. If $d_{A_1A_2} = 0$ than we introduce special distance given with:

$$\delta_{A_1A_2} = y_2 - y_1.$$

Lemma 3. *Distance and special distance are invariant under Galilean transformations.*

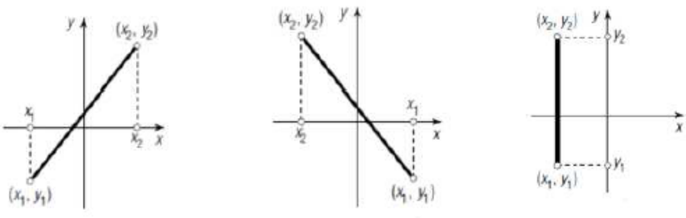

Figure 2: Positive, negative and zero distance

Proof. Look at Galilean transformation $g = [a, b, c]$, points $A_1(x_1, y_1)$, $A_2(x_2, y_2)$ and their images A_1' and A_2':

$$g(A_1) = A_1';\ \text{that is}\ g(x_1, y_1) = (x_1 + a, y_1 + bx_1 + c);$$
$$g(A_2) = A_2';\ \text{that is}\ g(x_2, y_2) = (x_2 + a, y_2 + bx_2 + c);$$

Now we can define distance between points A_1' and A_2':

$$d_{A_1'A_2'} = x_2 + a - x_1 - a = x_2 - x_1 = d_{A_1A_2}.$$

If $d_{A_1'A_2'} = 0$ then we look at special distance:

$$\delta_{A_1'A_2'} = y_2 + bx_2 + c - y_1 - bx_1 - c = y_2 - y_1 = \delta_{A_1A_2}.$$

□

3.3. Circle and angle

Circle S is a set of points $M(x, y)$ whose distances from a fixed point $Q(a, b)$ have constant absolute value r. If we follow this definition we can see that circle in Galilean geometry is special line. If $M(x, y)$ is a point of the circle $S(Q, r)$, notice equation of the circle is given with:

$$x - a = r.$$

Distance can be positive, negative or zero, so can radius r. If $r = 0$ then we define a special circle, a special line that contains center of the circle. Since, circles are special line, Galilean transformations map circles to circles.

Angle between two lines $l_1 \colon y = k_1 x + n_1$ and $l_2 \colon y = k_2 x + n_2$, that intersect at point $Q(a, b)$ is a segment on a circle $S(Q, 1)$. The angular measure of an angle between those two lines, is

$$\delta_{l_1 l_2} = k_2 - k_1.$$

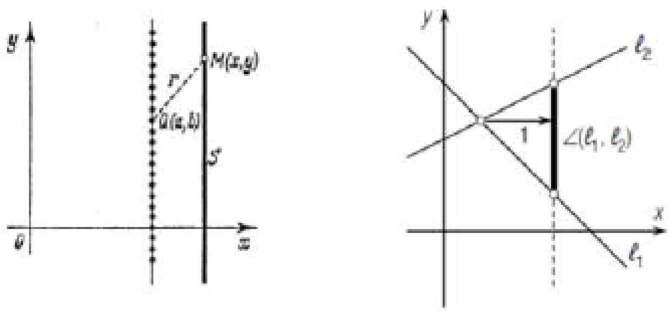

Figure 3: The circle and the angle

The angular measure can have positive, negative or zero value. If $\delta_{l_1 l_2} = 0$ then there are two possibilities either those two lines are parallel or they match. The angle between special and ordinary line is ∞.

Now let's observe point M and line l, $M \notin l$. There is special line m that contains point M and intersect line l in point P. Special line m is the line that is perpendicular to the line l, through point M. Special distance between points M and P is distance of the point M from the line l.

3.4. Triangle

Segment AB is a set of points of the line between points A and B, in Euclidean sense. Special segment A_1B_1 is a set of points of the special line between points A_1 and B_1.

If A, B, C are three non-collinear points then triangle ABC is a union of segments BC, CA, AB and none of these segments is a special segment.

With a, b, c we will denote the sides of the triangle:

$$|d_{AB}| = c, |d_{CA}| = b, |d_{BC}| = a,$$

and with A, B, C we will denote the angular measures:

$$|\delta_{bc}| = A, |\delta_{ca}| = B, |\delta_{ab}| = C.$$

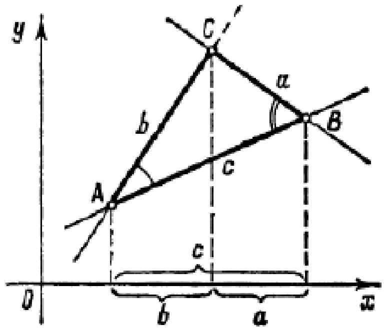

Figure 4: The triangle

Now let's look at the triangle ABC and its vertex coordinates:

$$A = A(x_a, y_a), B = B(x_b, y_b), C = C(x_c, y_c), (x_a < x_c < x_b).$$

Then

$$a = x_b - x_c$$
$$b = x_c - x_a$$
$$c = x_b - x_a.$$

Notice that in Galilean geometry we have equality of a triangle:

$$a + b = c, (a, b < c).$$

This equality is also valid for angular measures:

$$A + B = C, (A, B < C).$$

3.5. Cycles

In Euclidean geometry we can also define circles as the set of points from witch a given segment AB is seen at a constant directed angle α. In Galilean geometry set of the points described in this way is called cycle, Z. Equation of the cycle Z is:

$$y = px^2 + qx + r.$$

We can notice that this is equation of Euclidean parabola.

Like in Euclidean geometry the angle between a cord AB of the cycle Z and tangent line in points A and B equals α. If we look at the polygon line s, inscribed in arc AB, we can see that the length of that line equals to the length of the cord AB. Also, we can conclude that relation $\frac{s}{\alpha}$ is constant. The radius of the cycle Z is defined as $r = \frac{s}{\alpha}$. If the radius is positive than the cycle Z is convex, on the other hand, if the radius is negative the cycle Z is concave.

Lemma 4. *Galilean transformation g maps cycle $Z\colon y = px^2 + qx + r$ to the cycle which has same radius.*

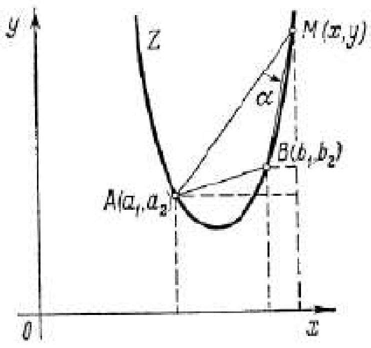

Figure 5: The cycle

4. The principle of duality

In Galilean geometry exists duality between certain terms. So the principle of duality is defined.

The principle of duality: *Interchanging the words "point" and "line", "distance" and "angle", "lies on" and "passes through" in any theorem in Galilean geometry yields another theorem.*

The main merit of the principle of duality is that it enables us to deduce new theorems from the known ones.

Example of this principle are dual equalities of triangle $a+b=c$ and $A+B=C$.

Due to this principle we meet some new figures in Galilean plane: co-parallelogram and co-trapezoid. Co-parallelogram is a figure dual to parallelogram, and co-trapezoid is a figure dual to trapezoid.

References

[1] Daril D. Holm, *Geometric mechanic. Part II: Rotating, Translating and Roling*, Mathematic Department Imperial College, London (2011).

[2] Dragović V., Milinković D., *Analiza na mnogostrukostima*, Matematički fakultet, Beograd (2003).

[3] Đorić M., Kalajdžić G., *Geometrija*, Beograd (2003).

[4] Hatchaturian A. V., *Galilean geometry*, Center for Continuous Mathematical Education, Moscow (2005).

[5] Yaglom I. M., *A Simple non-Euclidean Geometry and Its Phisical Basis*, Springer - Verlag, New York (1979).

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 121–135

Visual Mathematics in Education

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Abstract

The paper contains description of the course "Visual Mathematics and Design" organized at the Faculty of Information Technologies (Belgrade, Serbia). The course introduces students of graphical design to different areas of visual mathematics: symmetry in art and science, isometric symmetry groups, similarity symmetry, modularity, antisymmetry, colored symmetry, theory of proportions, theory of visual perception, perspective, anamorphoses, visual illusions, ethnomathematics, graph theory, and elements of knot theory. The output of the course is illustrated by representative original student works.

Keywords: visual mathematics, visual communication, symmetry, modularity, ethnomathematics

1. Introduction

The need for multidisciplinary courses in education is rapidly arising in the last few years. In the areas of applied sciences which are linked to different types of art expression and visual communication, we are using a variety of software. Different areas of visual mathematics can be used as a tool of visual communication, which is the universal and oldest way of communication. In order to use computers as creative tools in visual mathematics, it is necessary to be familiar with the basic ideas in behind.

Here we present the course Visual Mathematics and Design, established by the authors. It is one semester course in the first or the third year of the undergraduate study of Graphic Design at the Faculty of Information Technologies (FIT) in Belgrade. In this course we tried to connect two very different disciplines: mathematics and design. Mathematics, belonging to the exact sciences, uses a formal language of axioms, theorems, and proofs. Design, belonging to the sphere of the applied arts, requires a high level of creativity and intuition. At first glance, their only common point is the term „visual“. Thanks to the very rapid development of computers, visualization is becoming more and more important to scientific research. The period of time from the end of twentieth and the beginning of twenty-first century could be called „the era of computers“. Before that, mathematics was mostly a theoretical discipline, without using visualization for the study of abstract structures, like group theory or similar topics. Thanks to computers, today we are able to represent almost every mathematical structure by visual means. Design differs from pure art to the extent that it is strongly connected with human practical activities and needs. It needs to satisfy many ergonomic and other pragmatic requirements. To satisfy them, a designer need to use exact, reliable knowledge provided by natural sciences and mathematics. Hence, computer programs for 3-dimensional graphics and other software for visual modeling are the unifying elements of mathematics and design.

Our idea is to introduce students in various mathematical objects and their basic properties, visual message and visual identity, and get them familiar with different software to construct and manipulate with them. The course puts together subjects related to computer graphics, mathematics, design and some art and architecture disciplines. In this paper we consider only several main topics. Copyrights of all students' works presented here belong to the authors.

2. Symmetry in Art and Science

Symmetry in Art and Science is the starting point of the course, since the laws of nature and the objects of human creation are representations of symmetry. Symmetry in art reflects symmetry in nature. A.V. Shubnikov refers to this in his book "Symmetry in Science and Art". He defines symmetry as "the law of construction of structural objects". The scope and universality of the theory of symmetry can be noted by considering scientific fields in which it plays the most significant role: Mathematics, Physics (especially Solid State Physics, Particle Physics, and Quantum Physics), Crystallography, Chemistry, Biology, Aesthetics, Philosophy, etc. Owing to its universality and synthesizing role in the whole scientific system, certain modern-day authors give to the symmetry the status of philosophy category, thanks to its ability to express the fundamental laws of order in nature. The term „symmetry“ is connected to a whole spectrum of philosophic-aesthetic terms: harmony, proportionality, well-behaved form, etc.

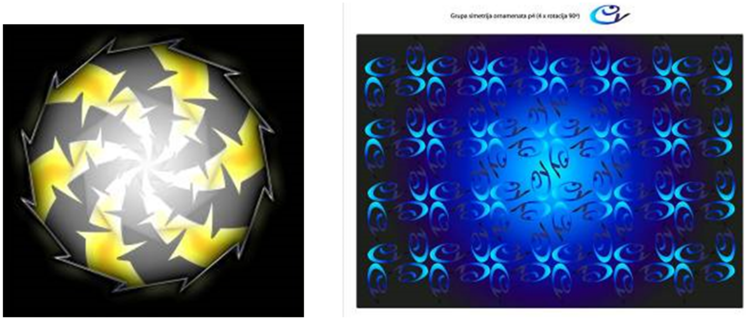
In our course, students start with symmetry exploration through the individual research project Symmetry everywhere, by making a set of photos with appropriate comments about symmetry. (discribing a kind of symmetry that is is present and visual effects it makes) (Fig.1). Starting with an informal concept of symmetry, they come to the formal, mathematically based concept of symmetry, plane isometries (reflections, rotations, translations and glide reflections) and their symbolic notation [5, 6, 15, 18, 21, 25, 26, 27].

Figure 1: A student investigation on the topic Symmetry everywhere (Sasa Petrovic)

The oldest examples of symmetry in art date from Paleolithic times (about 23 000 B.C.). The symmetry patterns from this time are very simple and mainly reduced to the repetition of some motif by two (mutually perpendicular) translations. We are not interested in the different initial asymmetric figures from which ornamental patterns are derived, but for the number of their different arrangements with regard to symmetry, since there are a limited number of possibilities: exactly 17 different symmetry types of plane ornaments. If we consider only linear groups of symmetries, the symmetry groups of friezes, line symmetry groups without invariant points, there will are only 7 symmetry types. In 3-dimensional space there is a total of 230 crystallographic symmetry groups.

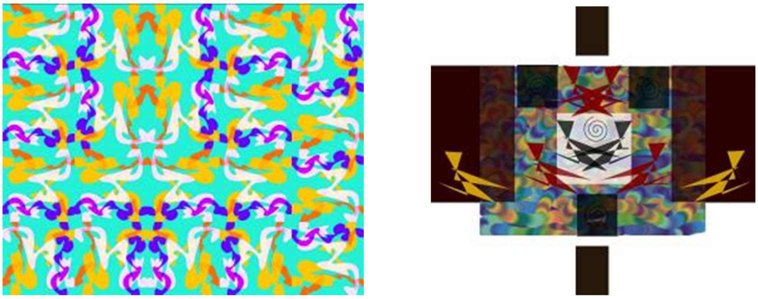
3. Isometric symmetry groups

In order to understand the variety of ornaments and their clasification, the next topic in the course is Isometric symmetry groups. We start with the beauty of rosettes, friezes and ornaments and the basic principles of their construction, introduce the concept of invariants and symmetry groups, and consider the symmetry groups of natural structures (crystals, regular and uniform polyhedra). By visualization, students are easily taken into the concept of a subgroup of a symmetry group, relations between groups and subgroups, and the meaning of the subgroup index in a group.The goal of this module is to learn how to find out and recognize construction methods by working on the examples from ornamental art [15, 18, 21, 25, 26, 27], and choose a basic element (tile, or prototile) for the perfect covering of a plane (without gaps or overlaps). During the study, students create rosettes, friezes and ornaments (Figs. 2 and 3) by using different software for explore symmetry groups and tessellations. At the end of this topic is presented the concept of tessellations by zoomorphic or antropomorphic motifs, with the special attention to M.C.Escher's works [12, 15,19].



a) b)

Figure 2: Students works on rosettes and ornaments: a) Milos Nikolic, b) Miroslav Zec



a) b)

Figure 3: Modern ornaments: a) Aleksandar Brzakovic, b) Ivana Jovanovic

4. Modularity

As a particular topic, we present the concept of Modularity, which can be treated as a generalization of symmetry and a manifestation of the principle of economy: possibility to crate variety of structures from one or few basic elements (modules) by their recombination. In modular constructions is desirable an equal use of the modules. Scientists always searched for the basic building blocks of nature ("atoms"), in Physics, Chemistry, Biology, and the other sciences. In Plato's philosophical treatise "Timaeus", the four basic elements in nature: earth, fire, air, and water, are identified with the regular polyhedra: cube, octahedron, icosahedron, tetrahedron, and the fifth regular polyhedron, dodecahedron, represented the Universe.

In physics, beginning from basic particles or elementary energy entities, the units of matter or energy, scientists try to explain nature by using modularity. A similar tendency occurs in art and design, especially ornamental.

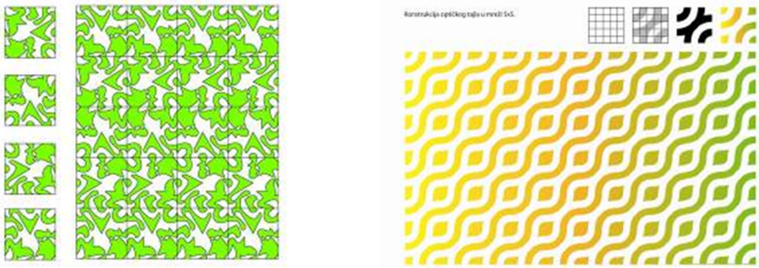


Figure 4: Students' works on modularity: a) Filip Roca, b) Strahinja Ivkovic, c) Miroslav Zec

The concept of modularity is introduced by recognizing modular structures in nature, art and science [13, 14, 15, 23, 24]. One of the simplest examples of a module is the Truchet tile– an antisymmetric and its variations. Truchet tile, was used beginning from Neolithic in the ornamental art of different cultures. A modification of the Truchet tile, that we call the Kufic tile, is the simplest Op-tile, a white square with one black diagonal strip, or it's negative. For one of the fundamental basic modules, Op-tile, a square with a set of parallel diagonal black and white strips and its negative recognized by S. Jablan, we also use the name "Versitile", proposed by the architect Ben Nicholson who found it analyzing Greek and Roman meander friezes and mazes. Key-patterns constructed from Op-tiles produce powerful visual effects of flickering and dazzle, thanks to the ambiguity which occurs due to the congruence between „figure" (the black part) and "ground" (white part) of the key-pattern. In the process of visual perception of such patterns, our eye oscillates between two equally probable interpretations. This kind of visual effect based on ambigiity is abundantly used in Op-art, which often uses unexpected visual effects of different geometric forms and colors producing visual illusions. Students investigate the choice of basic modules, modular constructions, the level of complexity of obtained structures, modular archetypes (Truchet tile), Op-tiles, Space-tiles, Knot-tiles [13,14,15], and explore by individual work the principles of recombination and visual identity, economy and diversity as the result of modularity (Figs. 4 and 5).

The previous work with Op-tiles can be used to present the concept related to the Theory of Binary Codes: bivalency as the basis of logical thinking. Antisymmetry is illustrated by construction of "black-white" ornaments, where students can perceive the relation between the figure and ground, the principle of duality, and visual dynamic of antisymmetric structures. In the lectures Antisymmetry Ornaments, students are getting familiar with 17 antisymmetry groups of friezes and with 46 "black-white" ornaments occurring in the history of ornamental art, from Neolithic until today [12, 19, 22, 23, 24]. Based on this knowledge, students exper-

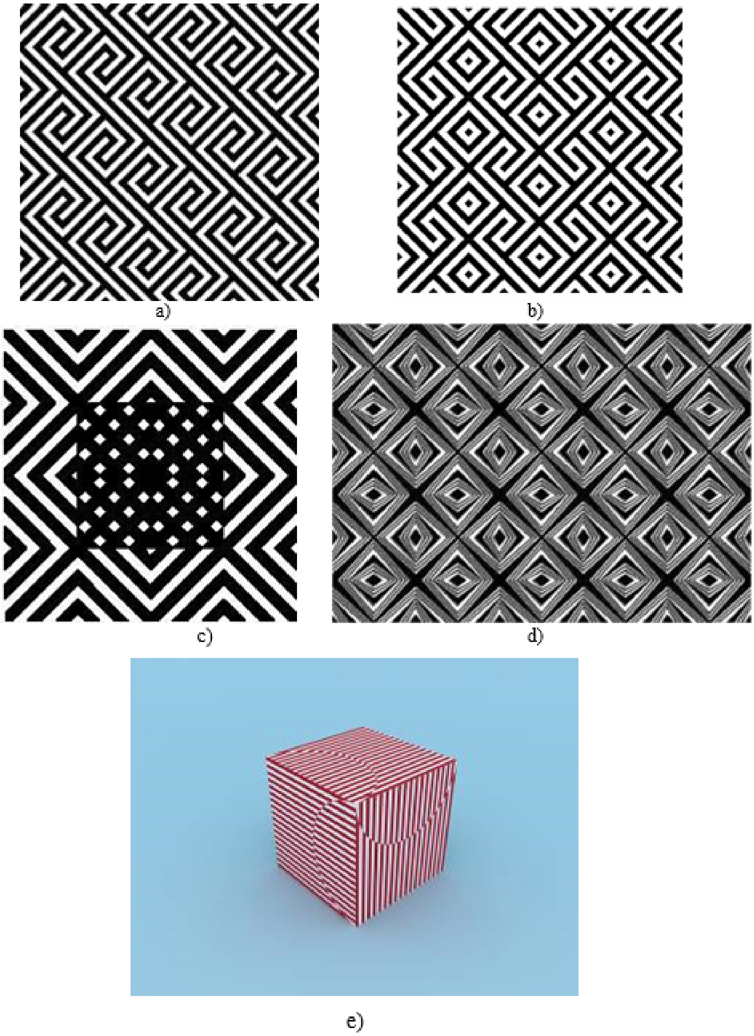


Figure 5: Modular structures based on OP-tiles: a) and d) Milos Nikolic, b) Marko Milanovic, c) Filip Milovanovic. e) Predrag Vidovic

imentate with antisymmetry and make their own antisymmetric constructions.

5. Theory of Visual Perception

The next topic Theory of Visual Perception is dedicated to the mechanisms of visual perception. We analyze visual perception of 2D and 3D objects, perspective and its special limiting cases– anamorphoses [4]. Students also experimentate with different structures obtained from a single mathematical object: square, circle, or triangle (Fig 6).

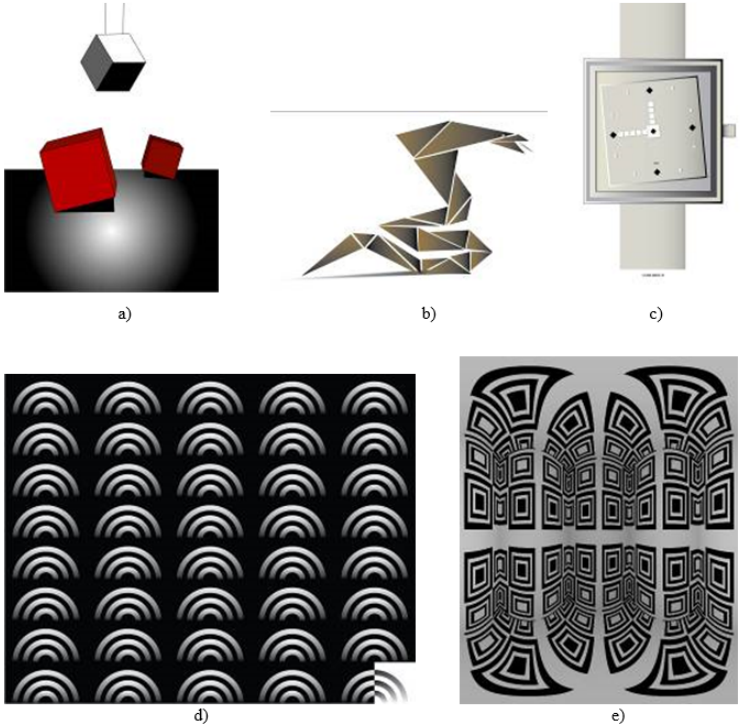


Figure 6: Different structures obtained using only one geometrical object: a) Filip Milovanovic, b) and c) Miroslav Zec, d) Predrag Vidovic, e) Irma Karisik

6. Visual Illusions

Investigatiing representations of 3D space in 2D plane, we analyze different ways of representing space in the history of art (in Prehistoric, Egyptian, Greek and Renaissance art), static and dynamic 3D perception (e.g., different phenomena which provide the illusion of a movement). In the section Visual Illusions are considered visual mechanisms responsible for them and illustrated by the examples of static and dynamic visual illusions (Fig 7, 8).

In particular, students analyze some well known impossible objects (Kofka cube, Penrose tribar, etc.) [8]. A basic building block of impossible objects is the Kofka cube: a regular hexagon divided into three congruent rhombuses. All three sides of

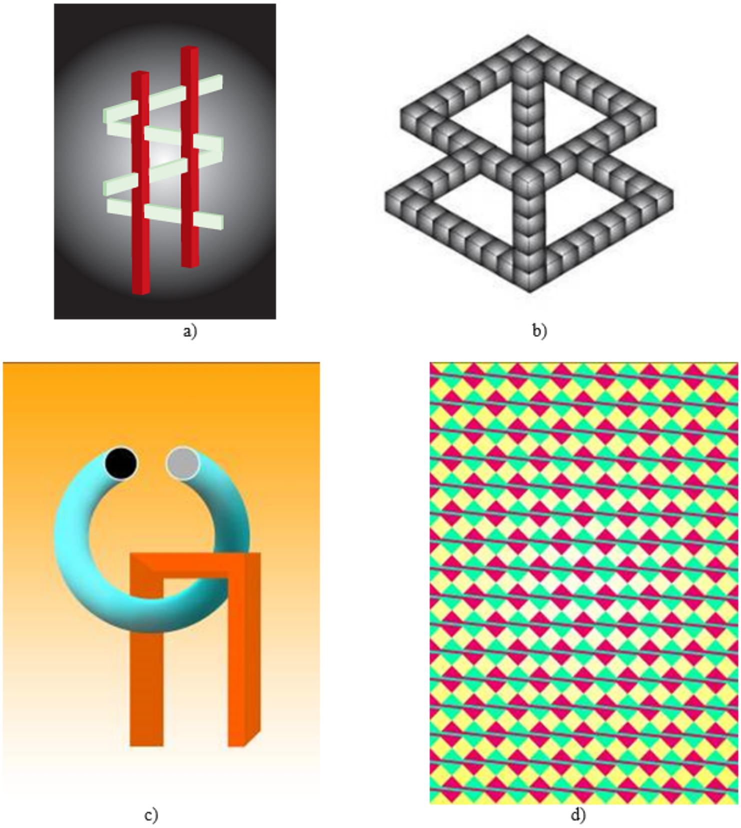


Figure 7: Impossible objects: a) Stevan Fililipovic, b) Filip Roca, c) and d) Aleksandar Brzakovic

the Kofka cube are identical, so we cannot tell from which of three equally possible points of view it is viewed, whether it is convex or concave, or even that it represents a 3D-object or a regular plane hexagon consisting of three rhombuses, which gives a rhombic tessellation. Many impossible figures can be constructed from Kofka cubes as 3D-puzzles. Impossible objects entered into the domain of mathematics and the visual arts in the twentieth century. The Kofka cube and Penrose tribar are the first examples of that kind. They became very popular and are used as the geometric basis for the construction of many artworks (e.g., graphics „Belvedere", „Waterfall", or „Ascending and Descending" by M.C. Escher).

Furher, in the topic Theory of Proportion students explore Golden section, Fibonacci sequence and similarity symmetry (dynamic symmetry) recognizing it in natural structures and biological process of growing [15, 18].

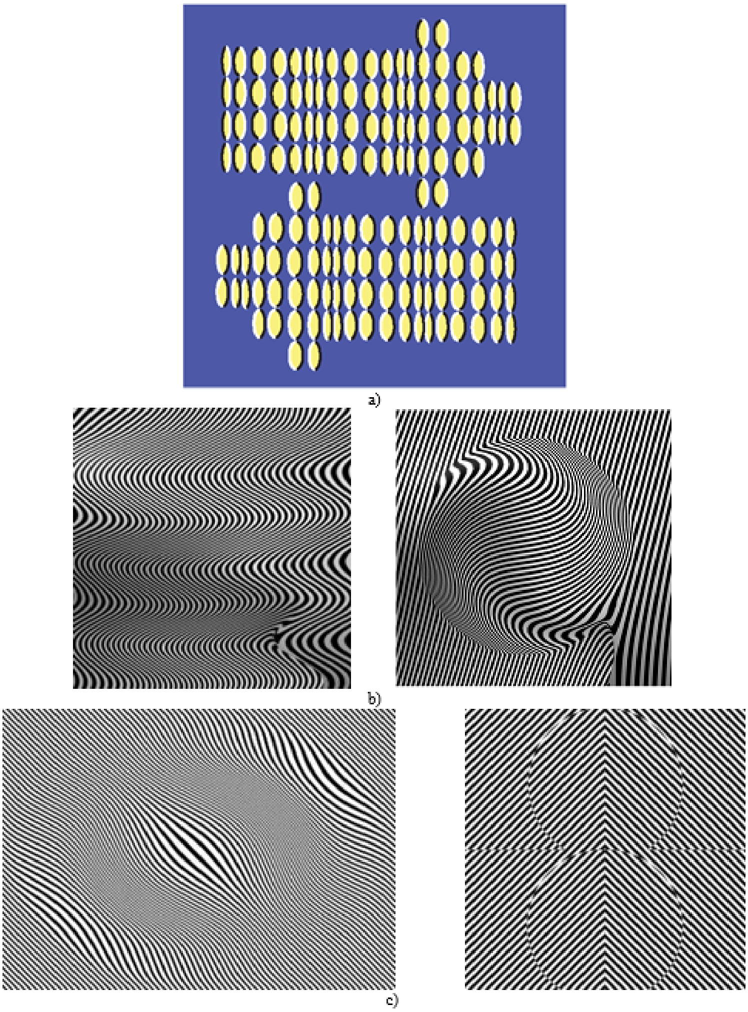


Figure 8: Student works inspired by works of A. Kitaoka and Op-art works: a) Zlata Mrkaljevic, b) Irma Karisik c) Filip Roca

7. Ethnomathematics

Ethnomathematics represents the connection between cultural anthropology and mathematics and utilizes mathematical modeling combined with visual design and cultural tradition, in order to solve real-world problems. As a part of Ethnomathematics, students learn the basics of Graph theory and its applications in visual mathematics. Graphs occurring in different cultures are reviewed and explained as a universal tool for illustrating relations, with a special attention to the application of graphs in visual representations of real-life models (traffic, telecommunication) [2]. In particular, students are introduced to the concept of Mirror Curves , 4-valent graphs obtained by drawing self-intersecting plane curves, through the examples of mirror-curves occurring in ornamental art [3, 10, 11].

Different cultures constructed mirror-curves: knot diagrams placed in plane tessellations, occurring in Tchokwe sand drawings, and Tamil and Celtic ornamental art. A perfect design (mirror-curve) has one continuous curve (component). Tamil curves, consisting of a single curve called „pavitram" („ring") or „Brahma-mudi" („Brahma's knot") represent a kind of the cultural ideal. Tchokwe sand drawings, called „sona" designs played a very important role in the transmission of knowledge from one generation to the other. Tchokwe children enjoyed drawing with their fingers simple stylized drawings of different animals, birds, or human figures, followed by stories and tales. More complex drawings were carried out only by the experienced story tellers („akwa kuta Sona" = „those who know how to draw"), which played the role of highly valued teachers, part of the intellectual elite of the Tchokwe society. We are interested in understanding the geometrical principle behind the construction of mirror curves.

The common geometrical construction principle of mirror curves, discovered by P. Gerdes, is the use of two-sided mirrors incident to the edges of a square, triangular or hexagonal regular plane tiling, or perpendicular to the edges in their midpoints [9, 10, 11]. The name "mirror curves" can be justified by visualizing them in a rectangular square grid of dimensions a, b ( ), whose sides are mirrors, and additional internal two-sided mirrors are placed between the square cells, coinciding with an edge, or perpendicular to it at its midpoint. In this grid, a ray of light, emitted from one edge-midpoint at an angle of , will close a component after a series of reflections. Beginning from different edge-midpoints and continuing until the whole rectangular grid is uniformly covered, we trace a mirror curve. This construction can be extended to any connected part of a plane tessellation. Every mirror curve can be converted into a knotwork design by introducing the relation "over-under" (Fig. 9) and combine different parts in order to obtain more complicated mirror curves [10, 17]

We can go further with basic concept of Lunda designs and fractals. Inspired by P. Gerdes' work, students made their own works on mirror curves, Lunda designs, Lunda fractals and knotwork lettering (Figs. 9, 10). A Lunda design can be obtained from a mirror-curve by denoting the sequence of its successive steps by 1, 2, 3, ..., and then taking all the numbers modulo 2. The result is a sequence of zeros and ones. By coloring all small squares (four of them contained in every cell), denoted by ones in black, and the others in white, we obtain „black-white", antisymmetric patterns called Lunda designs. More about Lunda fractals and Lunda designs can be found in [10].

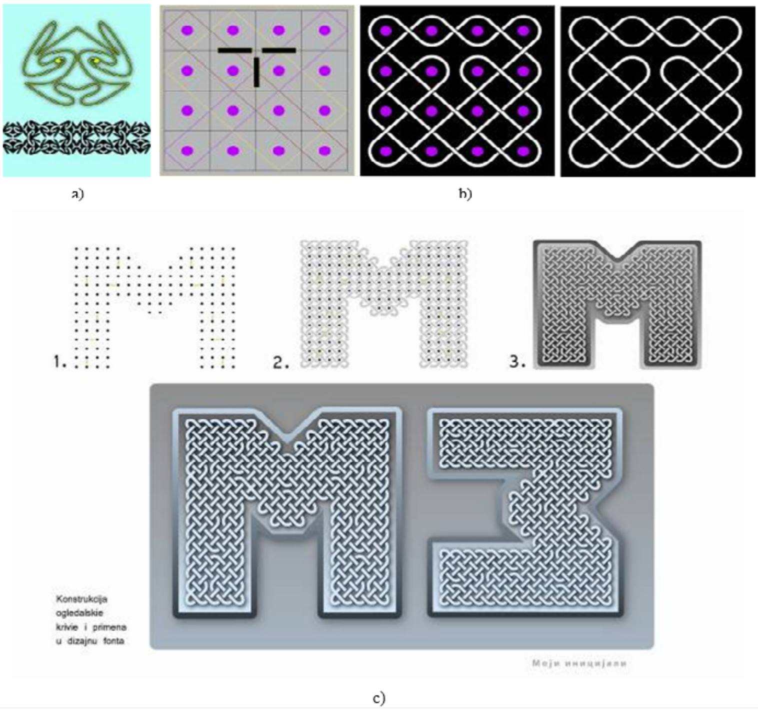


Figure 9: Mirror curves and Celtic knots: a) Aleksandar Brzakovic, b) Strahinja Ivkovic, c) Miroslav Zec

8. Knots and links

Knots and links are closely connected to ornamental art and interlacing patterns originating from basketry, matting, or plaiting that later are cipied to the stonger media (bone, stone, ceramics). Decorative knots have been used from prehistory till our days as a basis of different artworks, and they still can be an inspiration for artists and artisans. In the course we presented an introduction to knot theory and basic elements of topology (one and two-sided surfaces and their models: torus, Möbius band, Klein bottle and minimal surfaces). Within the theme Elements of Knot Theory students become familiar with the mathematical concept of knots and links, basic terminology and coding of knots: ambient isotopy, knot and link diagrams, Dowker and Gauss codes, Conway notation, Reidemeister moves, minimal diagrams, families of knots and links, tangles and basic polyhedra [1, 9, 20]. Students work with real knots, their models and drawings, and explore their properties (amphichirality, unknotting, relaxation, symmetry) by using knot theory software KnotPlot and LinKnot (Figure 11).

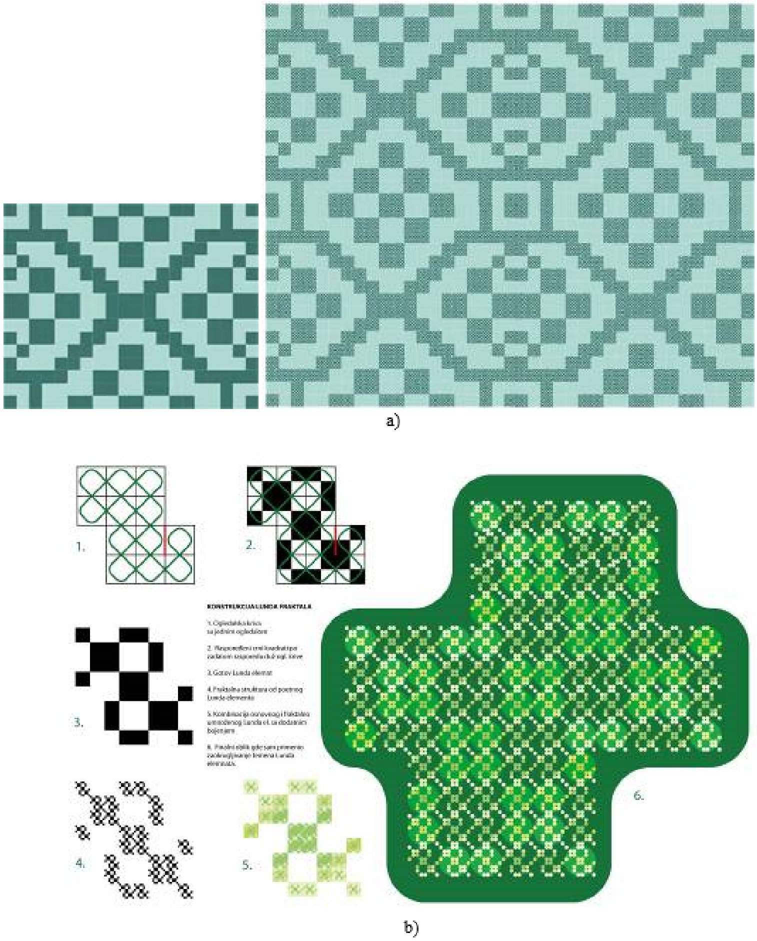


Figure 10: Lunda design and construction of Lunda fractals: a) Aleksandra Milic, b) Miroslav Zec

9. Fractals

Fractals are included in the course as the illustration of self-referential systems. Through the examples of Koch, Peano and Dragon curve, students become familiar with the basics of fractal theory and the concept of recursion, iterations and iteration series [20]. Moreover, students learned about L-systems (Lindenmayer systems), and natural recursive systems. As an exercise, they generate self-referential systems (Fig. 10) and some fractal images using free software (Fractint and Ultrafractal).

We also considered Symmetry in Architecture, symmetry of 3D structures, and

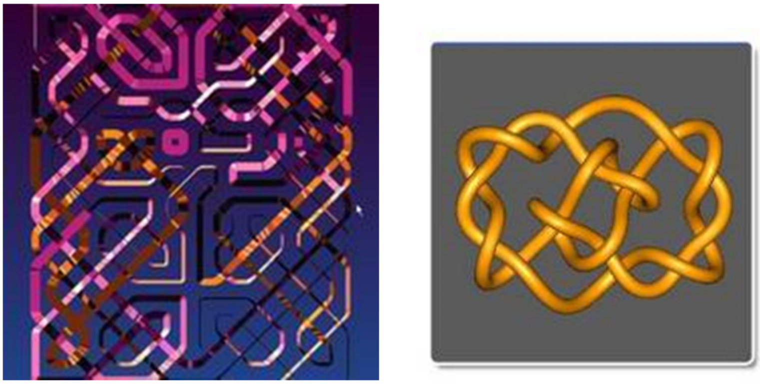


Figure 11: Exploration and construction of knots with KnotPlot: a) Dijana Dailoski, b) Marko Nikolic

Figure 12: Some of the final student's works – posters for the promotion of FIT (Graphic Design) by using elements from Visual Mathamatics course: a) Aleksandar Brzakovic b) Milos Lazarevic, c) Miroslav Zec, d) Strahinja Ivkovic

discussed static and dynamic architectural symmetry by analyzing classic and modern construction principles in architecture. In the modern architecture and design, thanks to the computer design and usage of new materials, the constructions of various new structures become possible, including organic-like structures. This is illustrated by examples of contemporary architecture (F. Gehry, M. Watanabe, H. Lalvani, and S. Calatrava). Students also learned about modular architectural design, multidimensional polytopes and their use in architectural projects (K. Mizayaki).

We are continuing to develop this course with the hope that it could be adapted not only for designers, but for architects, mathematicians, psychologists, teachers and many others. In the last year some parts of our course are used in the similar courses created in USA by J. Kappraff at NJIT, and at the Pecs and Kaposvar University in Hungary.

References

[1] Adams C.C.: The Knot Book, Freeman, New York, 1994.

[2] Ascher, M. Graphs in cultures: a study in ethnomathematics, Historia Math. 15, 3, 201-227, 1988.

[3] Bain G.: Celtic Art - the Methods of Construction, Dower, New York,1973.

[4] Barrett C.: Op Art. Studio Vista, London,1970.

[5] Coxeter H.S.M.: Introduction to Geometry, Willey, New York,1969.

[6] Coxeter H.S.M.: Regular Polytopes, 3rd ed., Dover, New York, 1973.

[7] Coxeter H.S.M.: A Simple Introduction to Colored Symmetry. I.J. Quantum Chemistry XXXI 1987. str. 455-461.

[8] Ernst B. : L'avanture des figures impossibles, Taschen, Berlin, 1990.

[9] Gerdes P.: Sona Geometry, Instituto Superior Pedagógico Moçambique, Maputo, 1994.

[10] Gerdes P.: Lunda Geometry– Designs, Polyominoes, Patterns, Symmetries, Universidade Pedagógica Moçambique, Maputo,1996.

[11] Gerdes P.: Geometry from Africa: Math. and Educational Explorations, Washington, DC: The Mathematical Association of America, 1999.

[12] Grünbaum B., Shephard G.C.: Tilings and Patterns. Freeman, San Francisco, 1987

[13] Jablan S.: Modular Patterns and Modular Games, Visual Mathematics, `http://www.mi.sanu.ac.rs/vismath/op/index.html`, 2001.

[14] Jablan S.: Modularity in Science and Art, Visual Mathematics, 4,1, 2002. `http://www.mi.sanu.ac.rs/vismath/jablan/cover.htm`

[15] Jablan S.: Symmetry, Ornament and Modularity, World Scientific, New Jersey, London, 2002. `http://www.emis.de/monographs/jablan/index.html`

[16] Jablan S., Sazdanovic R. : LinKnot, World Scientific, New 2007. (`http://math.ict.edu.rs/`).

[17] Jablan, S.V. (1995) Mirror generated curves, Symmetry: Culture and Science, 6, 2, 275-278.

[18] Kappraff J.: Connections: The Geometric Bridge Between Art and Science, World Scientific, 2007.

[19] Macgillavry, C.H.: Fantasy and Symmetry: The Periodic Drawings of M.C.Escher, Harry N.Abrams, New York, 1972.

[20] Mandelbrot B.: The Fractal Geometry of Nature, W H Freeman, New York, 1982.

[21] Martin G.A.: Transformation Geometry: An Introduction to Symmetry, Springer Verlag, Berlin 1982.

[22] Radovic Lj.: Antisymmetry and Multiple Antisymmetry, M.A. Thesis, University of Nis, Serbia, 1998.

[23] Radovic. Lj., Jablan S.: Antisymmetry and Modularity in Ornamental Art, Visual Mathematics 3,2, 2001. `http://www.mi.sanu.ac.rs/vismath/radovic/index.html`

[24] Radovic. Lj.: Reconstruction of Ornaments, Visual Mathematics 4, 1, 2002. `http://www.mi.sanu.ac.rs/vismath/radovic1/index.html`

[25] Shubnikov A.V., Koptsik V.A.: Symmetry in Science and Art, Plenum Press, New York, London, 1974.

[26] Washburn D.K., Crowe D.W.: Symmetries of Culture, University of Washington Press, Seattle, London, 1988.

[27] Weyl H.: Symmetry, Princeton University Press, Princeton, 1952.

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 137–142

Discovering the *Living Body of Mathematics* in Real Life

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Abstract

During the past 50 years the seminal ideas in mathematics education of problem solving, modelling and integration with other disciplines have finally led to the creation and use of real life themes in mathematics teaching, and thus to a truly humanistic vision of mathematics in society and in the Real World. This paper will look at some innovative international projects that have initiated this paradigm shift in our conception of mathematics, and how it should be taught. This paper will contrast mathematics as a skeleton collection of rules and axioms, with a *living body of mathematics* which derives from the multitude of ways mathematics is used in Society and in Real Life. Underpinning this analysis is the work of Polya, Kuhn, Lakatos and Wittgenstein.

1. A new paradigm shift for Mathematics and hence for Mathematics Education

The Mathematics in Society Project (MISP) began in 1980 as an international association of mathematics educators [3]. They realised there was a paradox in that mathematics is widely used and diffused implicitly in all industrialized societies (M2), but most pupils find school mathematics (M1) difficult and/or unpleasant. This table sharpens the contrast between these two kinds of mathematics:

Mathematics in school (Ml)	Mathematics in society (M2)
Often negative reaction	Positive/neutral reaction
Explicit	Implicit
Studied	Used
Often unsuccessful	Usually automatically successful
Often unpleasant	Usually neutral/pleasant
Not integrated with other things	Integrated

Figure 1: Contrasting two kinds of mathematics

Most schools and universities use a uniform and highly standardized syllabus for teaching mathematics (M1). On the other hand, no such uniform list of items exists to describe the manifold and implicit ways in which mathematics appears to be used in society. If we tried to compile such a list we should soon be overwhelmed by the enormity of particular detail, and the fact that even as we collect this data and try to record it, society is itself changing faster than we can construct our list. This realistic and holistic image of M2, mathematics in society, reminds us of the analogy that Wittgenstein made between language and a city: an enormous, complex, living, growing whole whose very nature makes it impossible to summarize or completely describe. This analogy between language and a city can be extended to include our two different conceptions of mathematics:

language = city = mathematics in society, growing, changing, like a body (M2)
dictionary = map = syllabus/curriculum, formal, fixed, like a skeleton (Ml)

And now we see, using the metaphor of a body and a skeleton, what the MISP project meant by the living body of mathematics. According to MISP, the reason why so many pupils find school mathematics difficult and/or unpleasant is because schools and colleges – institutionalized education – have based their concept of mathematics not on 'mathematics in society, M2' but rather on 'syllabus lists, Ml' and have tried to teach mathematics as though it only consisted of items on these lists.

It was as if the living body of mathematics in society (M2) was reduced to a 'skeleton' of syllabus items (Ml), and in the process institutionalized education lost sight of the fact that there ever had been, or ever could be, such a living body for mathematics (M2). It is not surprising then, pursuing this metaphor, that the study of a skeleton without knowing of the existence of a body has proved to be much less interesting to our pupils, especially as the real purpose of a skeleton is its role in relation to a body!

From the viewpoint of Ml it may seem that this new conception of mathematics is un-analytic, unclear, 'against method' as Feyerabend would say. From the viewpoint of M2, however, it is the old conception (Ml) which is seen as disadvantageous, since it reduces a living, integrated subject (M2) to a lifeless and intrinsically un-real and unmotivating list of things which can, and should, exist together in a more holistic way.

The newly formed MISP project spent many years after 1980 discovering and hence better comprehending this living body of mathematics in a practical and concrete way, by creating a large amount of original materials for mathematics students. Unlike previous attempts to 'apply' mathematics (M1 again) we literally forgot about the skeleton (Ml) and instead spent many years wandering amongst the highways and byways of what was obviously a new and fascinating city: mathematics in society (M2), collecting personal and stimulating experiences from the people in our society, and in real life, who were using (implicitly) this living body of mathematics.

MISP eventually created, tested and distributed hundreds of original thematic units (booklets with a real life theme) which were used by students to learn mathematics (M2) implicitly in a new and much more motivating way. Examples of such real life themes include: Survival, Travel, Housing, Machines, Art, Architecture, Agriculture, Magic, Water Consumption, etc – in fact the whole range of human activities and interests including, for example, history (the Story of Xenophon, The Merchants of Venice, Captain Cook) and creative fiction (Sherlock Holmes, Gulliver). Two other features of these thematic units have been the strong use of visualisation, and the personalisation of some themes, in which a fictional character, derived from reality, is introduced to communicate the real life theme: Business Woman's trip to Hong Kong, A Tale of Two Cities, Astronomer Jan, Nurse Justyna, A lorry Driver, Running a Pizza Shop, and so on. These MISP booklets have been adopted in many schools all over the world, in Australia, New Zealand, Singapore, Brazil, Uruguay, the USA, and Europe, and were a major stimulus to many subsequent and parallel projects that shared the MISP vision.

2. Kuhn, Lakatos and Paradigm Shifts

The term *Paradigm Shift* was created and popularised by Thomas Kuhn in his seminal book *The Structure of Scientific Revolutions* [1]. A Paradigm Shift is a revolution, a fundamental change in our world view which changes even the way reality is perceived and understood. We strongly recommend to our readers Kuhn's book and the equally fundamental and seminal work of Imre Lakatos: *Proofs and Refutations: The Logic of Mathematical Discovery* [2]. It will come as no surprise to admirers of George Pólya and his book *How to Solve* It that he was a strong influence on the development of Lakatos's revolutionary ideas applied to the History of Mathematics. In both Kuhn's and Lakatos' work the crucial idea is that of a revolutionary (as opposed to an evolutionary) Paradigm Shift.

Kuhn and Lakatos explain that a paradigm shift occurs when the new conception resolves problems and paradoxes which the old conception fails to resolve. The success of the new MISP conception was confirmed by the enormous increase in interest and motivation shown by our students when they were immersed in real life/society re-creations or simulations, in which they implicitly learned not only the syllabus mathematics (M1) but they appreciated its place in the living body of mathematics (M2), just as a skeleton is meant to function inside a real body!

3. Further progress globally of this new paradigm shift

In Holland, as a result of the highly innovative and thematic work of many of their national projects based at Utrecht and elsewhere, a publisher in 1986 (Meulenhoff Educatief, Amsterdam) actually produced possibly the first truly thematic text book (Exact Wikunde) in which we also saw the idea of personalisation of stories and themes, which MISP had also pioneered. The text book series was too far ahead of its time (as MISP had been in the UK in the 1980s) but ThiemeMeulenhoff, Utrecht have recently re-published an almost identical thematic textbook (Pascal Wiskunde) which is having much more success because of the changed attitude globally to thematic work.

A *Mathematics in Society Unit* has for many years been part of the official mathematics curriculum for the state of Queensland in Australia. Also in Australia, in the 1980s, the MISP work was the inspiration for a continuing Year 6 Australia Pioneer Settlement Project in Scotch College, Melbourne where ALL the curriculum (not just mathematics) was taught via practical work on a large plot of land adjacent to the school (building, gardening, looking after animals, exploring etc) re-enacting the work and activities of the early Australian pioneers! This certainly demonstrated how much more successfully our curriculum in ALL subjects could be taught if it was transformed into totally realistic activities and themes, the *living body of the whole curriculum* as it were. A similar holistic approach was used successfully by the Rudolf Steiner school in Eltham, Victoria, Australia, with an emphasis on the historical, cultural and humanistic heritage of our present day society.

In Poland from 2004 onwards, Margaret Fryska and her national team of regional groups of writers also produced two years work of draft chapters for a thematic textbook of the future in Polish, inspired by MISP's new conception of mathematics. These chapters still exist as resource units in schools in Poland in the expectation of eventual publication, but only when Polish teachers feel ready for such a giant quantum leap or paradigm shift.

In Germany, starting as early as 1977, the innovative teacher association MUED has created thousands of problem solving and modelling activities, some of which also featured real life themes, often with a socially constructive theme. The most successful implementation of the living body of mathematics in schools, however, is the fully thematic MatheLive series published by Klett and inspired by the MUED group, who provided the principal authors for the series. MathLive is perhaps the most perfect embodiment so far of our new conception of mathematics. We see the living body of mathematics now being taught by ordinary teachers in many normal schools and to a growing proportion of the whole school population in Germany. Our only regret is that this series so far only exists in German despite our efforts to interest an English publisher to translate and adapt it for students in English and eventually, we hope, in other languages too. A close study of the MISP and MatheLive materials is perhaps the best way to fully appreciate what we mean by the living body of mathematics, and it is also important to actually see the success of these materials in schools to fully understand how this new and exciting paradigm shift works in practice.

4. To the future

This new conception of what mathematics is (M2) not only encourages the production of real life themes. We have also incorporated many new and important teaching methods that we feel enhance student motivation and effective learning. There is only space here to mention some names, but each of them deserves careful study: Continuous Assessment, Self-Assessment, Group Work, The Expert Method, Co-operative and Active Learning, Integration with other school subjects, Use of Technology and so on. These vastly improved techniques of teaching/learning compare favourably with the *talk and chalk* that so often accompanied M1 in the past. Like our new conception of mathematics M2, they also derive from the Real World and hence must enhance the teaching of the living body of mathematics itself.

As a direct result of many years of cooperation between MISP and MUED, a new project called Developing Quality in Mathematics Education (DQME) was proposed in 2003 and realised in our European Union Comenius Projects DQME I (2004–2007) and its continuation project DQME II (2007–2010). Both projects produced and disseminated many practical materials and also new ideas for learning and teaching mathematics in the classroom as the living body of mathematics. Our Webpage `http://www.dqme2.eu/` has further information (in 11 languages) on the history and development of the two DQME projects.

It is the main objective of a new project formed in 2010, DQME3, that we continue to produce, translate and disseminate as much new and useful material as we can in many countries throughout the world as a continuation of DQME I and II. The 12th International Conference of the Mathematics Education for the Future Project: The *Future of Mathematics Education in a Connected World* was held from September 21–26, 2014 at the Hunguest Hotel Sun Resort, Herceg Novi, Montenegro and included a re-launching of DQME3 as part of this now global paradigm shift in mathematics education called the living body of mathematics.

5. Concluding remarks

We are convinced that these innovations lead to the more effective learning of "mathematics". But this "mathematics" now means not only the traditional mathematics syllabus (M1) Our students can now appreciate mathematics not as a subject restricted to school learning, but rather as a widely used and successful tool to solve, often implicitly, many real life problems in society. Thematic mathematics teaches them that mathematics is an integral part of many people's everyday life. We have been convinced over and over again, throughout the years, of the success of showing students that NOT only mathematicians, mathematics teachers, accountants and informatics specialists use mathematics, but also (eg) lorry and taxi drivers, hospital nurses, restaurant managers, woodcutters, artists and people from a whole host of other human activities and occupations who routinely and successfully use mathematics as part of their normal realistic life.

This is the essence of thematic mathematics: to illustrate in a visual, motivating and communicative way the reality of the living body of mathematics, mathematics as used in our society, and hence also in human history, in human culture, and human creative fiction. In this sense the paradigm shift we have described is in all its facets a humanistic revolution in the teaching and learning of mathematics, reminding us of the seminal work of Kuhn and especially Lakatos who showed the history of the development of mathematics was NOT just about logic and reason, but a record of human creativity, and ... may we also add ... the triumph of the human spirit.

References

[1] Thomas S. Kuhn, *The Structure of Scientific Revolutions*, Phoenix, 1972

[2] Imre Lakatos, *Proofs and Refutations*, CUP, 1976

[3] Alan Rogerson, *MISP - A New Conception of Mathematics*, Int. J. Math. Educ. Sci. Technol. 17.5. 1986

[4] Alan Rogerson, *A Brief History and Guide to Thematic Mathematics*, Real Life Themes and Mathematics in Society, Growing Mathematics, Second annual publication of the Comenius-Network Developing Quality in Mathematics Education II (ed Hans-Wolfgang Henn and Stefanie Meier) TU Dortmund Sep 2009

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 143–149

Mathematical Workshops, Learning and Popularization of Mathematics

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Abstract

On the example of mathematical workshop launched for pupils of upper grades of primary school, we discuss the benefits of workshops related to the improvement of teaching methods and students' knowledge and understanding of mathematics.Through illustrative examples from the workshops we have shown different techniques and possibilities for realization of teaching process in the upper grades of primary schools mathematics.Also, the concept of mathematical workshops is described in detail and the effects of the workshops on pupils knowledge of mathematics, the reactions of mathematics teachers and the popularization of mathematics, are analyzed.

1. Introduction

Teaching of mathematics, and in particular its modernization and improvement, always presents a challenge not only for teachers of mathematics, but also for people from other professions where mathematics is used. Mathematics is characterized by most as complicated science, and something that understands only a small percentage of people. Also, mathematics is rarely viewed from the perspective of science applicable in every aspect of real life, which is the main reason for its low popularity.

In order to popularize mathematics and present it in its true light, as interesting and applied science, and to get closer to primary school pupils at the beginning of their development path, a mathematical workshop has been designed.

The paper introduces and describes activities conducted on a mathematical workshop launched in Zrenjanin, by the author of the paper, in cooperation and with the support of the Cultural Centre of Zrenjanin.

2. Mathematical workshop

Workshops of various types and purposes have been present for a long time with us, and they generally encounter with positive reactions from participants. By examining the types of workshops which take place in our area, it was determined that the contents of these workshops are related to various topics and sciences, but there was not one dedicated to mathematics. Each type of extracurricular activities in mathematics are related to the practicing of problems solving, preparing for graduation and entrance exams, and they include the application of traditional teaching methods (black board, oral presentations...). The idea of the author of this paper, for the establishment of the mathematical workshop, has just emerged from these results.

Mathematical workshop as devised by the author of the paper, should have some basic objectives. The first objective relates to the popularization of mathematics as a science, since it is a widespread attitude by children, and most of adults, that mathematics is something that is incomprehensible and difficult. Providing opportunities for children to socialize, play and get acquainted with mathematics in a new light, to learn something new, repeat materials, is also one of the goals of mathematical workshop. The third objective is to demonstrate new teaching methods such as active learning and mathematical modeling, learning on real world examples and throughout the games, which results in another objective of mathematical workshop: presentation of mathematics as interesting science applicable on situations from real world environment.

Having regard to all objectives set, it was designed a mathematical workshop for pupils of upper grades of primary school, called - Matemationica.

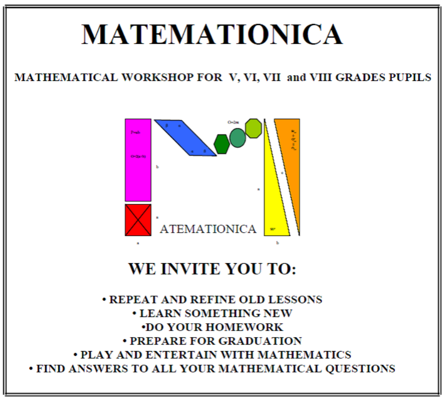

Figure 1: Poster with Matemationica logo

3. Matemationica

Matemationica is the result of many years of work and the aspirations of author to enclose, popularize and present mathematics in whole new light to pupils. It is designed for pupils of upper grades of primary school, and for the teachers and professors of mathematics.

3.1. Organization of Matemationica

Matemationica is completely no charge workshop, and pupils attend the workshop at their free choice. It is designed for all pupils in upper grades of primary school, regardless of the score, and math skills. Matemationica was held weekly, in duration of 90 minutes.

Also, Matemationica processes contents connected with themes of regular mathematics classes, at the time. Due to continuous work and dedication to the topics to be covered, each week one grade attended the workshop. So, Matemationica began working with the fifth grade, the next week attended sixth grade, ... By this kind of organization dedication to each senior grade separately can be achieved.

Although Matemationica primarily was designed only for pupils, it changes it's orientation also towards teachers and professors, thru demonstration of new teaching methods and techniques that can be used in mathematics education.

3.2. Contents

Matemationica was conducted in cycles, where one cycle consisted of four weeks in a row. Thus, one cycle attended respectively pupils of V, VI, VII and VIII grade. This organization is deliberately created, because of the contents devoted to each cycle.

I Cycle

The first cycle included topics that were connected with topics student were learning at that time at mathematics classes at school, separately for each grade, except that each subject was treated in a different way - through games.

The pupils were presented with a variety of popular games, but with the mathematical content. Thus, fifth grade pupils solved crossword puzzles where the issues horizontally and vertically were related to divisibility of numbers, least common multiple and greatest common divisor, which they were learning at the time in school. Sixth graders were offered the game Discover the picture, where everyone reveals a new box, with the task on integers, which when properly resolved, reveals one part of the big picture. Seventh graders had also played the game Discover the picture, but on algebraic expressions. For eighth graders has been designed popular game Sorry!, but with slight modifications. In fact, when someone puts a figurine, automatically raises the question connected with the field. If you answer correctly, you stay on the field, otherwise the figure is moved one field back and respond to a new question ... This game contained questions concerning the area of plane figures and surface area and volume of prisms, since the pupils that day at school had the control task with topics: area and volume of prisms.

Pupils have reacted very positively to these games. During each game they were divided into groups, usually those were teams of pupils from one school, so each game had also a competitive character. It was very interesting to watch their reactions throughout the games, their dedication to the tasks, and their impressions that they are not dealing with mathematics, but with something that is very fun.

Teachers have also had very positive impressions of the workshop, and were especially enthusiastic about the teaching methods used in the workshop. Active learning, learning through games and real world examples were very highly rated.

This cycle of Matemationica can perhaps be best described by the comment of one of the fifth graders: *"Teacher, in my life I have never had a more beautiful lesson in math!"*.

II Cycle

The second cycle of Matemationica was dedicated to making posters on selected topics. Pupils from each grade made posters with topics related to the content of the regular school mathematics lessons.

Pupils by themselves created posters from collage paper, painted them with crayons and markers ... They were split into teams, one team pro poster, and it was very cute and nice to look at them how they jointly agreed upon look of their poster, how they shared responsibilities related to the creation of posters, ... This type of work inspires team spirit in children. Also, the poster art provided a different approach to learning mathematics, learning thru visualization, because all formulas, theorems and concepts with which the students encountered in teaching process revived and get its visual form on posters.

For even greater popularization of mathematics, at the end of the school year was held an exhibition of posters created by the pupils. The exhibition was attended by media, and that made pupils, authors of posters, even more happy.

III Cycle

Each workshop of third cycle was devoted to origami. Pupils were introduced to the world of origami, but with an emphasis on creation of geometric figures such as cube, cuboid, ... This was very interesting to students because they were extremely attracted to origami as an art, and on the other hand, they tend to reveal mathematics itself in origami. Constructing an origami figure pupils master the basics of geometry, they easily detected characteristics of figures, and most important, they could see figures in three dimensions, and thus better understand the formulas related to the surface area, volume, ... Also, pupils practiced precision thru origami, saw the dependencies associated with angles, properties of line, various geometric figures, which in the teaching process is generally avoided, and pupils learn without understanding because they do not have a real sense how geometric figures looks like in reality.

Figure 2: Games, posters and Origami

IV Cycle

Since the fourth cycle of Matemationica was held during April and May, this cycle has been designed as a summary of all previous activities with intention to present to wider public what knowledge students acquired on Matemationica. Therefore was designed a mathematical fashion show with the theme "Fashionmatics". During the fourth cycle pupils were making their models, which they will present to the whole city at the fashionshow to be held at the end of the school year. All the models were in the spirit of mathematics, each pupil chose a geometric figure that his/her model will represent. Thus, we had models of kite, circles, cubes, squares, coupe, sphere, ... Each pupil had task to wear his/her model on the stage, and to present it by saying something about this geometrical figure, its size, area, perimeter, volume, ...

Children approached this cycle of Matemationica very seriously, and they were very interested for making models. Particularly pleasant surprise was their research of geometric figures and imagination with which they fitted research data into their models.

3.3. Impressions

Although originally designed for groups of fifteen pupils per workshop, Matemationica already at beginning attracted between twenty and thirty interested pupils per workshop, which was a pleasant surprise considering the popularity of mathematics among pupils.

The atmosphere at each workshop was active and positive, pupils were happy to participate in all planned activities. Teachers attended workshops gladly, justifying their interest with interesting content, ideas and presentation of new teaching methods with an emphasis on active learning.

Effects of Matemationica were reflected thru further activities of pupils who attended workshops. In collaboration with their teachers, they visited pupils of other grades in their schools and they presented some interesting features that were processed on Matemationica, which confirms pupils interest for this kind of work, especially for learning through games and real-life examples.

4. New Years Matemationica

Following interest of pupils for Matemationica, emerged the idea for special edition of workshop, which resulted in the New Years Matemationica, held in last working week of the first semester. Topic of the New Years Matemationica was QUEST FOR LOST FORMULA, a game designed in the form of mathematical associations.

New Year's Matemationica involved pupils of most primary schools in Zrenjanin. In accordance with the theme of the special edition of Matemationica, the association game was designed which consisted of different tasks, selected to equaly represent mathematical contents covered in the V, VI, VII and VIII grade. Each school chose its team of pupils to represent it, following condition that each team must have at least four members, one from each class (V, VI, VII and VIII), so that everyone is given the opportunity to demonstrate their knowledge. Also, all school participants had equal opportunities in answering.

Association contained a total of nine tasks. First, pupils opened slide with the task and they were given time to solve the task. Team which would solve the problem first, represented their solution. If the solution is correct, the team gets one point and opens a slide with a detailed solution of the task and slide of the trail. After solving all tasks, we opened all traces and pupils solved the association. Winning team was the team of the school with the highest number of points.

New Years Matemationica began with the arrival of Santa Claus, who introduced himself as a mathematical Santa Claus, and said to the students that he had lost a very important formula, without which he can not find holiday gifts he brought, and therefore prays to pupils to use their knowledge and help him find the formula and hidden presents. This was followed by solving problems and the quest for the lost formula.

New Years Matemationica gathered about 300 pupils of upper grades of the primary schools in Zrenjanin, which was, by the comments of present parents and teachers of mathematics, unexpectedly much. Pupils demonstrated great interest in mathematics, represented in this way. Atmosphere of New Years Matemationica was more than positive, students competed, cheered for their teams, and even asked their teachers to prepare them for this special edition of Matemationica after regular lessons in school.

Succes of New Years Matemationica was confirmed by the words of a parent of one pupil who participated in the competition: *"I can not believe that you gathered children in such great number, and that the reason is - MATHEMATICS."*

5. Conclusions

The effects produced by Matemationica have met the objectives set as guiding idea for creating mathematical workshop.

Popularization of mathematics as a science has been largely fulfilled through Matemationica, and confirmed by pupils interest for this extracurricular activity. Also, students are given the opportunity to gather on Matemationica, teamwork and usefull spend of free time was encouraged. New teaching methods were demonstrated, with an emphasis on active learning and teaching through examples from real life. That way mathematics was presented as a dynamic, interesting and applicable science.

Matemationica continues its work, with always fresh ideas, effective, interesting examples, and new special editions of Matemationica that are yet to come. Future directions that will lead Matemationica tend to further improvement of the mathematical knowledge of pupils and modernization of mathematics teaching techniques in schools.

References

[1] W. Blum, P. Galbraith, H. W. Henn & M. Niss,(eds.), Modelling and Applications in mathematics education, The 14th ICMI study, (The new ICMI Study Series),Vol. 10, New York, Springer, 2007.

[2] H. W. Henn, Modelling in school - chances and obstacles, The Montana Mathematics Enthusiast, Monograph 3, pp. 125-138, 2007.

[3] G. Kaiser, K. Maaß, Modelling in Lower Mathematics Secondary Classroom - Problems and Opportunities, In Blum, W., Galbraith, P., Henn, H.-W. & Niss M.(eds.), Modelling and Applications in Mathematics Education, New York, Springer Academics (former Kluwer Academics), 2006.

[4] T. Sekulić., The role of active learning and mathematical modeling in modern mathematics education, Computer algebra and dynamics systems in mathematics education-CADGME 2012, International conference, University of Novi Sad, June 2012., Novi Sad, Serbia

[5] `http://public.carnet.hr/~ahorvate/`

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 151–156

Platoground-game that connect teaching mathematics and physical education

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Abstract

Learning environments affect student achievement. Games create enjoyable environment. In this paper we will give an idea how an outside game can connect teaching of mathematics and physical education. We will describe Platoground-game that we have created in order to connect these two subjects and we will give some ideas for future research.

1. Introduction

Motivational and cognitive activity of pupils depends on the organization of the teaching process. The lack of these activities in the learning process makes pupils believe that mathematics is boring and pointless. As a consequence of this kind of teaching there is a lack of motivation, low level of subject competence, mathematical anxiety (that is a product of low self-esteem and fear of failure) [1,2]. According to [1] learning environment can affect the interest and motivation of pupils, the emergence of mathematical anxiety. Application of specific teaching methods can contribute to reducing mathematical anxiety [2]. One of consequences of the mathematical anxiety is low self-esteem, so it is necessary to help pupils to raise the level of confidence in themselves and in some cases that can be achieved by increasing physical activity [3]. Numerous studies provide different results about the impact of physical activity on academic attainment [4], but according to study [5] results indicate that there is a long-term positive impact of moderate-to-vigorous physical activity on academic attainment in adolescence, so school should be encouraged to promote physical activity. We came up with an idea that it would be good to connect mathematic and physical education because significant number of pupils spend their spare time mostly passively, their physical activity reduces over time, physical activity contributes to a better psycho-physical condition by reduction of stress, anxiety and so on [6,7].

The Van Hiele Theory

The Van Hiele's model of the development of geometric thought [8,9] has five levels (Table 1).

Level 0 (visualization)	the lowest level that starts with nonverbal thinking; figures are identified by physical appearance; it is based on recognition
Level 1 (analysis)	descriptive level, properties of figures are recognized
Level 2 (informal deduction)	there is logical order among properties (one leads to another); pupils can understand and formulate definition by using properties of figures, follow and give informal arguments
Level 3 (deduction)	pupils understand axiomatic system; proofs can be constructed by using axioms, postulates, definitions, theorems
Level 4 (rigor)	the highest level that is signed by abstraction; pupils can understand and work with different axiomatic systems, non-Euclidian geometries

Table 1: Levels of geometric thought given by Van Hiele and their descriptions

In the paper [10] Van Hiele's model of the development of geometric thought was applied in learning solid geometry and it was noticed that this level structure is not strictly linear and that at certain moments it comes to the interweaving levels.

2. Platoground

Knowing that learning environment can increase pupils' motivation, that increasing of self-esteem can reduce mathematical anxiety, that physical activities have a positive impact on self-esteem and that in the process of growing up pupils reduce their physical activities, we came up to an idea to connect mathematics and physical education. In our opinion learning mathematics through the game in new environment (outside, on playground) will have a positive impact on learning process (it is less strict, traditional and formal way of learning, so pupils are more relaxed). Since mental image is important for development of abstract thinking and it is based on experience [11], our game-lecture has to be based on gaining experience by using mathematical models.

In Serbia, pupils meet Plato solids in 3rd grade of secondary school (age approximately 17-18) and it is mostly informatively. We thought that it would be great if pupils could meet Plato solids even earlier, so we created a game called Platoground (this game was specially created for Family day, Visuality & Mathematics: Experimental education of mathematics through visual arts, sciences and playful activities that was held in Belgrade on 19th July 2014).

Platoground (see Figure 1) is educational and social game that can be played outside and inside (in large room like physical gym). Depending on the age of the players rules can be modified so everyone can play it. The number of players can be 2, 3 or 4. Each player has his/her own start point represented with circle on game's ground scheme (see Figure 1). Before the game starts each player gets five different colored Platonic solids (tetrahedron, hexahedron, octahedron, dodecahedron, icosahedron) for example first one gets Plato solids that are yellow, second one blue, ...) that have to be near start field-position. There is a marked dodecahedron (dodecahedron that has written numbers from 1 to 12 on it). Order of throwing marked dodecahedron during the game has to be established before playing it. It can be done by throwing it, so that the order of received numbers determines who will be first, second, third and fourth. There are 16 fields on the ground that player can step during the game (equilateral triangle, square, regular pentagon, circle, four of each mentioned kind). The number of steps (number of crossed fields) depends on number received by throwing marked dodecahedron. The last stepped field (after one throwing) determines the way of moving on the ground in the next round and the number of winning points.

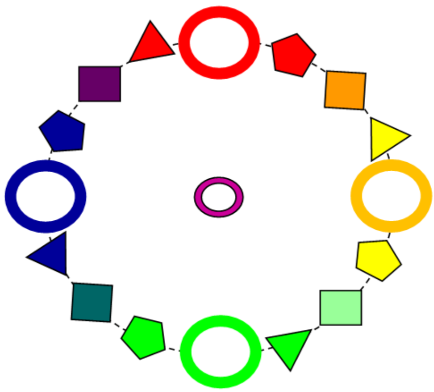

Figure 1: Scheme of Platoground

If a field is equilateral triangle then in the next round player will have to move by one-leg jumping, if it is square than he/she has to move by jumping (two legs jumping), if it is pentagon than by walking and if it is circle, player can choose the way of moving on the ground.

The number of winning points depends on solid that is connected to the field. Each field has one solid (pentagonal field has dodecahedron, square field has hexahedron) except triangle fields (each triangle field has three solids-tetrahedron, octahedron and icosahedron) and circle fields (each circle field has one player's collection of the Platonic solids). These solids are called field solids. Player wins point by throwing appropriate field solid into basket that is in the central part of the ground (marked as purple ring, see Figure 1). The number of field solid's sides is the number of winning points. In the case of the triangle and circle field (where the number of field solids is larger than one), the player has a right to choose which field solid. If the field solid has already been thrown into the basket, than there are no winning points, unless the player has adequate field solid in his/hers collection of Plato solids (that can be used as a wildcard field solid). In the case of lack of adequate wildcard field solid or unsuccessful throwing field solid into the basket, there are no winning points for the player (in that round) and field solid has to be returned to corresponding field.

Circle field is a field of knowledge. In order to choose field solid from the collection, the player has to answer a question or solve a mathematical problem from a pulled-out card (problem is given by a teacher). In case of correct answer, player gets extra points (extra icosahedron) and a right to throw chosen field solid into the basket. In case of incorrect answer, he/she has to skip to the next round.

Square field is a field of sports. To win extra points (extra decahedron) the player has to fulfill the physical task given on the card that was pulled-out (the task is given by a teacher). If the task cannot be fulfilled then the player has to skip next round. The game is over when there are no solids on the ground. The winner is the player that has the highest score (number of points).

Figure 2: Platoground, Family Day during the Summer school, University Metropolitan, Belgrade, 2014

This game requires physical and mental activity. Its level of difficulty depends on players and it has to be determined by mathematics and physical education teachers. Certainly this game helps pupils to learn mathematic through the game, in more-relaxed situation (they learn which figure can be side of Plato solid, how many Plato solids there are, how many sides each Plato solids has etc.). Depending on pupils' age, teacher can give a task to pupils to make models of Plato solids, so that they will become familiar to solids' nets. Raising the level of knowledge and level of physical activity can be done thanks to fields of knowledge and fields of sport. Requests should be harmonized with the capabilities and abilities of the pupils and should be gradually increased.

3. Conclusion

If we want to make learning mathematics more interesting, we need to create new methods. Since learning mathematics is more inside activity, it would be great if it becomes an outside activity from time to time. We need to create and apply various learning methods in order to increase pupils learning motivation. That doesn't mean that we have to leave traditional methods, it means that we occasionally have to apply a new method so that we avoid monotony of our lessons. Platoground is one idea that provides us such new methods and helps connecting two subjects-mathematics and physical education. Our future research will be directed towards assessment the effects of this learning method in primary and secondary school.

References

[1] Mathew Mitchell, Judy Gilson: Interest and Anxiety in Mathematics http://files.eric.ed.gov/fulltext/ED412116.pdf

[2] Ayatollah Karimi, S. Venkatesan: Mathematics Anxiety, Mathematics Performance and Academic Hardiness in High School Students http://www.krepublishers.com/02-Journals/IJES/IJES-01-0-000-09-Web/IJES-01-1-000-09-Abst-PDF/IJES-01-01-033-09-012-Karimi-A/IJES-01-01-033-09-012-Karimi-A-Tt.pdf

[3] Mark S Tremblay, J. Wyatt Inman, J.Douglas Wilims: The Relation between Physical Activity, Self-Esteem, and Academic Achievement in 12-Year-Old Children, Pedagogic Exercise Science, 2000, 12,312-323 http://extranet.nuorisuomi.fi/download/attachments/3245041/the+relationship+between+physical+activity,self-esteem,+and+academic+achievement+in+12-year-old+children.pdf

[4] Dawn Podulka Coe, James M. Pivarnik, Christopher J. Womack, Mathew J. Reeves, Robert M. Malina: Effect of Physical Education and Activity Levels on Academic Achievement in Children, Medicine & Science in Sport & Exercise, 2006 http://www.tahperd.org/HOME_PDFs/Activity_Achievemnt_Scores_article.pdf

[5] J. N. Booth, S. D. Leary, C. Joinson, A. R. Ness, P. D. Tomporowski, J. M. Boyle, J. J. Reilly: Associations between objectively measured physical activity and academic attainment in adolescents from a UK cohort, BJSM, 2013 http://bjsm.bmj.com/content/early/2013/10/04/bjsports-2013-092334.full.pdf+html

[6] Zoran Djokić: Procena fizičke aktivnosti učenika uzrasta 11 godina, Tims.Acta: naučni časopis za sport, turizam i velnes, 2014, Vol. 8, Br. 1, 61-69 http://www.tims.edu.rs/wp-content/uploads/2014/03/No.7_Djokic_Procena-fizicke-aktivnosti-ucenika-uzrasta-11-godina.pdf

[7] Davor Vuković: Pregled istraživanja o povezanosti fizičke aktivnosti i gojaznosti dece uzrasta od 5 do 10 godina https://fedorabg.bg.ac.rs/fedora/get/o:6616/bdef:Content/get

[8] Mary L.Crowley: The van Hiele Model of the Development of Geometric Thought http://psych.stanford.edu/ jlm/pdfs/Crowley87ThevanHeileModel.pdf

[9] Pierre M. van Hiele: Developing Geometric Thinking through Activities That Begin with Play, Teaching Children Mathematics 6, 1999, 310-16 http://print.nycenet.edu/NR/rdonlyres/0EFD73D4-340A-42E2-8EB4-3BC2A6B05603/38319/30vanHielePlay.pdf

[10] Angel Gutiérrez, Adela Jaime, José M. Fortuny: An alternative paradigm to evaluate the acquisition of the van Hiele levels, Journal for Research in Mathematics Education, 1991, Vol.22, No.3, 237-251 http://www.uv.es/gutierre/archivos1/textospdf/GutJaiFor91.pdf

[11] Milosav Marjanović: Metodika matematike 1, Učiteljski fakultet, Beograd, 1996

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 157–162

Visualization of fractional calculus

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Abstract

The visualizations of definite integral for area problems are considered. The functions defined by definite integral, the convolution and the composite of integrals are visualized and analyzed. The definitions of fractional integral and derivative are introduced and visualized by using GeoGebra Packages. The presented visualizations were used in teaching calculus for the students, mayor physics, at the Faculty of Science, University of Novi Sad. The students were tested after the presentations and they show good results.

Keywords: fractional calculus, visualization, integral

1. Introduction

Nowadays many books, surveys, journals and papers address problems involving fractional calculus, i.e., fractional derivatives and integrals with appropriate initial or boundary conditions.

The reason for this lies in the fact that the fractional calculus got wide use in numerous physical and other applications, as viscoelastic materials, fluid flow, diffusive transport, electrical networks, electromagnetic theory, probability and others.

Fractional calculus can be considered as the generalization (extension) of the well known differential and integral calculus of noninteger order.

The origins of the fractional calculus go back all the way to the end of the 17th century. L'Hospital asked in Leibniz about the sense of the notation:

$$\frac{D^n}{Dx^n} \quad \text{if} \quad n = 1/2$$

i.e., the fractional derivative of order 1/2.

Then Leibniz's answer was: *"An apparent paradox, from which one day useful consequences will be drawn."*

In the continuation of the paper we shall visually introduced fractional integral, J^a, of order $a > 0$, in the sense Riemann-Liouville and fractional derivative, D^a, of order $a > 0$, in Caputo sense (rather than the Riemann-Liouville one), because it is more suitable for application to the problems with initial and boundary conditions.

2. Visualization of definite integral

The application of definite integral for determining the corresponding area between the graph of the considered function and x axes is well known. Interactive presentation of the area is presented on Figure 1. Click on Figure 1 and you will get the interactive file. By changing the values of lower and upper limits, with the slider, the values of integral $I := \int_a^b f(x)dx$, for $f(x) = -x(x-4)$, and the values of corresponding area P are changing

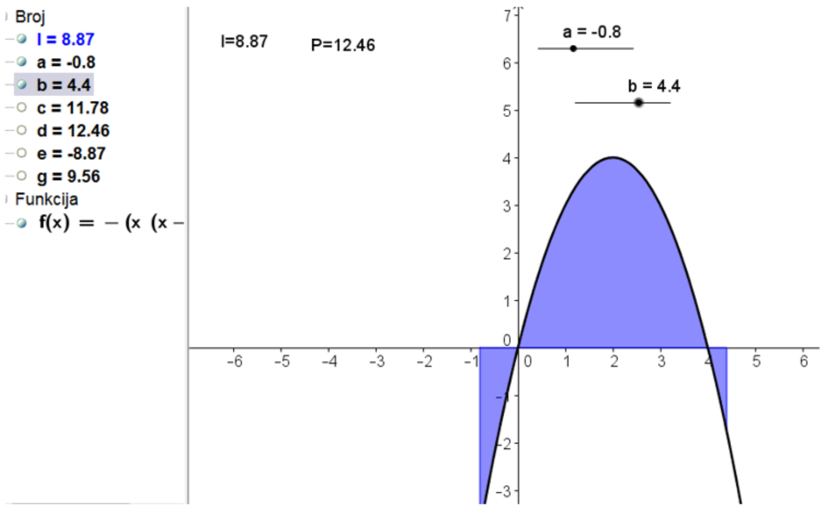

Figure 1

3. Visualization of the function given by integral

The function F, defined by definite integral:

$$F(t) = \int_a^t f(x)dx,$$

can be visualized and analyzed in the sense of previous considerations of definite integral and its applications. The properties of the function F is conditioned by the function f, because it is its primitive function. As an example, we consider the function $f(x) = \frac{1}{x}$, $x > 0$, and the function

$$F(t) = \int_1^t \frac{1}{x} dx,$$

for $t > 1$, and $t < 1$. By the click on Figure 2 the interactive applications can be obtained. The function $\ln t$ is defined for $t > 1$, and $t < 1$, and the points A (on the left hand side, for $t > 1$) and C (on the right hand side $t < 1$) belong to their graphs, but their y-coordinates are the values of the function F. Coordinates of these points can be followed on Figure 2, by using sliders t and t_1, respectively.

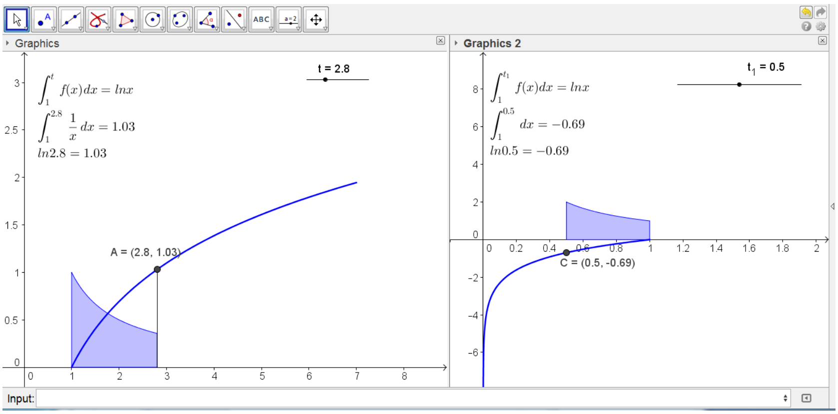

Figure 2

4. Visualization of the convolution

The composite of integrals $F(F(t))$ of the function $f(x) = x$ is evaluated as:

$$F(F(t)) = \int_0^t \left(\int_0^\tau x dx \right) d\tau = \int_0^t \frac{\tau^2}{2} d\tau = \frac{t^3}{3!}.$$

The same result can be obtained by using the formula called convolution of two same functions $f(x) = x$:

$$\int_0^t (t - \tau)\tau d\tau = t \int_0^t \tau d\tau - \int_0^t \tau^2 d\tau = \frac{t^3}{2} - \frac{t^3}{3} = \frac{t^3}{3!}.$$

In general the convolution of two functions f and g is given by:

$$(f * g)(t) = \int_0^t f(t-\tau)g(\tau)d\tau.$$

On Figure 3 the convolution of the functions $f(x) = x$ and $g(x) = e^x$ is visualized. Let us remark that the convolution is the function given by the corresponding integral. Therefore the visualization is presented analogously as in previous cases. The point A belongs to graph of the function representing convolution and its y-coordinate is the same as the area under the graph of the function $p(x) = e^{t-x}x$. By changing of the value t, the values of corresponding area and convolutions are changed. By the click on Figure 3 the interactive applications can be obtained.

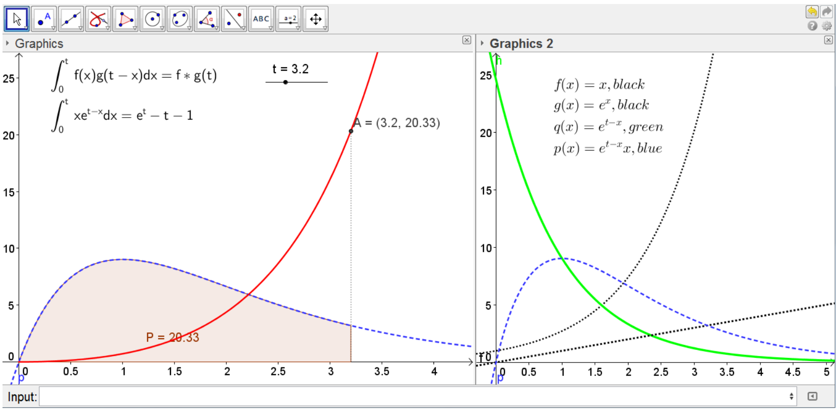

Figure 3

5. Visualization of the fractional integral

The fractional integral, J^a, of order $a > 0$, in the sense Riemann-Liouville, is defined as the following convolution:

$$J^a f(t) = \frac{1}{\Gamma(a)} \int_0^t (t-\tau)^{a-1} f(\tau)d\tau.$$

We put

$$J^0 f(t) = f(t).$$

The main properties of fractional integral operators J^a, of order $a > 0$, are:

$$J^a J^b f(t) = J^{a+b} f(t) \ (a, b > 0), \ \ J^a J^b f(t) = J^b J^a f(t), \ \ J^a t^c = \frac{\Gamma(c+1)}{\Gamma(a+c+1)} t^{a+c}.$$

On Figure 4, the fractional integral of the function $f(x) = x$, is visualized. By changing a with the slider, from 0 to 2, the graph of fractional integral is changing. For $a = 1$, the fractional integral is primitive function for the function $f(x) = x$, i.e., $h(x) = \frac{x^2}{2}$, and for $a = 2$, it coincide with the function $g(x) = \frac{x^3}{6}$. By the click on Figure 4, the interactive applications can be obtained.

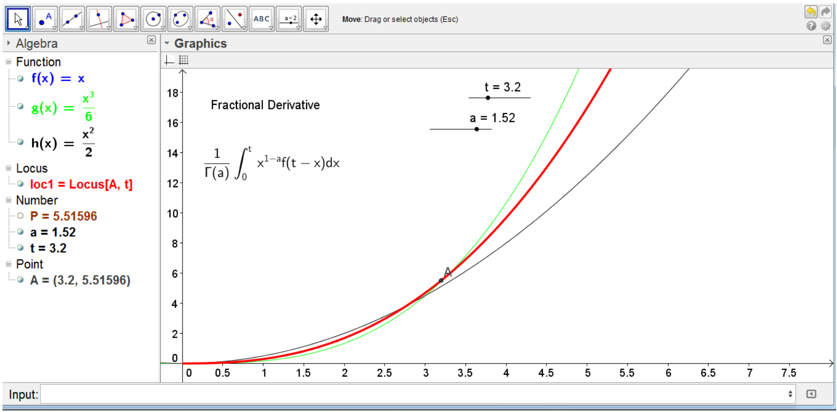

Figure 4

6. Visualization of fractional derivative

In this paper we shall consider the fractional derivatives in Caputo sense (rather than the Riemann-Liouville one), because it is more suitable for application to the problems with initial and boundary conditions. The Caputo fractional derivative of order $\alpha > 0$ is defined by:

$$D^a f(t) = \begin{cases} \frac{1}{\Gamma(m-a)} \int_0^t (t-\tau)^{m-a-1} f^{(m)}(\tau) d\tau, & m-1 < a < m, \\ \frac{d^m f(t)}{dt^m}, & a = m, \end{cases} \quad (m \in N,\ t > 0).$$

It is important to note that the following relation between J^a and D^a from holds for $m - 1 < a < m$,

$$D^a J^a f(t) = f(t);$$

$$J^a D^a f(t) = f(t) - \sum_{k=0}^{m-1} f^{(k)}(0^+) \frac{t^k}{k!}.$$

On Figure 5, the fractional derivative of the function $f(x) = x$, is visualized. By changing a, by using slider, from 0 to 1, the graph of fractional derivative is changing from the graph of function $f(x) = x$, for $a = 0$, to graph of constant for $a = 1$. By the click on Figure 5, the interactive applications can be obtained.

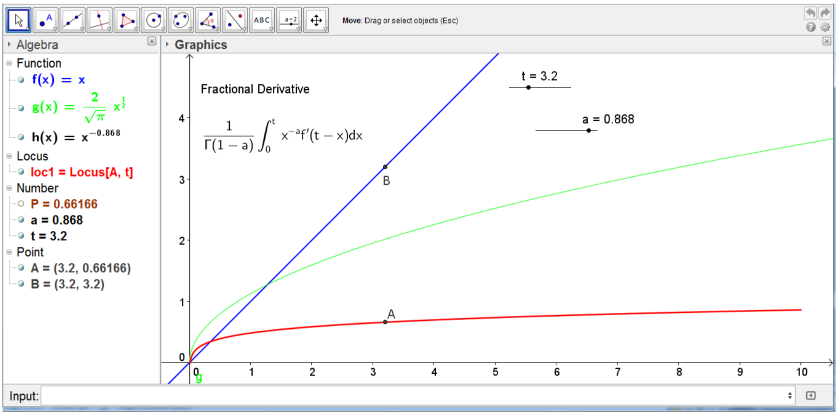

Figure 5

References

[1] Caputo, M., Linear models of dissipation whose Q is almost frequency independent-II, Geophys. J. Royal Astronom. Soc., 13, No 5 (1967), 529-539, (2008).

[2] Kiryakova, V., Some operational tools for solving fractional and higher integer order differential equations: A survey on their mutual relations, American Institute of Physics - Conf. Proc. 1497 Proc. 38th Intern. Conf. AMEE' 2012), 273-289; doi: 10.1063/1.4766795, (2012).

[3] Ross, B., A brief history and exposition of fundamental theory of fractional calculus, Fractional Calculus and Its Applications (Proc. 1st Internat. Conf. held in New Haven, 1974; Ed. B. Ross), Lecture Notes in Math. 457, Springer-Verlag, N. York, pp. 1-37, (1975).

[4] Takači, Dj., Takači, A., Štrboja, M., On the character of operational solutions of the time-fractional diffusion equation, Nonlinear Analysis: 72, 5, 2367-2374, (2010).

[5] Takači, Dj., Takači, A., Takači, Aleksandar, On the operational solutions of Fuzzy fractional differential equations, FCAA, Vol17, No4, (2014).

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 163–167

Mathematical modeling of Illusions

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Abstract

The mathematical modeling processes of different illusions are analyzed in a view of new technology. Starting from illusions as real problems, all stages and transitions from one stage to another are considered. The cognitive activities following the whole process of mathematical modeling of illusions are analyzed. The packages Mathematica and GeoGebra are used for the geometry constructions of illusions.

Keywords: mathematical modeling, ilusions, *Geogebra*

1. Introduction

The process of mathematical modeling and mathematical models together are often represented in mathematics, sciences, educations and other real life situations. Nowadays, the new technologies involve lot of changing in all segments of human activities, especially in the process of mathematical modeling. Interactive presentations are the subject of numerous studies confirming their great challenge and positive impacts on education. Mathematical modeling connects mathematics with the real world problems and vice versa. In particular, it is very important to analyze all cognitive activities following mathematical modeling process in teaching.

Model can be considered as an image of reality, simplified and idealized. Its purpose is to create a better understanding of the real system and process that occurs in it. The process which leads to the model is referred to modeling. Modeling process is considered as a method of solving problems. During the modeling process, it is necessary to identify and distinguish those elements and characteristics of the system that are relevant for a given survey, while other elements and characteristics will be neglected.

Models can be physical (prototype aircraft or cars), symbolic (chemical formulas, maps, schemes), mathematical and others.

The process of mathematical modeling in teaching is widely represented in the literature related to the teaching of mathematics. According to the results of G. Stillman (see [3], [4]) the whole mathematical modeling process can be visualized

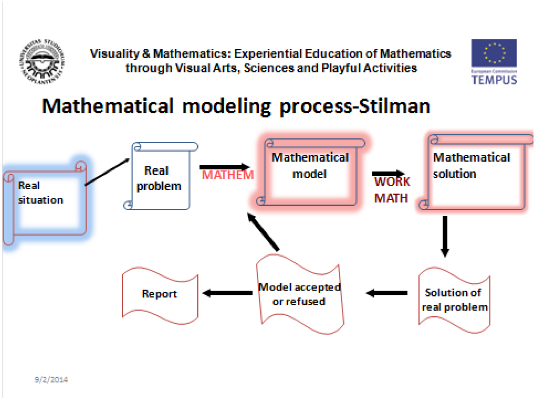

Figure 1

by the schema on Figure 1. The transitions from real problem to mathematical model and from mathematical model to mathematical solution are mainly applied in teaching mathematics. Therefore it is important for students to follow the whole process, in order to deepen their knowledge.

The mathematical modeling process, starting from Ebbinghaus, Hering, horse-frog and rabbit-duck illusions, is presented in this paper.

2. Ebbinghaus illusion

The real problem is the optical illusion of relative size perception, Ebbinghaus illusion (`http://en.wikipedia.org/wiki/Ebbinghaus_illusion`). It can be seen that the two orange circles are exactly the same size, however the one on the right appears larger. The task is to determine the mathematical model of this illusion, by using *Mathematica*.

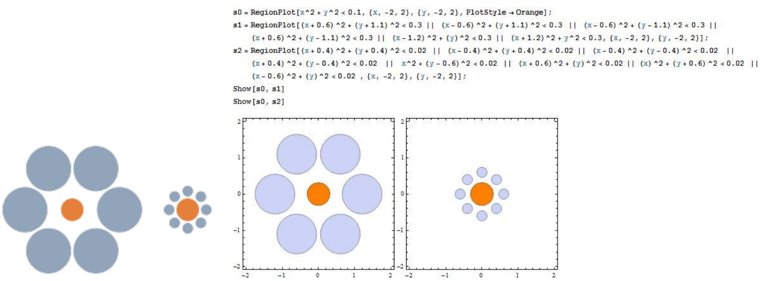

Figure 2

The student should construct the mathematical models of objects in Figure 2 by using package *Mathematica*. In order to get these geometric objects, the student has to do some mathematical considerations (the number of circles, their centers, radiuses and others) and of course to handle that with *Mathematica*. It can be easily done by using function *RegionPlot*.

3. The Hering illusion

The real problem is picture of the geometrical-optical illusion discovered by the German physiologist Ewald Hering in 1861. It can be seen that two straight and parallel lines look as they were bowed outwards. The distortion is produced by the radiating pattern and was ascribed by Hering to an overestimation of the angle made at the points of intersection. It is interesting that what yields is the straightness of the parallel lines and not of the radiating lines, implying that there is a hierarchical ordering among components of such illusion.

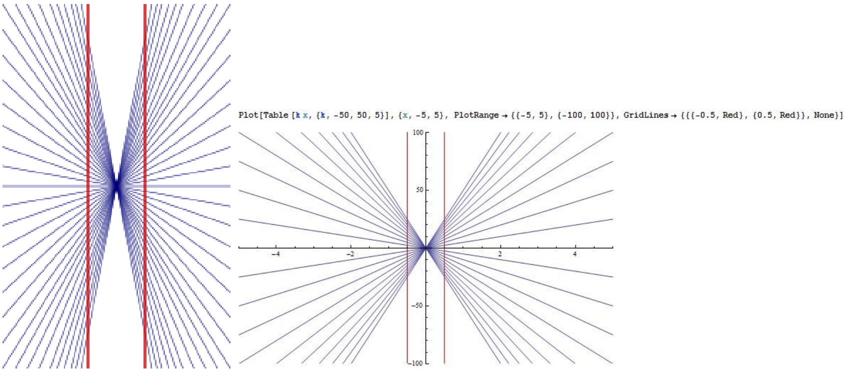

Figure 3

The student should construct the mathematical model of the object on Figure 3 (right) in package *Mathematica*. The easiest way to visualize this illusion is to consider the family of straight lines considering them as the graphs of the functions $y = kx$ for $k \in (a, b)$ $a < 0,\ b > 0$. It can be done in a simple way by using functions *Plot* and *Table*.

4. Frog and horse illusion

The next illusion is visualized with *GeoGebra* package. The student inserts a picture and applies the rotation. The main discussion is about the angle of rotation. The common way is to apply the rotation with the angle of 90 degrees, but using sliders for changing the angle of rotation is more interesting way, which is given in the following application:

By the click on Figure 4, the interactive *GeoGebra* file is obtained. Applying the full circle rotation of the figure, it can be seen that the frog becomes horse

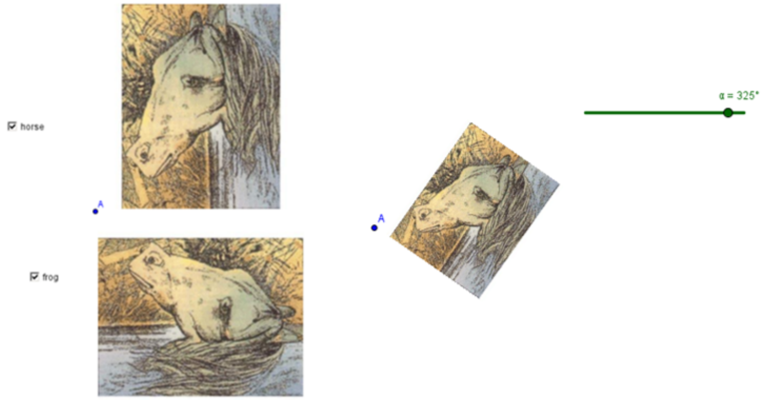

Figure 4

vice versa. Let us remark that we have two pictures in Figure 3 and one picture in Figure 4. This picture is obtained by rotation, but the original one was made invisible.

5. Duck and Rabbit illusion

The illusion duck and rabbits is constructed in package *GeoGebra*, by using rotation. The interactive *GeoGebra* file is obtained by the click on Figure 5. The figure rotates with the whole circle and the ducks become rabbits and vice versa, which looks very nice.

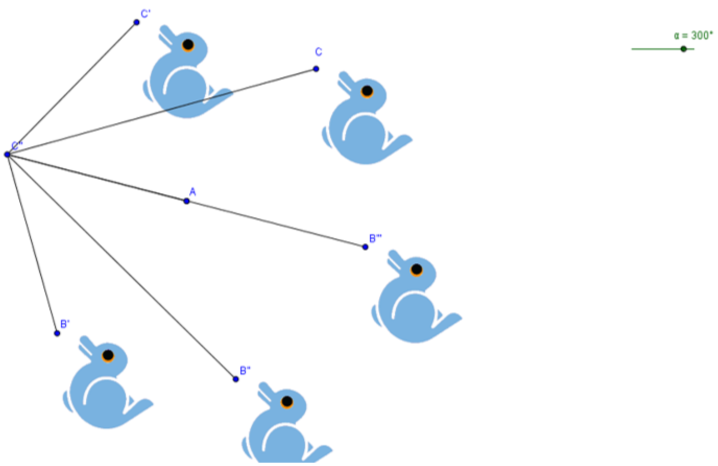

Figure 5

References

[1] Boaler, J., Mathematical modelling and new theories of learning, teaching mathematics and its applications, vol. 20, issue 3, 121-128, (2001).

[2] Doerr, h., English, L., A modelling perspective on students' mathematical reasoning about data, journal for research in mathematics education, 34(2), 110-136, (2003).

[3] Galbraith, P., Stillman, G., Brown, J., Edwards, I., Facilitating middle secondary modelling competencies. In C. Haines, P. Galbraith, W. Blum, S. Khan (eds.), Mathematical modelling (ICTMA12): edu., Eng. Economics, chichester, UK: horwood, 130-140, (2007).

[4] Korenova, L., The use of a digital environment for developing the creativity of mathematically gifted high school students, The 12th international congress on mathematical education, proceed.creativity in mathematics education, Coex, Seoul, Korea, (2012),.

[5] Stillman, G., Galbraith, P., Towards constructing a measure of the complexity of applications tasks. S.J. Lamon, W. A. Parker, & S. K. Houston (Eds.), Mathematical modelling: A way of life (pp. 317-327). Chichester, UK: Horwood (2003).

[6] Takači, A., Mathematical and simulation models of traffic flow, Proc. Appl. Math. Mech., GAMM 5, 633-634 (2005).

[7] Takači, A., Skripta iz Matematičkog modeliranja, PMF Novi Sad i WUS, (2006).

[8] Takači, A., (ed.) Develoment of Computer-aided Methods in teachin Mathematics and Science, Project 06SER02/02/003, (Takači Arpad), Proc.of School of Intensive courses in Novi Sad, April 4-8.(2008.)

Educational tools of teachers

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 171–177

Sierpinski triangle and pyramid

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Abstract

Sierpinski pyramid is one of the most beautiful structures that allow us to hide mathematics in game and art creation by letting students enjoy in exploring fractals, its patterns and properties. That way they become more interested in rich mathematical explorations and connections. Although the primary school mathematics curriculum is not anticipated with fractals as teaching lesson, it can be very usable in many different domain of elementary school mathematical teaching, such as fractions, area, similarity, and so on. In eight grade, it plays a big role in developing students knowledge of tetrahedron and pyramid properties.

1. Introduction

Some of the main purposes of mathematics teaching are the formation of positive attitudes towards mathematics, development of critical thinking and spirit of inquiry, mastering the cognitive processes necessary for an understanding of mathematical concepts, acquisition of mathematical knowledge and skills necessary for modeling and solving real problems and development of cooperative learning and mathematical communication skills. Teachers should enable students to learn how to use mathematical skills in daily life and ensure that children will not be discouraged by mathematics. But in reality it's not that easy to achieve established aims. Overall achievement in mathematics is considered quite low and students have difficulties in comprehending mathematical concepts, and with the time, they become turned off from mathematics. Therefore, in order to entice students to embrace math and inspire them to learn and discover its beauty, teachers should use visualization, creativity and modeling as suitable tools for students' learning process.

Teachers of mathematics in elementary school, considering the years of their students, need to build a connection between mathematics and artistic projects, and to provide children with a chance to participate in such activities so they can accomplish much more in mathematics. Opportunity to be a participant of both International Summer Schools for Visual Mathematics and Education (Summer University and Experience Workshop) organized in the frame of the TEMPUS IV Project "Visuality & Mathematics: Experiential Education of Mathematics through Visual Arts, Sciences and Playful Activities" (2013. and 2014.), become a springboard of my searching for possibilities to integrate art, science or play-centered learning into mathematics teaching programs, to investigate its pedagogical results, in order to develop or improve my students knowledge and make it more meaningful and accessible.

In 8th grade of primary education students are learning about pyramids, investigating it both geometrically and algebraically. Among computational problems facing later, at the very beginning of studying pyramids, many students happened to struggle with drawing and constructing representations of three-dimensional geometric objects in two-dimensional plane, and also with its net representation. Although in most cases teachers are using a variety of tools and showing models to class, still it seems to be not enough to build a strong and appropriate understanding.

2. Sierpinski gasket in Classroom

The conversation with students about Swedish packaging company “Tetra-Pak”, the root of its name and packaging invented in 1952., the one made of tetrahedron-shaped cartons, raised their interest in dealing with geometry and mathematics. Seeing that students are very interested in something that they could talk about with their parents, also seen in an exhibition dedicated to the time of their parents youth, I realized that the best way to motivate them is to find a model they could work on.

Sierpinski pyramid and its tetrahedron structure was a good way to integrate art and play into studying mathematics. It is found that fractals generally shows complex geometrical character and shape, set up on simple mathematical foundation.

Fractals are not part of curriculum at the elementary level of math teaching. They are well known for their nice computer graphics pictures, but fractals are best known as spirals and examples from nature like fern leaf, broccoli, river networks, snowflakes, etc. Therefore, fractals can be easily connected with other school subjects. All this makes fractals very interesting for investigation. I organized my students in groups and gave them a task to research topic connected with fractals. First group of students had to discover what fractions are, where we can find them in a world around as and to show some interesting examples. The next two groups had to investigate Sierpinski triangle and Sierpinski pyramid. Fourth group had to find out about Waclaw Sierpinski, a Polish mathematician who created Sierpinski triangle, also known as Sierpinski gasket, in 1915. All groups presented their researches to the other students.

What the fractals are? There is not exact mathematical definition, but we could

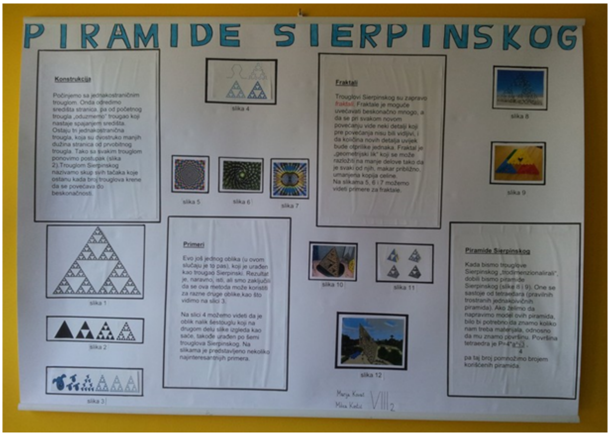

Figure 1: Project preparation

say that it is a geometric shape in which an identical pattern repeats itself. It means that mathematical fractals are iterative; they are formed by applying procedure over and over again. Its other geometric characterization is self-similarity: the shape is made of smaller copies of itself. Model of Sierpinski triangle is prototypical representative of the fractals in two dimensions. It can be constructed from an equilateral triangle in two different techniques, in both cases extremely simple to make. The first one is to repeatedly subtract smaller and smaller triangles. That way the original triangle is divided into four equal triangles and the middle one is removed. Each of the remaining is divided the same way, and this process is carried on indefinitely. The other method also starts with an equilateral triangle but repeatedly adds new triangles to the construction. In each step three identical constructions are grouped together into a larger one. The second method is maybe more accurate to connect with making sculptures, but the first one is showing fractals beauty from both mathematical and visible point of view.

My students have discovered it by drawing iterations on white board, all the way till continuous patterns could be noticeable. At the end they had a task to count, noticing that a number of triangles used as patterns increase by a factor of 3 each step. That way they relate geometric ideas to numbers. (See Figure 2.)

Sierpinski triangle is very intricate, and yet so simple to understand. Knowing how to create repeating and growing patterns, understand relations, and functions doesn't necessary mean that students won't make mistakes. Noticing and correcting them is also important part of learning process. (See Figure 3.)

2.1. Objectives

Depending on tasks we put in front of our students we can expect them to be able to visualize, build, and draw geometric objects, apply appropriate tools and formulas to determine measurements, develop precision in measuring lines and in

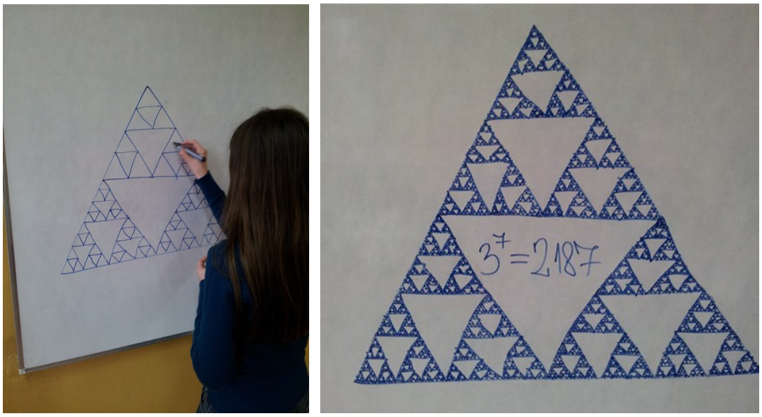

Figure 2: Exploration of Sierpinski triangle on the board

Figure 3: Sierpinski triangle project

finding or constructing midpoints, build, understand and use formulas for areas (any shaded triangle at the n-th stage) and for that need construct, analyze and interpret tables, perform or describe line symmetry, develop mathematical arguments about all geometric relationships, solve problems involving fractions, ratios and proportions, discover fraction that represent part of triangle and calculate the fraction of area which is (not) cut out, for any step, etc.

3. Sierpinski pyramid

As part of lecture preparation, a teacher gives each group or pair of student material for work and explains what their assignments are. By using models students cut tetrahedron nets out of paper. Then, they have to fold them so they could make pyramids - tetrahedrons. Students have an assignment (already knowing what we are going to make) to count how many tetrahedrons they need to make a structure, for each level, so that become 4 iteration Sierpinski pyramid.

They counted that it would be needed 256 basic tetrahedrons. After putting 3 of them at a bottom and fourth above, so that its base vertices touch others 3 tops, students have made 64 new structures. Using founded pattern and gluing smaller pieces together, students got 16, later 4 and at the end one beautiful structure - work of art and also 3D math model. All the time, teacher is supposed to lead and monitor the progress of building.

Figure 4: Sierpinski pyramid project

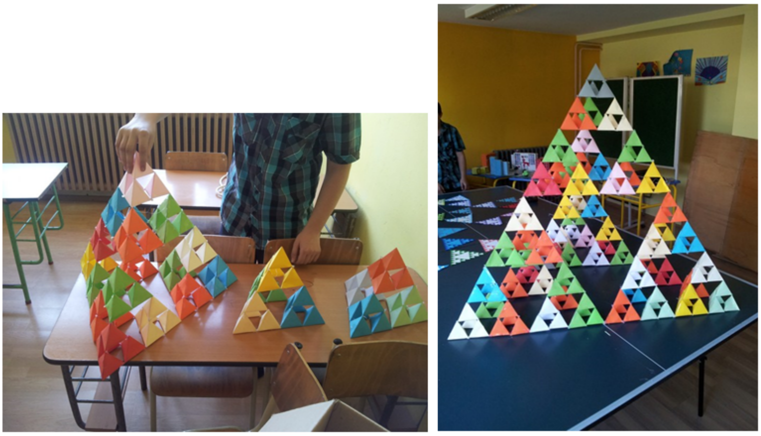

Figure 5: Sierpinski pyramid project

3.1. Objectives

In order to explore the main fractal concepts students could do the investigation on their own. By building this structure students become an active participants in their learning, motivated for discovering properties of tetrahedron and pyramid in general. One of the main goals is teaching students to use mathematical models to represent and understand quantitative relationships. It helps them to identify and describe the attributes of pyramid, and specially tetrahedron, based on various attributes (e.g. faces, edges, and corners), to identify and analyze pyramid nets, find it and calculate by formula for height, area and volume of pyramids at any given step - different iterations. Also they can count the area of paper used to make structure, and area of inside "holes".

When students start playing with pyramid they will be able to discover its different shadows - plane figures (triangle, square, rectangle) and recall some of the math concepts they learned before, such as writing plane formulas and counting by it. Using model makes easier the process of discovering pyramid symmetries, scale factor, similarities, fractions, ratios and proportion. Also, they will be able to use it for coding, to represent and understand quantitative relationships, to calculate number of tetrahedrons, used in each iteration of the pyramid (they could figure out the algorithm 4^{a}, for a-th step).

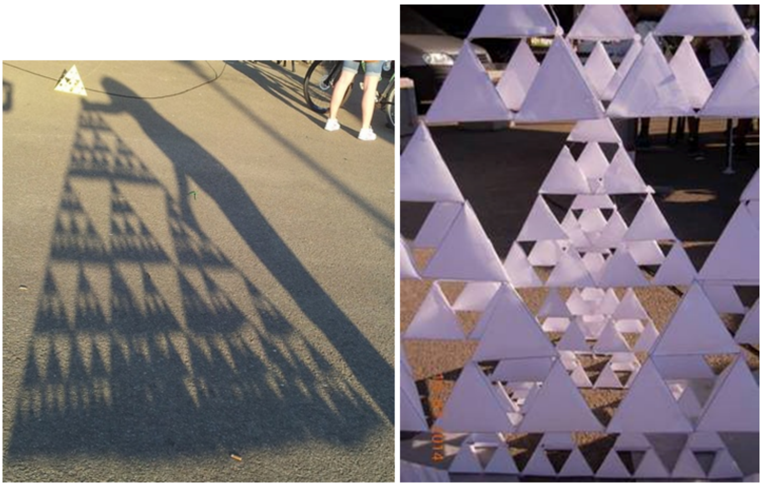

Figure 6: Sierpinski pyramid on Family day in Belgrade during Summer School 2014

In many cases students who have trouble with math don't want to try to learn it, mostly because they just don't understand it or have no motivation to even try. A lot of students take an attitude that math is too hard, they don't need it in life, or they don't even care. But in reality, students actually don't know how to learn something. If not being able to visualize some representation or rule, they loose interest in dealing with it. This challenge gives students an opportunity to beat those problems. Even more, they can enjoy in beauty of their mathematical, playful and artistic deed, that is, as they say, „even more beautiful in real life then on a photo or video presentation”.

Figure 7: Sierpinski pyramid-result of the project

References

[1] `http://fractalfoundation.org/`

[2] `http://math.rice.edu/~lanius/frac`

[3] `http://paulbourke.net/fractals/symmetry/MathEd_PDF_PresentIt.pdf`

[4] `https://www.chatham.edu/pti/curriculum/units/2006/Sill.pdf`

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Belgrade, Serbia, September 17–19, 2014. pp. 179–188

Axial reflection and plane mirror reflection in analytic geometry

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Abstract

The idea for this work came from experience acquired through International Summer Schools on Visual Mathematics (in Eger 2013. and in Belgrade 2014.) in the frame of the TEMPUS IV. Our main idea was to join thematics in three school subjects: mathematics, physics and art, and to find corresponding mathematical models. In the field of isometric transformations and analytic geometry, math is connected with physics and art through plane mirrors, mirror curves and lot of other interesting phenomena and examples in everyday situations. We presented this work on a school lesson through practical work of our students.

1. Methodological basis

We compared the curricula of three school subjects (mathematics, physics and art) as well as corresponding standards for ours high schools and we have found many possibilities to work interdisciplinary. We have created the lesson “Axial reflection and plane mirror reflection in analytic geometry”. Through this lesson students can develop mathematical and science competencies (PISA) and interdisciplinary approach using examples from art and everyday life. Math competencies are mathematical thinking skill, mathematical argumentation skill (connections and integration for problem solving), modelling skill (translating “reality” into math model), representation skill, symbolic, formal and technical skill, interdisciplinarity (translating from natural language to symbolic/formal language), communication skill (this includes expressing oneself, in a variety of ways, on matters with a mathematical content, in oral as well as in written form, and understanding others’ written or oral statements about such matters), aids and tools skill (knowing about, being able to make use of aids and tools that may assist mathematical activity).

Science competencies are explaining phenomena scientifically, evaluating and designing scientific enquiry and interpreting data and evidence scientifically.

In this lesson, we achieved all competencies. We planed group work to. Every group got an activity to solve. At the end, every group presented their solution.

2. Activities for students

Plane mirror in the Cartesian coordinate system

Working instructions: When a ray of light is reflected on a flat mirror, the reflection obey the following laws of reflection. The incident ray, the normal line and the reflected ray belong to the same plan. The angle between the incident ray and the normal, called the angle of incidence, equals the angle formed by the reflected ray and the normal.

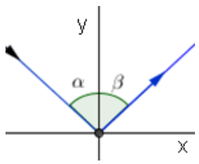

Figure 1: The law of reflection

Group 1:

Task 1 Plane mirror is on the position $y = b$. How to set a laser beam so that, after mirror reflection, hits a given point A? Calculate the angle φ. $(b > 0, 0 < m < b, n > 0)$

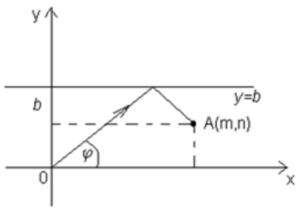

Figure 2: Task 1

Answer: The angle of reflection is equal to the angle of incidence. Based on the properties of axial symmetry, one can obtain that $\operatorname{tg}\varphi = (2b - n)/m$.

Task 2 The laser beam parallel to the y-axis hits the mirror at point A. Calculate the angle x at which is needed to tilt the mirror so that the laser beam hits the origin after reflection. Find the equation of straight line that represents a mirror in that position.

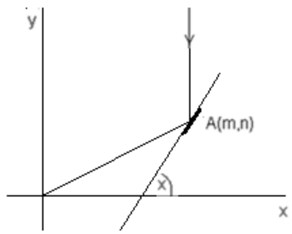

Figure 3: Task 2

Answer: Triangle ΔAOV is isosceles ($OA = OB$). Based on the Pythagorean theorem:

$$OB = OA = \sqrt{m^2 + n^2} \quad \text{and} \quad AC = \sqrt{m^2 + n^2} + n,$$

so

$$\operatorname{tg} x = \frac{\sqrt{m^2 + n^2} + n}{m}.$$

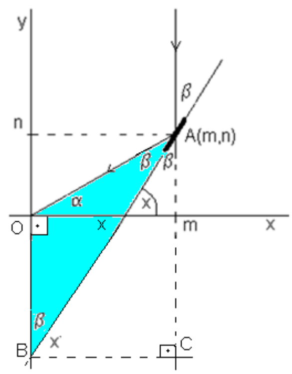

Figure 4: Illustration for task 2 solution

It is necessary that the students use a mathematical model for the real life situation: laser beam and flat mirror are represented as a straight lines in coordinate system. Through solving this task student will develop modeling skills, interdisciplinarity, mathematical thinking skills and scientific enquiry.

Group 2:

Mirror curves in the Cartesian coordinate system in a plane (1)

Working instructions: A mirror curve is a closed polygonal line reflected at the sides of a rectangle and possibly at one or more double-sided mirrors placed horizontally or vertically, midway, between neighbouring grid points belonging to a rectangular grid. The polygonal line makes angles of 45° with the sides of the rectangle and internal mirror. It can be notice that an incident ray and a reflected ray belong to the same plane. Square grid with mirrors is given in a coordinate plane.

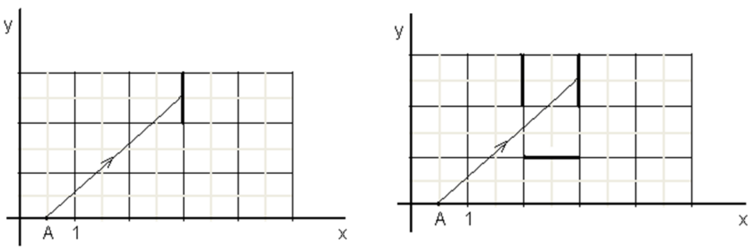

Figure 5: Square grid

Task 3 Continue drawing as it is started. Be aware that the angle of reflection is equal to the angle of incidence. In this case it is 45°. Wt is the result? Answer: Monolinear curve.

Task 4 Calculate the sum of all gradients of straight lines whose parts form a mirror curve. Answer: All the gradients are 1 or -1. Becouse of monolinearity sum is 0.

Group 3:

Mirror curves in the Cartesian coordinate system in a plane (2)

Working instructions: A mirror curve is a closed polygonal line reflected at the sides of a rectangle and possibly at one or more double-sided mirrors placed horizontally or vertically, midway, between neighbouring grid points belonging to a rectangular grid. The polygonal line makes angles of 45° with the sides of the rectangle and internal mirror. It can be notice that an incident ray and a reflected ray belong to the same plane. Square grid with mirrors is given in a coordinate plane.

Group 4:

Task 5 Continue drawing as started. Be aware that the angle of reflection is equal to the angle of incidence. In this case it is 45°. Answer: Monolinear curve.

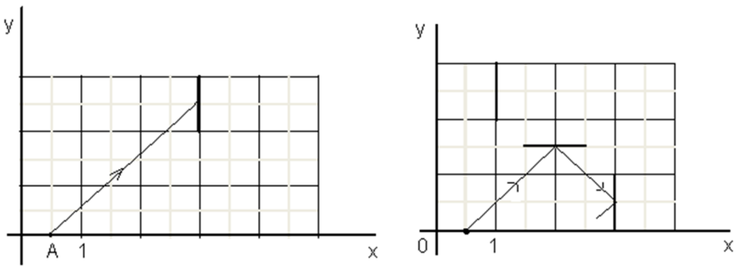

Figure 6: Square grid

Task 6 Calculate the length of the resulting mirror curve. Answer: Formula for distance betwin two points $A(x_A, y_A)$ and $B(x_B, y_B)$ is

$$d = \sqrt{(x_B - x_A)^2 - (y_B - y_A)^2}.$$

Students have opportunity to work on mathematical problem using interdisciplinary approach. Knowledge from the sphere of arts can enrich the mathematics and vice versa.

Group 5:

Number of images

Working instructions: Look at the picture below. Two plane mirrors are set at acute angle. To see the dependence between measure of the angle and the number of images you can construct some cases using a compass and ruler. Recall the characteristics of axial symmetry.

Figure 7: The materials

Task 6 Connect two mirrors using adhesive tape. Does the number of images depends on the angle between the mirrors? Explain.

Answer: Number of images: $n = [360°/\alpha] - 1$

ANGLE (α)	Number of mirrors observed	Number of images observed
10^0		
20^0		
60^0		
90^0		
120^0		
150^0		
180^0		

Figure 8: Table for obtained results

Task 7 Two mirrors (a and b) are placed at angle of 45°. Point A is located between two mirrors. How many images have point A? (y-axis is the image of x-axis in respect to the mirror a, point D is image of point C in respect to the mirror a, and so on.) Explain the connection between mirror reflection and axial symmetry? Calculate the coordinates of all the images.

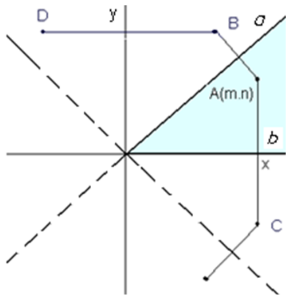

Figure 9: Illustration for task 7

Answer: Look at the figure 10.

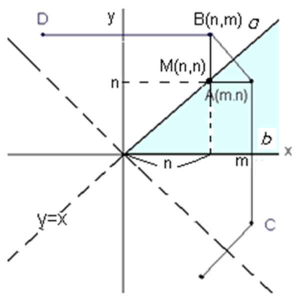

Figure 10: Illustration for answer 7

Similar situations in every day life can be found in the amusing parks, movies, fascinating toys like kaleidoscope and instruments like periscope to the dressing rooms. Working on this problem students will develop their tools skills (knowing about, being able to make use of aids and tools that may assist mathematical activity) and science competencies (explaining phenomena scientifically, evaluating and designing scientific enquiry and interpreting data and evidence scientifically).

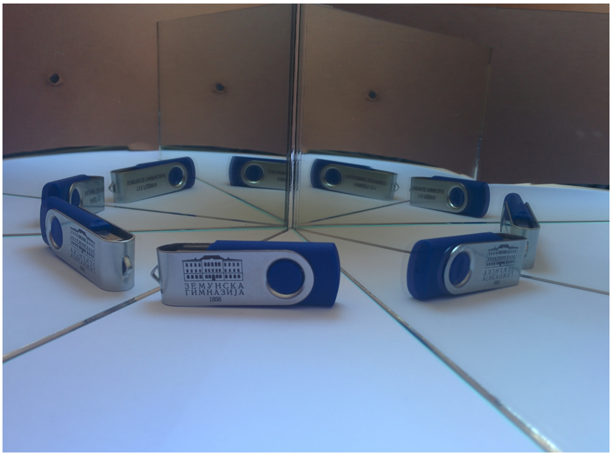

Figure 11: Illustration for answer 6

Group 6:

How to set the plane mirror

Working instructions: What is the middle line of the triangle?

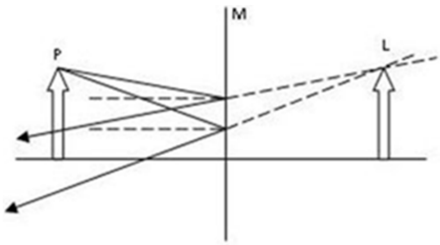

Figure 12: Creating image in a plane mirror

Task 8 The flat mirror should be put on the wall. What must be the length of the mirror and how high it should be set, so that man, 192 cm high, can see himself in full?

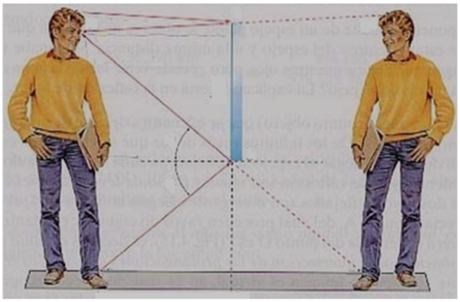

Figure 13: Illustration for task 8

Answer: Using the properties of treangles ΔDD_1B_1 and ΔDA_1D_1 and their midle lines lenght of mirror must be at least 192/2 cm . Point M should be aligned with the line of the eyes (FM).

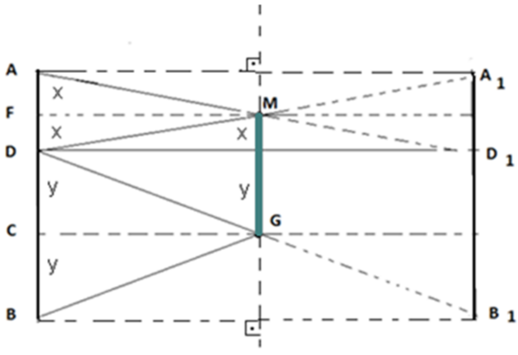

Figure 14: Illustration for answer 8

Group 7:

Task 9 If F1 is the reflected image of the object F from the plane mirror, where the mirror should be placed? Draw and determine the coordinates of the endpoints of the shortest such mirror.

This is an example that can be solved in many different ways. Selecting the method of analytic geometry task is appropriate for our students and their curriculum. The same problem can be used in work with the students of different ages, from elementary school like playful activity and in different situations in everyday life.

Homework: Homework was the same for all the students. They got instructions how to make a kaleidoscope and periscope. It was suggested to make some photos

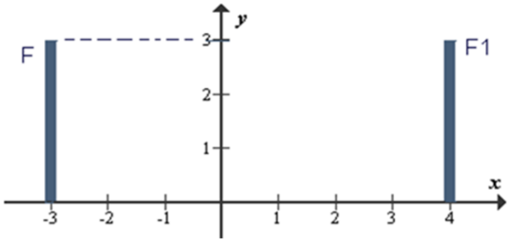

Figure 15: Illustration for Task 9

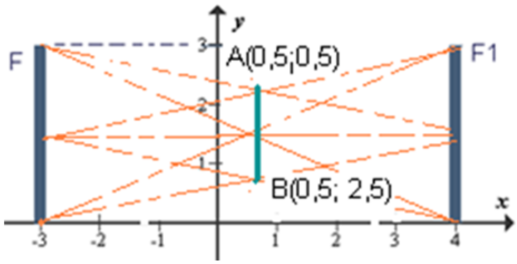

Figure 16: Illustration for answer 9

of their own products and experiments that could be used for school exhibition. On the Figure 16 is photo made by Stefan Stojicic, student at Zemunska gimnazija high school.

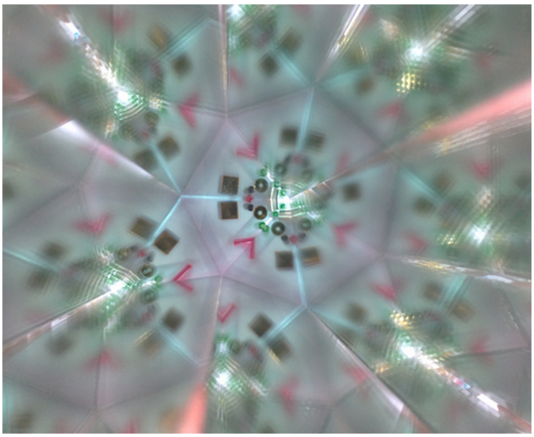

Figure 17: Kaleidoscope made image

3. Summary

Reasoning and sense making is the core of all mathematical learning and understanding. Reasoning is the process of conclusions based on evidence or stated assumptions-extending the knowledge that one has at a given moment. Sense making develops understanding of a situation, concept, or context by connecting it with existing knowledge. A high school mathematics and science learning based on reasoning and sense making provides more qualitative knowledge to the students.

We think that this could be achieved by coordinated activities which connect different school subjects, incorporating examples from everyday life and focusing activities on outcomes, as it is presented in our lesson plan.

References

[1] OPTIKS Light, Color, and Their Uses; An Educator /s Guide With Activitis In Science and Matehematics; NASA http:spacelink.nasa.gov

[2] Stanislaw Schukajlow and André Krug, Do Multiple Solutions Matter? Prompting Multiple Solutions, Interest, Competence, and Autonomy. Journal for Research in Mathematics Education, 2014, 45, 497–533.

[3] Timo Tossavainen, Three theories and a teachnig model - Tools for better teaching, Summer University and Experience Workshop, 13 July 2013 - 25 July 2013 Hungary, Eger, EKF

[4] Dragica Pavlovic Babic, Aleksandar Baucal, Matematicka pismenost, PISA 2003 i PISA 2014, Beograd2009, Republika Srbija, Ministarstvo prosvete, Zavod za vrednovanje kvaliteta obrazovanja i vaspitanja, Institut za psihologiju Filozofskog fakulteta Univerziteta u Beogradu

[5] PISA 2009 ASSESSMENT FRAMEWORK – Key competencies in reading, mathematics and science © OECD 2009 `http://www.oecd.org/pisa/pisaproducts/44455820.pdf`

[6] ZUOV Pravilnik o nastavnom planu i programu za gimnaziju, Sluzbtni glasnik RS-Prosvetni glasnik 2014

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 189–193

Characteristics of Quadrilaterals and Treasure Hunt

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Abstract

This text is about another way of visualization in mathematical teaching. This teaching method connects mathematics and real life and making math more interesting. For better memorizing mathematical facts, using imagination, teacher could create a math story. For example, I will show two stories that I have created for my pupils. In the conclusion will be discussed about effects of this method.

1. The story about telling stories

There are many ways for using visualization in math teaching. In my work I am using models, pupils workshops, didactic tools that allow visualization. I motivate my pupils for doing mathematics through the unusual contents and exhibitions. This paper is about another way of visualizing mathematical contents in elementary school, and that is: putting mathematical facts in an interesting story for helping kids to memorize those . There are two stories that illustrate this method and make math more fun. First one is designed to make it easier to memorize properties of quadrilaterals. The second story shows how practicing mathematical procedures can be more interesting.

2. The first story: Characteristics of Quadrilaterals

I used this story for teaching lecture "Characteristics quadrilaterals", which is a very boring lecture if it is taught in classical and traditional way. I thought: if it's so boring to me to teach, how it will going to look like to kids? So, I created a story about quadrilaterals family and their relationships between cousins. While

I'm telling the story I'm drawing genealogy of this family on the board. Here is the story:

„Once upon a time there was an old grandfather quadrilateral. Lets look at him! Try to describe him! How many sides and angles has he got? The sum of the angles of a quadrilateral is 360°, as we already have learned. He has got two sons: Trapezius and Deltoid. Deltoid was a black ship in the family, and we'll discuss about him a little bit latter. As we already know genetics is a miracle! Every next generation has inherited characteristics of their father. So, what characteristics have trapezoid and deltoids got? (Children are answering). But, every next generation is more successful than previous. Trapezoid is more advanced than his father. He acquired a new trait. Which one? Well, look at those parallel sides! Aren't they nice? He also has a secret about his angles, but I'll tell you: the sum of the angles that lie on non-parallel sides is always 180°. Let's meet his son now. It is Parallelogram. Look at that pretty boy? Can you see what is new about him? Yes, he's got two equal and parallel sides! Are there something interesting about his angles? (Kids can find it successfully). He was much more advanced than his grandfather. He's got two sons: rectangle and rhombus. Have they the same characteristics as the elder members of the family? They prospered in large steps. Let's discuss about them. (I suggested to the children what is important to examine: sides, angles, diagonals, symmetry).

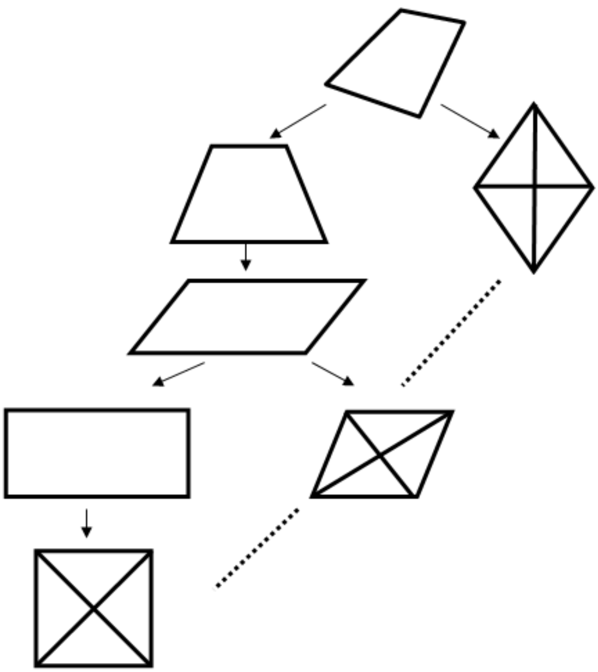

Figure 1: Genealogy of quadrilaterals family

And finally, here it is! The perfect one: The Square! He's got everything as his father, grandfather and great-grandfather and much more. What you can tell me about him? (Kids are answering and discuss.) And what about deltoid? He was on his hands and a very weird. He was completely different from the other family members. All of his cousins have got at least one or even two parallel sides. But, oh no, not him! He hasn't got parallel sides, he has two identical pairs of neighboring sides! He was more interested in it's inner space than exterior. He built normal diagonals, and this characteristic inherited rhombus and square. While I'm talking this story in class, the interest of children raise. This story involved the whole class. At the class it is always lively and fun, but for me more important is that through this genealogy they could remember better the important characteristic of quadrilaterals. For permanent visualisation I have a board in the classroom that is constantly on the wall.

3. The second story: A Treasure Hunt

*For a good math*ematical knowledge it is very important to have a lot of practice. That could be very boring for children. Especially if the task is: solve the equation or calculate the expression. We wouldn't expect that it will be fun for kids? The better way is to connect mathematical problem with real life. That will make class more interesting, and it will show that maths is everywhere around us. Meaning, learning maths is a necessary skill in a modern world. Also, it's very important to learn to understand what is read.

In the 7th grade, children learn coordinate system and percentage. So, I created a task for practicing those lectures in the form of story. Pupils have done this task divided into groups during two school classes. Here is a story:

Explorer Joca searched the attic of his grandfather and he found an old map. The map was faded out from long standing, but he could see that this is the map of the hidden treasure on the Procentus island. He couldn't see very well every detail, but the map was useful. He always loves challenges and decided to go treasure hunting.

First of all, he had to go shopping. He didn't want to spend a lot of time for finding cheaper stuffs that he need for hunting. He looked for the help of the internet and looked through catalogs. Here is what he found (some of the items have reduced price for several percentages):

	SPADE	PICKAX	SHOVEL	HATCHET	SLEEPING BAG	SMALL TENT
INTER-KOMERC	399^{99}	270	280	412	1870	3899^{99}
TRADE COMPANY	~~450~~ -5%	290	300	420	~~2300~~ -18%	~~4200~~ -8%

Answer the question:
1. HOW MUCH MONEY HE WILL SPEND IF HE DOES SHOPPING IN THOSE SHOPS? WHICH ONE IS CHEAPER?

He also had to by some food. Here are the prices of required groceries in the super markets:

	CAN OF BEANS	CAN OF PEAS	WATER	BREAD	CHOKOLATE	SAUSAGE /КГ
INTEREX	256	222	127	85	42	875
TEMPO	250	265	130	87	40	956

Considering that was a Saturday, in the TEMPO was effective discount of 12

Answer the question:
2. WHERE SHOULD HE DO SHOPPING TO SAVE MONEY?

So, he bought everything he needed, packed up and went to the road. When he reached the coast, he realized that he had to rent a boat or a scooter to go to the Percentus island. But first of all, he had to change money. Fortunately, there was a currency exchange where for one eki (eki is the official currency of the country) you give 2.5 dinars. The prices for renting were:

	1 hour	for each following started hour
BOAT	300 eki	+ 250 eki
SCOOTER	500 eki	+ 350 eki

The motor boat moves at the speed 20 km/h, and scooter even 50 km/h. On the local map he saw that the distance between coast and island is 45 km. The island isn't so big, so he meant that 3 hours will be enough to finish his job and start to go back.

Answer the questions:
3. WHAT TIME WILL HE NEEDS TO GET TO THE ISLAND IF HE USES BOAT, AND WHAT IF HE USES SCOOTER?
4.WOULD IT BE CHEAPER TO RENT A BOAT OR A SCOOTER?
5. HOW MUCH MONEY HE HAVE TO CHANGE FOR RENTING A CHEAPER BOAT?

When he arrived on the island he began to interpret the map. As far as map was a bit damaged, he had to drew the necessary objects. He read the instructions:

„Start with the point named P (the palm tree) and go right to the east for 15 steps. Then turn to the north, walk 8 steps straight ahead and there you will find the well W (although, when you read this, the well maybe no longer be there, so you must carefully count the steps). After that you have to find a big rock R. You will find it as a centrally symmetric point of point W relative to P. Go 4 steps north of the rock, and then 35 steps right to the east. And there you are! There is a treasure chest buried in the ground (point C). When you dig it up do not try to break the chest because it is made of very solid materials. Try to open it by the secret cod. The code consists of the following numbers: the distance from the palm to the well, the absolute value of distance between the rock and the horizontal axis, 60% of distance between the well and the vertical axes, 25% of distance between the well and the x-axes counted as absolute value, the difference between coordinates of point where you find the chest. Good luck! Enjoy the treasure!"

Answer the questions:
6. PUT THE POINTS P, W, R and C IN THE COORDINATE SYSTEM ON THE DEFINED PLACES.
7.WHAT IS THE SECRET COD?

CONGRATULATION! MATHEMATICAL KNOWLEADGE BROUGHT YOU TO THE TREASURE!

4. Conclusions

Effects of this method are:

- Kids were very interested in finding solutions of the tasks
- They were motivated to rich the end of the task
- They have done a lot of tasks, much more than usual
- They have spontaneously discussed in the group and between groups
- Everything looked like a game

Putting mathematical facts in an interesting story:

- helps children to memorise mathematical facts
- it is useful method for visualization
- makes class much more interesting
- introductions math in real life
- provides a motivation for further learning mathematics

References

[1] Milorad K. Banjanin, Milica Radmilović ,Vizuelizacija u nastavi matematike, http://www.ffuis.edu.ba/media/faculty/documents/2013/03/04/vizuelizacija_u_nastavi_matematike.ppt

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 195–199

Plato solids — lesson plan that connects mathematics to ecology, electrical engineering and economy

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Abstract

Base on knowledge and experience that we gained in European Summer School for Visual Mathematics and Education, we came to an idea how to implement our experience gained in working on topic "Plato solids in ecology-Ecology, waste recycling and energy" (project: Energy is all around us 2013) in secondary school curriculum. In this paper we will describe that idea.

1. Introduction

Education should follow changes and needs of society. Modern society requires young people that are creative, corporative and flexible. So teaching of mathematics should not be only directed toward developing logical skills and conquering mathematical knowledge but also toward using knowledge and (logical, creative, corporative, communicative and other) skills in order to solve and/or overcome some practical problem [1].

1.1. Modelling in education

Modeling in teaching mathematics has seven phases (1st phase: topic explanation, 2nd phase: real-life problem situation, 3rd phase: creating mathematical model, 4th phase: solving mathematical problem, mathematical answers, 5th phase: problem solution, practical understanding, 6th phase: analyze and results' evaluation, model's value and limitations, 7th phase: report) [2, 3].

Our vision of modeling in teaching of mathematics requires solving environmental problem by using mathematical knowledge and knowledge gained in learning other subjects (such as ecology, economy, computer science, electrical engineering science...). It also requires corporation between pupils from different schools (branches) because they need to see that corporation with colleagues from other branches is necessary in order to overcome some problem.

2. Lesson plan

European Summer School for Visual Mathematics and Education showed us that visualization in teaching mathematics and implementation of existing teaching experience into school curriculum are very important. So, by using that knowledge and previous experience gained in working on topic "Plato solids in ecology-Ecology, waste recycling and energy" (project: Energy is all around us 2013) modifying it and adjusting it according to the knowledge from summer school, we came to an idea to create this lesson plan that can be used in secondary school (3rd grade, age approximately 17-18). It requires corporation of three schools one that has experience with ecology, one with economy and one with electrical engineering and computer science (in our case that was technical school, economic and trade school and electrical engineering school).

First lesson is dedicated to introduction to Plato solids. Working in groups and using a computer, tablet or mobile phone pupils have to find more information and interesting details about Plato solids. Pupils of each school (together) have to prepare one short presentation about Plato solids that will show to friends-colleagues from other schools via Skype. After that they will present their work and select best one.

Second lesson should start with motivation movie about importance of preserving environment (recommendation: movie about the girl who fell silent the world for a 6 minutes [4]). Pupils-ecologist (in our case- pupils from technical school) talk about importance of preserving environment, types of pollution, waste recycling options, and energy management by using the previously prepared this presentation. After their lecture, all pupils discuss about the importance of preserving the environment, finding solutions for waste recycling and energy management. They also discuss about whether mathematics can be applied in ecology and how.

Homework: All pupils have to consider possibility of using Plato solids for classification and recycling of waste. In order to do their homework they can corporate and discuss via social network (Facebook), Skype and/or mobile phone.

Third lesson starts with presenting and changing ideas about waste classification by using Plato solids. All pupils are connected via Skype, they discuss about ideas and choose the best one. Then realization of the best idea starts by creating solution models out of paper, wire or something else (see Figure 1 and Figure 2).

Pupils-engineers (in our case, pupils from electrical engineering school) have to find some new technical solution that will make solution of the environmental problem more attractive, useful etc. Based on experience gained during creating models out of different material and by using different model nets, pupils discuss about minimal waste material. They also discuss about importance of the model colour. Material that can be used for this discussion has to be based on some researches about perception of colours (recommendation: paper about perception of the colors [5]).

Homework: Pupils-engineers (pupils from electrical engineering school) have to calculate the area and the volume of Plato solids and amount of material that will be used for creating realistic model (they can discuss about problems with friends-colleagues from other schools via social networks or mobile phone). Pupils-engineers have to send final calculation needs to colleagues-economics (pupils from economic and trade school) so that they could do financial calculation of average material costs. Pupils-ecologists (pupils from technical school) have a task to implement existing ecological standards into the model solution.

Fourth lesson represents summary of all work that has been done. Pupils-engineers present their calculation on area and volume of Plato solids. Their also present their solution of using material for creating realistic model. All pupils discuss and choose the best solution.

Pupils – ecologists present implementation of ecological standards into the models. After their presentation there should be discussion about importance of the standards and how they are created.

Pupils - economists present financial calculation on average costs of making real model. They explain to their friends-colleagues invoice.

Figure 1: Models made out of paper

This lesson plan requires different pupils' activities. Working this way, pupils will increase their internal motivation, develop ecological competencies and raise awareness about environmental protection, develop logical thinking and the ability to apply knowledge to new situations, develop social and communication skills, train for research work.

It also supports reciprocal peer education which has a positive impact on all pupils involved in this learning process [6]. Although, we haven't work the same way as we created lesson plan (modifications and improvements are made), we noticed that pupils enjoyed working and learning together.

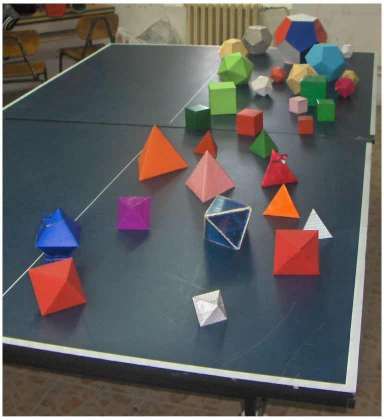

Figure 2: Models made out of paper

3. Conclusion

Teaching of mathematics should be connected with teaching other subjects. Pupils need to see how they can apply mathematical knowledge in real-life situation, they need to see and feel a joy of being able to solve a problem by applying mathematical knowledge, because that is the way to increase their internal motivation. By applying reciprocal peer education between pupils from different schools, pupils learn how to corporate, they can develop their social and communication skills, gain new knowledge.

References

[1] Pravilnik o izmenama i dopunama Pravilnika o nastavnom planu i programu za gimnaziju http://www.mpn.gov.rs/images/content/dokumenta/NPP/sr/gimnazije/SVE_ZAJEDNO_ZA_SAJT.docx

[2] Natalija Budinski, Rihterova skala kao logaritamski model u GeoGebri, International GeoGebra Conference for Southest Europe, Novi Sad, 15-16 January, 2011

[3] Hugh Burkhardt (with contributions by Henry Pollak), Modelling in Mathematics Classrooms: reflections on past developments and the future http://www.researchgate.net/publication/226388981_Modelling_in_Mathematics_Classrooms_reflections_on_past_developments_and_the_future/file/72e7e52d40e23b1b12.pdf

[4] Girl who fell silent the world for a 6 minutes http://www.youtube.com/watch?v=HlEqTnCUNIg

[5] Svetlana Čizmić, Brankica Župunski: Boje i geometrijske forme u kodiranju informacija u procesu rada, Psihologija, 2006, Vol. 39 (3), str. 327-342 http://www.doiserbia.nb.rs/img/doi/0048-5705/2006/0048-57050603327C.pdf

[6] Marlynn M. Griffin, Bryan W. Griffin: An Investigation of the Effects of Reciprocal Peer Tutoring on Achievement, Self-Efficacy, and Test Anxiety, http://files.eric.ed.gov/fulltext/ED383756.pdf

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Belgrade, Serbia, September 17–19, 2014. pp. 201–207

Mathematical modeling in teaching

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Abstract

This paper considers the problem of how to teach mathematics in an easier way, by using modeling. First, we teach pupils to add and subtract in the set of natural numbers. For that we can use a special ruler with windows. With this ruler they can solve equalities or inequalities, and they can check and see the results. Second, we teach them to add and subtract in the set of integers. For that we can use the model with two circles, which represents two sets of integers and we can calculate while rotating the circles. Third model is for multiplying integers. Now students can see why minus times minus gives plus. Fourth model is for presenting the fractions and angles. Fifth model is for multiplying integers. With these educational toolkits we can process the basic knowledge of mathematics and algebra. It may also be helpful in working with children with disabilities.

1. Introduction

Nowadays, when modeling is mentioned, people usually think of computer modeling. Modeling is a necessary teaching tool. I have been working for 20 years in education and whenever I need to explain something, I use modeling. I like the idea of implementing art in maths. The idea was to replace learning definitions by heart (e.g. Unknown addend is calculated by subtracting the known addend from the sum.). Instead of that, students can use models to solve a problem by simply counting and then see the results. “Inspired” by problems with solving equations and inequalities with high school students, my friend Gorana Zakic and I wrote a guide in 1999 for using the Ruler and it was printed in 1000 copies. We named it LITTLE MATHEMATICIANS. For more difficult problems, such as addition of integers, we use the Round tool. With the help of these educational tools students meet the numerical rules in the set of integers. If each student has a tool, they can solve problems outside the classroom, without the use of computers, notebooks or textbooks. The point is to teach maths in a more interesting and easier way.

2. Ruler

The first educational tool is the ruler. Inspired by problems of solving equations and inequalities with high school students, I made a ruler with windows, which can assist teachers in working with younger students who are beginning to learn mathematics. It may also be helpful in working with children with disabilities.

Figure 1: One side of the Ruler with a window

The other side of the ruler represents a two-digit number, so we have a repetition of digits from 0 to 9. The number 64 will be represented in this way:

Figure 2: The other side of the Ruler with two windows

Equations solving can be connected to a direction. At the same time students meet the numerical rules in the set of natural numbers and the notion of the opposite operations. For example:

$$X - 12 = 6$$

Set the window to the value of the result (in our case the number 6)

Figure 3: Moving the window and counting

Move the window for 12 in the opposite direction from the sign in front of number 12 (the opposite operation).

Figure 4: The result $X = 6 + 12 = 18$

If a number is greater than 20 pupils must use the other side of the ruler with double digit numbers, and so they will be able to understand the usefulness of *positional number system*. Two digit numbers have a digit of the tens and a digit of the units. The problem arises when they need to increase a double digit number with another double digit number. For example:

$$27 + 15 = (27 + 5) + 10 = (27 + 3) + 2 + 10$$

When adding up numbers sometimes they will have the transfer of tens e.g. $27 + 5$, when they increase 27 with 3 and then they "take" the next ten, and add 2.

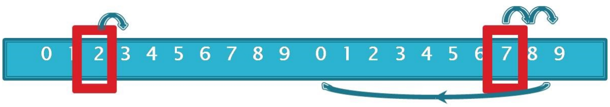


Figure 5: The transfer of tens $27 + 3$

Now we just add two units and one ten.

Figure 6: $(27 + 3) + 2 + 10$

In a case of subtraction, if there are not enough units, we will borrow from tens. For example:

$$63 - 27 = 50 + 13 - 7 - 20$$

Figure 7: $63 - 7$

Figure 8: $63 - 7 - 20 = 36$

Solving equation $23 + X = 64$

Figure 9: $X = 64 - 23 = 41$

3. Round tool

Most students have more trouble with mathematics when they learn addition in the set of integers. With this teaching tool it is much easier to explain and teach them the rules of addition. It is made of two circles with a common center. Both circles represent the real line and the numbers from -20 to 20 (it depends on the size of the circles and the font used in the preparation) are written on them. This tool can be made by students from a plastic foil. This Round tool is used for the addition of integer numbers. For example $-7+3$ can be calculated in this way: set zero in the inner circle, below number -7 in the outer circle and then look at number 3 in the inner circle and read the result in the outer circle.

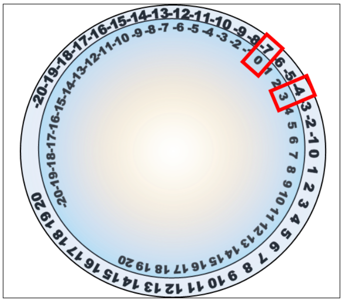


Figure 10: Adding $-7+3=-4$

3.1. Solving equation and inequality by using the Round tool

When we are solving equations (or inequalities), for example, we will just do the inverse operation of the one we did with an unknown, the same as we did with the ruler. For example $-4+K=-7$

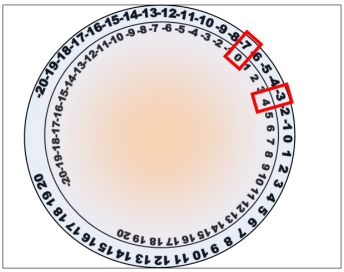
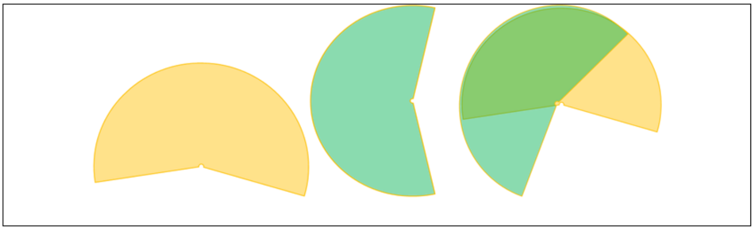


Figure 11: K is an element of the set $\{-4,-5,-6,-7,-8...\}$

When number -4 is added to k, we added a positive number 4 to both sides of the equation, (since there is a double change of the direction) On the left side we get $0 + K = -7 + 4 = -3$ (In the case of inequalities $-4 + K < -7$ the method is the same, and we can see the results, which in this case are numbers less than -3.)

4. Angle tool

We can make a similar tool that can measure the angles. We can make it from these parts and use it to demonstrate the addition and subtraction of angles.



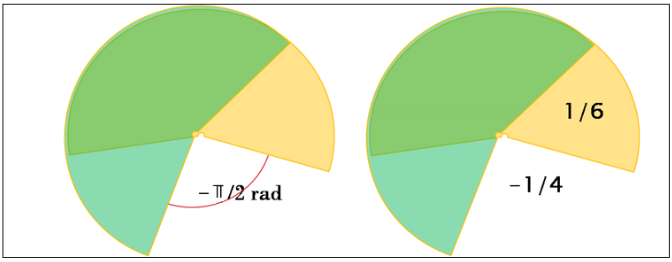


Figure 12: It can be also used for presenting the angles in Radians, or for presenting fractions

5. Fractions

There is another known way of presenting fractions and their least common denominator. (See Figure 13.)

Each of these tools students can make from the plastic foil. By making tools, they will understand fractions better.

6. Multiplication of integers

By using the table of Figure 14, students can connect multiplication, with the notion surfaces of area. They can see that the surface of square is associated with the operations of squaring and square root (diagonal Y=x). In this table it is easy

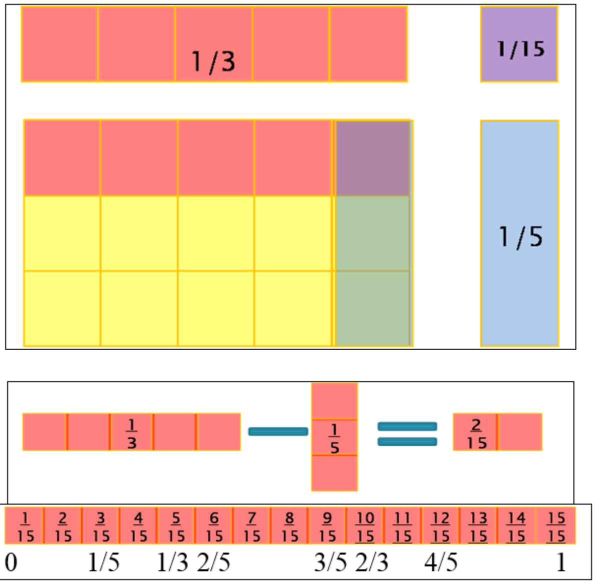


Figure 13: Connecting fractions with the order of the real line

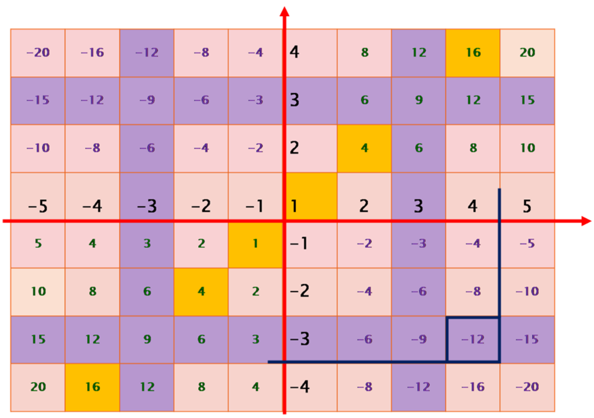


Figure 14: Table multiplication of integers placed in the coordinate system

to notice the numbers divisible by 3, for example. How can we solve equations as

$$X \cdot (-3) = -12?$$

Find the number -12 in the table and the number 3 on one axis. Putting this "L" tool, the second arm will show the result on the second axis.($X = 4$) If students have a Multiplication table and a Round tool for add I think that it would be easy to recognize the difference between multiplication and adding of integers.

7. Summary

The above mentioned teaching tools, ruler, round tool, angle tool, and multiplication table of integers represent the basics of algebra and with their help we can perform mathematical operations in the easiest way possible.

- For the addition of positive integers and the positional number system we use the ruler.
- For the addition of integers and also for solving equalities and inequalities we use the round tool.
- To present angles or fractions we use the angle tool.
- For divisibility of numbers, squaring, and for calculating area of the figure we use the multiplication table of integers.

The Closing Conference of the project Visuality & Mathematics
Belgrade, Serbia, September 17–19, 2014. pp. 209–213

Angle-a-trons (angle models made of paper)

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Abstract

This class is for 11 years old students in 5th grade elementary school. I got inspiration for this class from a video on Khan Academy called "Angle-a-trons" (see [1]). I remembered when I was a student that my teacher showed us how to make angle-a-trons. We usually don't carry around a protractor, so this can be useful to the students because they can make their own angle models. In the same time, students can see the size of the most used angles 180°, 90°, 45°, 22.5°; ...,60°, 30°, 15°,... I think that they will especially like the drawing activity using rulers and angle-a-trons made of paper.

1. Lesson plan

Exercises, matters, parts of the lesson. In the introduction of the class, ask students are there any angles in the rectangular piece of paper. What about irregular piece of paper? With one fold we can turn it into 180° angle-a-tron. Every student should make angle-a-tron of 180°. Then, they make 90° angle by folding it on the half, then 45°, then 22.5° and so on...We can get angle of 60° by folding 180° on three equal parts, then by folding it into half we get angles of 30°, 15° and so on...

We can add them together. We can put two angles of 60° together to get 120° angle-a-tron. We can get angle of 135° by addition (90°+45°) or by subtraction (180°-45°).

Students check the angles with their protractors after they make the models.

The students are divided into four groups. Every student draws a picture, using just one ruler and angle-a-trons which they made. If they have time, they can color the drawings. Students can help each other.

Every group chooses the best drawing which will be displayed on the classroom wall.

Methods and forms of student activities. Individual work (making angle-a-trons), group work, exhibition. The students are divided into four groups:

- Group 1: Use an angle-a-tron of 90°
- Group 2: Use an angle-a-tron of 90° and 45°
- Group 3: Use an angle-a-tron of 60°
- Group 4: Use an angle-a-tron of 120°

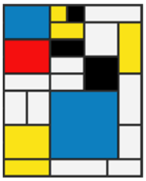
Developable competencies. Making use of aids and tools such as:

1. Knowing the existence and properties of various tools and aids for mathematical activity, and their range and limitations;
2. Being able to reflectively use such aids and tools.

2. Supplements

Used materials. For this class we need several papers for every student (for making angle-a-trons), markers, protractor for measuring angles, rulers, paper A4 for drawing, crayons for coloring the pictures.

Photos. Tasks for group work:



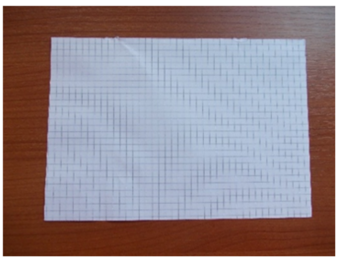
Group 1:

Group 2:

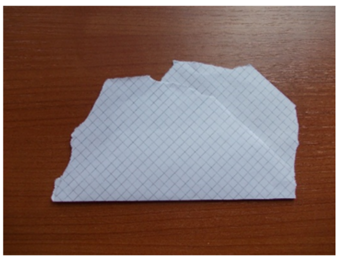
Group 3:

Group 4:

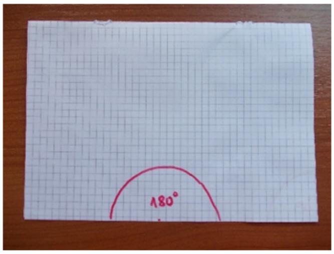
Pictures of the angle-a-trons:



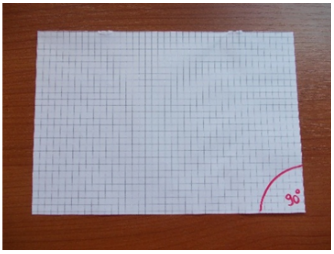
Rectangular piece of paper



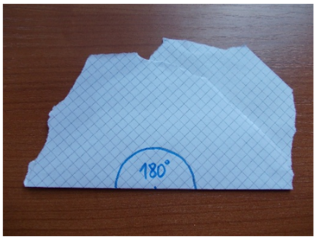
Irregular piece of paper

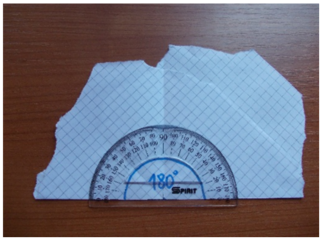


An angle of 180°

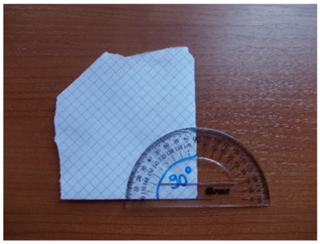


An angle of 90°

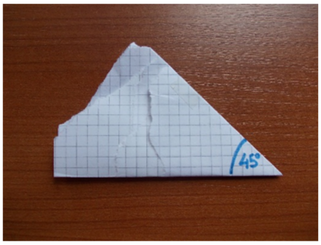


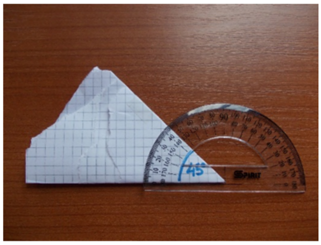


Angle-a-tron of 180°

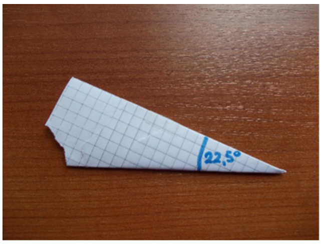


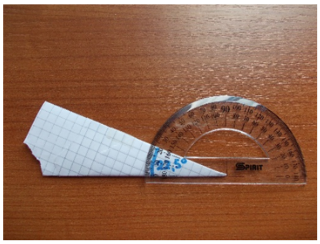
Angle-a-tron of 90°



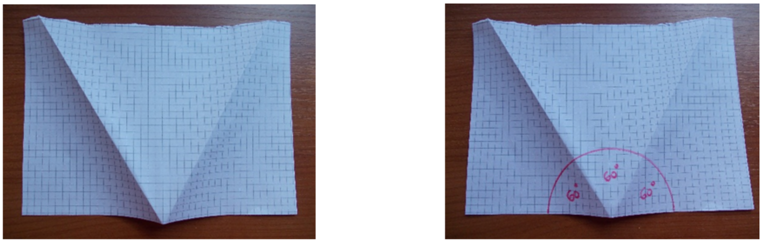


Angle-a-tron of 45°

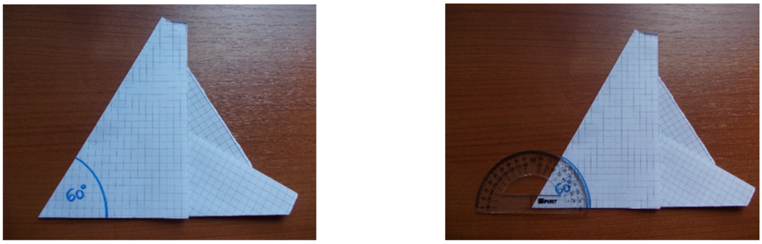




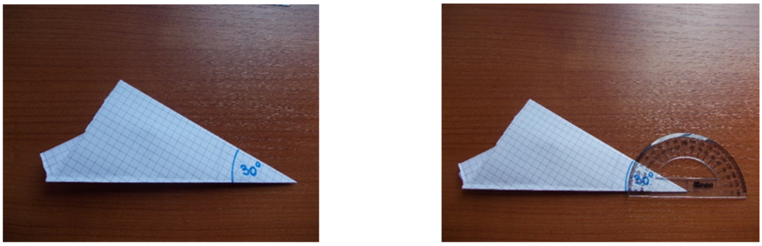
Angle-a-tron of 22.5°



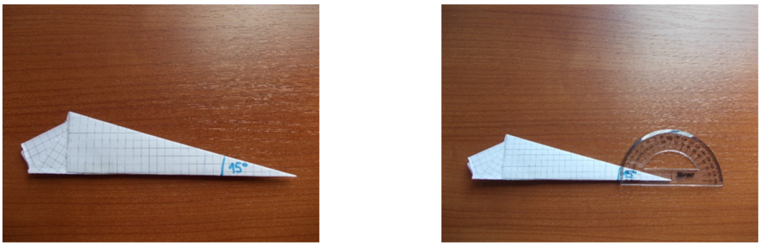
Making an angle-a-tron of 60°



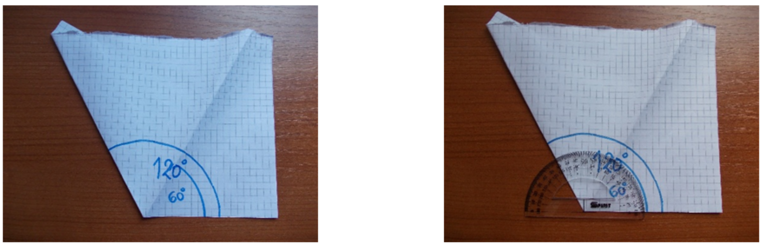
Angle-a-tron of 60°



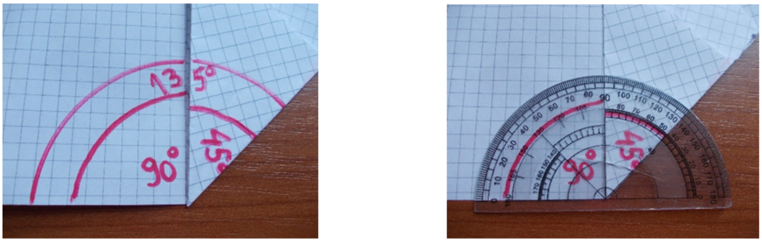
Angle-a-tron of 30°



Angle-a-tron of 15°



Angle-a-tron of 120°



Angle-a-tron of 135°

References

[1] Video, Angle-a-trons, Khan Academy, `https://www.khanacademy.org/math/recreational-math/vi-hart/spirals-fibonacci/v/angle-a-trons`

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Golden Ratio

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You have 10 new tools for determining the golden section (sector, spiral, rectangle, triangle) whether it be math, photography, nature, architecture, design, . . . depends on you! Enjoy!

Tutorial: http://tube.geogebra.org/material/show/id/148772

Geogebra material: http://tube.geogebra.org/material/show/id/148786

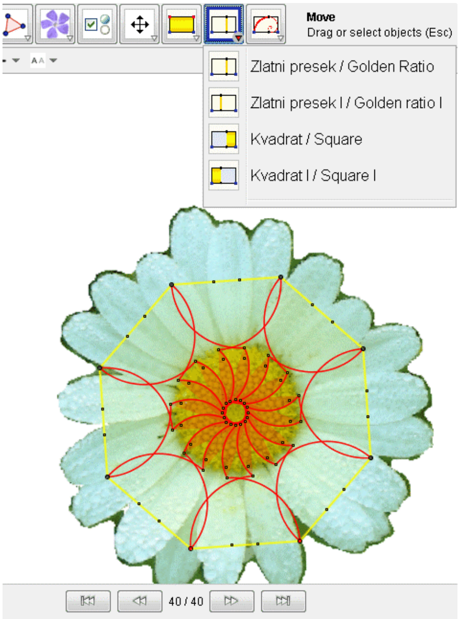


Figure 1: Golden ratio and rectangle(square)

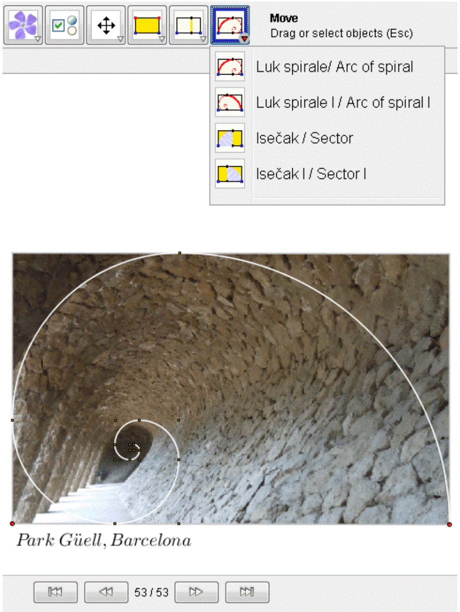


Figure 2: Golden arc of spiral and sector

Exhibitions

Slavik V. Jablan

Mathematician

Slavik V. Jablan (born in Sarajevo on 10th June 1952) graduated in mathematics from the University of Belgrade (1971), where also gained his M.A. (1981), and Ph.D. degree (1984). Participated in the postdoc scientific programs in Kishinev (Moldova, 1985), USA and Canada (1990). Fulbright scholar in 2003/4. Published a few monographs (*Symmetry, Ornament and Modularity* (World Scientific, 2002), *LinKnot- Knot theory by computer* (World Scientific, 2007) and webMathematica book *Linknot* (`http://math.ict.edu.rs/`), more than 80 papers on the knot theory, theory of symmetry and ornament, antisymmetry, colored symmetry, and ethnomathematics, and participated at many international conferences (Bridges, ISAMA, ISIS Symmetry Congresses, Gathering for Gardner, knot-theory and mathematical crystallography conferences) and created visual-mathematics course at FIT (Belgrade). The co-editor of the book *Introductory Lectures on Knot Theory* (World Scientific, 2012). The co-Editor of the electronic journal "VisMath" (`http://www.mi.sanu.ac.rs/vismath/`). As a painter and Math-artist has more than 15 exhibitions and the award at the International Competition of Industrial Design and New Technology CEVISAMA '87 (Valencia, Spain).

Exhibitions

Solo exhibitions

2004 Dom kulture Studentski grad, Belgrade, Serbia

2010 Gallery "Cella Septichora", Pecs, Hungary

2010 Gallery of the Kaposvar University, Kaposvar, Hungary

2013 George Kepes Foundation, Eger, Hungary

2014 Mikser House, Belgrade, Serbia

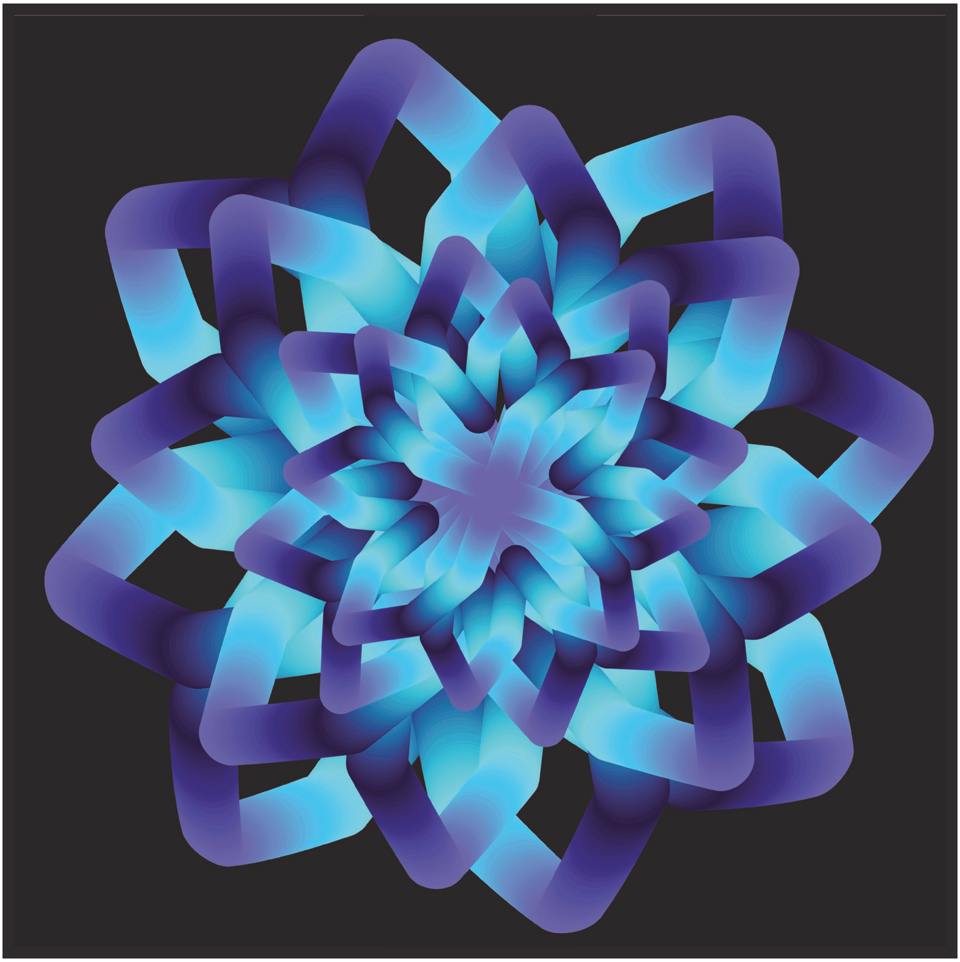
Collective exhibitions

AMS Meeting, San Antonio, USA

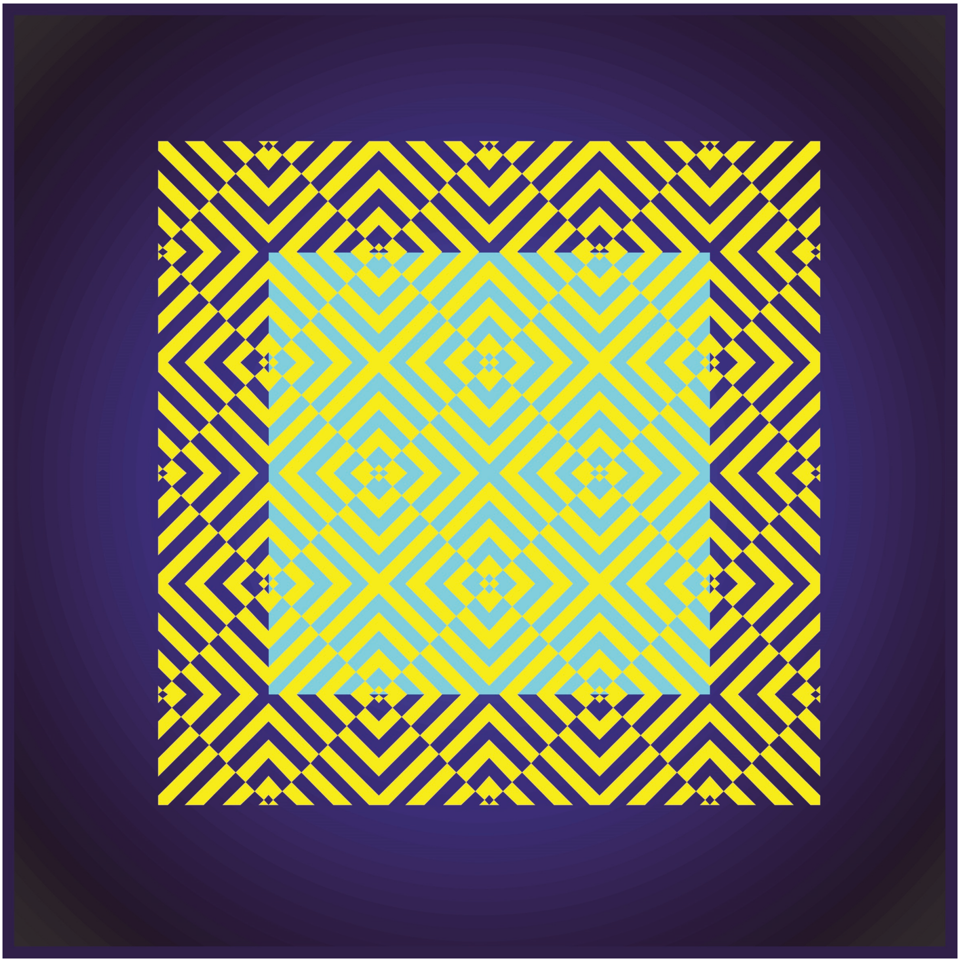
Ringling School of Arts, Sarasota, USA

ISIS Symmetry Congress, Gmuend, Austria

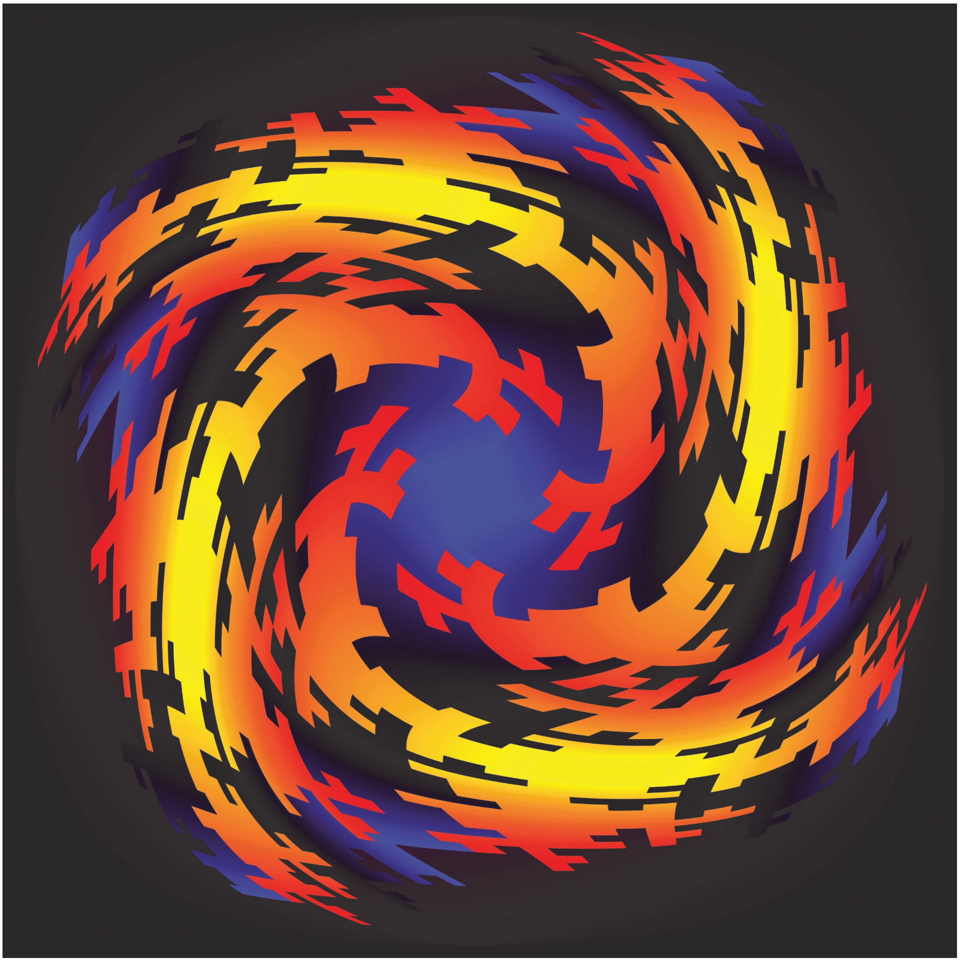
May- month of Mathematics, Belgrade, Serbia



Title: Neon Flower
Author: Slavik V. Jablan
digital print, 50 × 50 cm



Title: Oop
Author: Slavik V. Jablan
digital print, 50 × 50 cm



Title: Koch
Author: Slavik V. Jablan
digital print, 50 × 50 cm

Description

Digital print "Neon Flower" is based on the similarity symmetry: a dynamic kind of symmetry characteristic for the processes of growing in nature.

Digiral print "Oop" is inspired by the works of Op-art and uses the basic element so-called Op-tlle: a square with the set of parallel lines. The important property of this work is the use of complementary colors.

Digital print "Koch" is inspired by fractal Koch curve. This curve is usually based on regular triangles, but this one is based on square and deformed by using geometrcal effects from the program *Paint Shop Pro*.

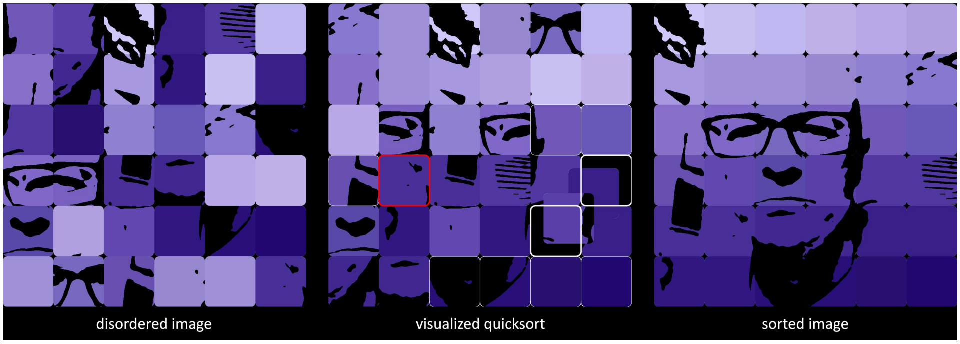
All original graphics are created in the program *Corel Draw*.

Contact: sjablan@gmail.com.

Gerhard Funk

Art-, Mathematics and Informatics Teacher, Artist, Professor at the University of Art and Design Linz

Gerhard Funk was born in 1958. He studied Mathematics and Art Education in Linz und received his Ph.D. in Theoretical Computer Science. As a high school teacher he taught art education, mathematics and informatics. In parallel he worked as assistant and researcher at RISC Linz (Research Institute for Symbolic Computation). In 1993 he transferred to the University of Art Linz, where he established an education programme for digital media and developed the eLearning platform "Digital Media for Artists – DMA". Since 2004 he is a full professor at the Institute of Media and the head of the bachelor's degree program "Timebased and Interactive Media" that he has conceptualized. Additionally he leads the branch of study "Web Art & Design" of the master's degree program in "Web Sciences" that is offered together with the Johannes Kepler University Linz.



Title: Image sorting – a tibute to Donald E. Knuth
Author: Gerhard Funk

Description

This artistic work tries to visualize in endless loops very popular sorting algorithms in an attractive, more sensual way by sorting images which are taken in real time from a camera.

All iteration steps of the algorithms are animated slowly. So the visitors can follow the sorting process and by watching these sequences of iteration steps they should be able to extract for themselves the underlying sorting principles without any further explanation. The final, correct image emerges successively during the sorting process.

For a preview see: `http://youtu.be/hEHJsa9iuxs`

Searching and sorting are fundamental tasks in almost every larger program and software system. This is the reason why Donald E. Knuth, the great computer scientist, has dedicated one whole volume to the topic of sorting and searching in his famous lifelong project "The Art of Computer Programming", first published 1973. But nowadays the different concepts of sorting are deeply hidden behind standard build-in sort-functions. We use them in our high-level programming languages without knowing what's going on behind the function calls. My art work makes the attempt to bring back sorting on the stage and to visualize several strategies of sorting.

Of course there exist hundreds of visualisations on YouTube, but mainly for Computer science students. On contrary to these prosaic versions I tried to develop a more artistic approach for an exhibition context. Sorting is mostly demonstrated very technically with rows of numbers or letters. Internally every image is also stored as an array of pixels and every pixel is described by numbers. Depending on the colour model e.g. the brightness of a pixel is stored between 0 and 255. So it was obvious to visualize the sorting process using images as input. The advantage of images is that the beholders can perceive the whole image at once. Therefore they can immediately recognize the state of order and disorder and the underlying sorting process.

At the moment I visualized for this artistic work four very popular sorting algorithms: Bubble sort, insertion sort, quicksort and a parallelized version of bubble sort. Bubble and insertion sort have the average complexity of $O(n^2)$, where n is the number of elements to be sorted, which is slow and not very efficient. But their strategies are straight forward and easy to understand. Quicksort is faster, has a better average complexity of $O(n \log n)$ and is the mostly used algorithm. But it is a little bit harder to understand the underlying concept looking at the visualized sorting process. And finally a parallelized version, which is fast with a complexity of $O(n)$, but needs a chain of n processors. This algorithm is impractical for computers, but is very efficient to order a group of people by size, because each person has a brain, which can be considered as a processor.

My intention was to visualize the algorithms in an attractive, more sensual way and to animate each iteration step of the algorithms. The animations should run slowly. So the visitors can follow the sorting process and by watching this sequence of iteration steps they should be able to extract for themselves the underlying sorting principle without any further explanation.

My first idea was to start with an image (given or taken from visitors by a camera in the exhibition) and to sort the pixels of this image using the brightness. In this case the image will be deconstructed. But the minimum size of this image must be 50 x 50 px, because in a smaller resolution it is impossible to recognize faces and objects. So we have at least 2500 pixels to sort with 6.250.000 iteration steps. To watch all this steps is boring or it must be done so fast, that nobody can follow a single iteration. So I decided to change the approach.

Now I take a gray-scale image for a camera, blur it a little bit and transform it into a black and white image using a threshold parameter. Then I colorize the white parts by changing the brightness of the used colour from light in the upper left corner to dark in the lower right corner. Finally this image is split into 6 x 6 tiles and these tiles are rearranged randomly. So the sorting process is reduced to 36 elements (a square of 6 x 6), which starts from a disordered image and ends with the correct image. Now the visitors can follow the algorithm, see how the image is reconstructed and what is to see in this image - the watching visitor. I think, this makes the installation more attractive and motivates the beholders to stay longer in front of the installation. Because they want to see the final, unknown result, the original image. After the image is sorted a new picture is taken from the camera and the process starts from the beginning. It's an endless loop.

Additionally I use white markers to emphasize an iteration step. This helps to understand the sorting principles easier. In quicksort I also use red marker for the pivot element. It is possible to hide these markers, but it's harder to see what's going on.

The programme is written in Processing.

The presentation of this art work is scalable. It is interesting to see how the final sorted image emerges during the sorting process and this is totally different for these four algorithms. Therefore the best form to present the work in an exhibition is to show all four sorting processes in parallel on screens (e.g. iMac, 27") using a camera for the input. The screens should be placed side by side on sockets or on the wall. In a reduced form the work can also run on one iMac with its build-in camera where in each sorting cycle the algorithm is changed.

If no camera is available I use a modified image of Donald E. Knuth, taken from his webpage.

Contact: `gerhard.funk@ufg.ac.at`

Kunstuniversität Linz, Institut für Medien
Reindlstrasse 16-18, A-4040 Linz, Austria

Monica Gross Meinhart

Costume Designer for Contemporary Dance, Artist, Illustrator, Textile Design Teacher

Monica Gross Meinhart was born 1974 in New York, USA. Lives and works in Vienna, Austria.

Education and Qualifications: Master of Arts in Textiles - Free, Applied and Experimental Artistic Design, University of Applied Arts, Vienna. Secondary school for Fashion and Clothing Design with an additional course in Costume Tailoring, Vienna.

Vocation: Costume Designer for Contemporary Dance (Rose Breuss, Milli Bitterli, Liz King, Georg Reischl, Catherine Guerin, LEDies, Austria). Freelance Artist and Illustrator. Experience in various Fields of Fashion (including Azzedine Alaia, Lee Kyle, Paris; Life Ball, Kids in Fashion, Vienna)

Title: EMA Endless Memory Archive
MDF, Paper, Cardboard, Bookbinders Linen
39,7 x 38 x 11 cm (corpus) 100 x 150 cm (when folded out)
Author: Monica Gross Meinhart

Description

Personal experiences, communicative memory and the inexhaustible flood of media imagery compose themselves to endless amounts of memories within each indivdual.

EMA makes an attemp to categorize and archive these memories in order to rehabilitate them later. Visualized memory sequences are linked together to form associative chains and are then archived. Their subsequent reception is alienating as the observer is caught between personal associations and artificially constructed memories. The associative behavior of the memory sequences and the resulting infinity are examples of fractal properties.

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Austria

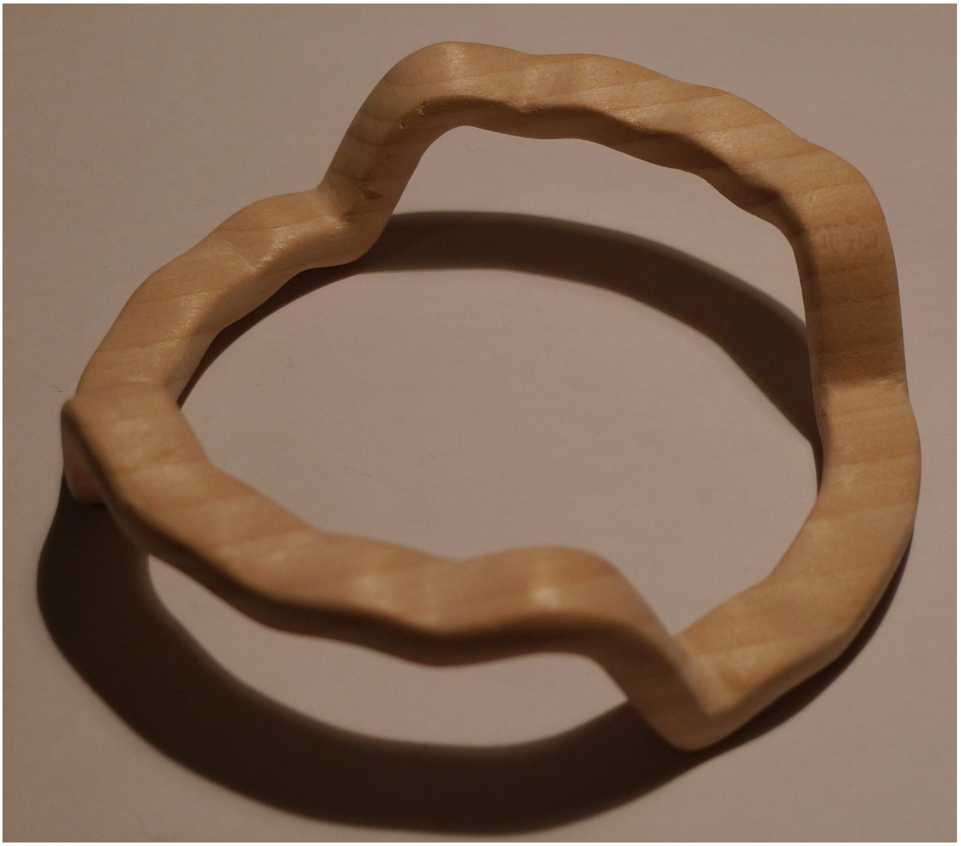
Holunder Manuel Heiss

Digital Artist

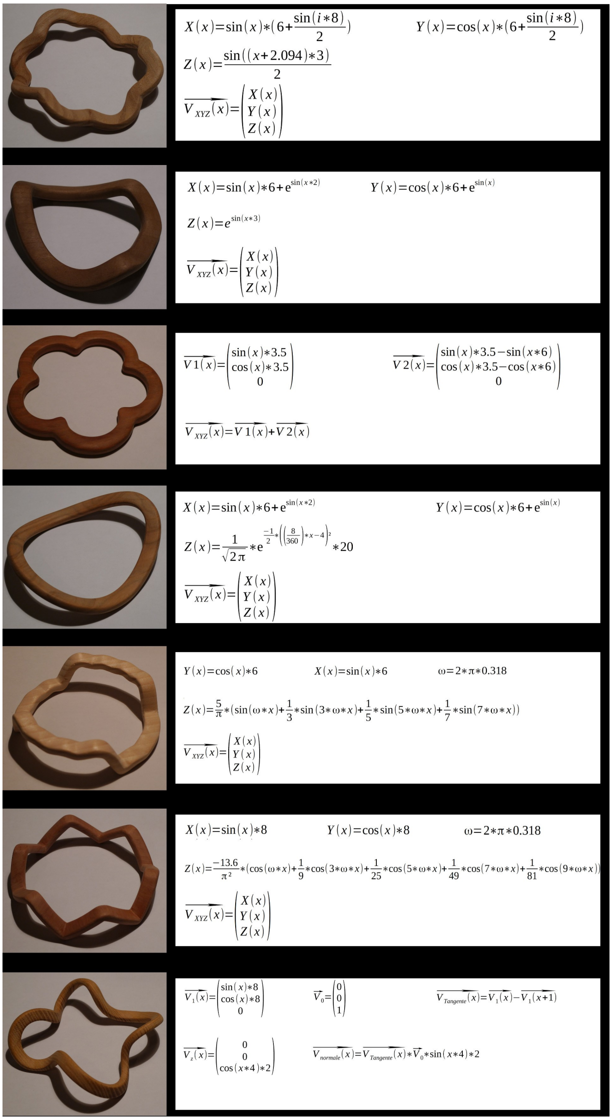
Holunder Manuel Heiss was born 1983 in Innsbruck (Tyrol). Lives and works in Vienna and has been studying Digital Art with Prof. Ruth Schnell at the University for Applied Arts Vienna since 2009.

Public artworks:

"RC-Car Painting" at Elektrofachadel Club Celeste
"Face Projection" at `www.date-an-artist.com`
"Blackbox" at Y/OUR/SPACE
"GPS-City Tagging" at open house digital art class
"SternenTransporter" and "EILudwig" at digital frictions – Weisses Haus Wien



Title: Mathteething
Wood, Digital Prints
7 Rings with 110mm Diameter and 7 Prints A3 or A4
Author: Holunder Manuel Heiss



Title: Mathteething
Wood, Digital Prints
7 Rings with 110mm Diameter and 7 Prints A3 or A4
Author: Holunder Manuel Heiss

Description

The artwork deals with two different aspects.

One aspect is the transformation from formula over 3D-Renderings to an completely analog medium like wooden objects for children.

The other thing is the question about how much should we force the education of our children. In many countries they tell you that it is really important to start teaching children and try to grow their intellectual mind with music, languages, paintings,... as early as possible. My Teething rings are some kind of ironical statement to this pedagogic approaches. They are made by mathematical correct formulas and the child should get mathematical education when it takes them into its mouth. Actually the mouth is one of the first and most used sense that babies are using to find out more about their surrounding world. So it should be the best sense to start teaching mathematics.

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Manuel Holunder Heiss
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1050 Wien

Ulrich Kühn

Sculptor, Media Artist, Musician, Teacher at the University of Applied Arts Vienna

Ulrich Kühn was born 08. 03. 1982 in Amstetten/Lower Austria.

Ulrich Kühn works as sculptor, media artist, musician, teacher and lecturer at the university of applied arts Vienna. He is working at the border between media art, performance and experimental digital film production - specifically real-time visualization and interactivity.

He received his academic/artistic training at the University of Applied Arts Vienna in art and design education and sculpturing, multimedia.

Artistic work, employment

since 2002 working as a free artist/musician/free software programmer
2007-2009 freelancer at cat-x - conceptional art technology
since 2009 teaching in schools
since 2009 lecturing at the university of applied arts: digital/analog interfacing

Most important prizes/awards received

agitas Kunstpreis, Vienna
Alice Schwarzer Preis, VIenna
Ursula Blickle Preis des MUMOK, Museum of modern Arts, Vienna

Most important invitations to performances, presentations, exhibitions

Triennale linz at LENTOS linz
participation an organization of "independent 3D cinema", austrias first non-commercial 3D cinema festival
performance at the "kofomi" komponistenformum mittersill, austria
screening at the videofestival bochum, germany
2009 performance at the ars electronica festival, linz

Name and institution of key international cooperation partners in the last 5 years

ORF Radio Ö1, kunstradio
Lentos museum
Museumsquartier Wien
ima - institue for media archaelogy
mica music information center austria

Separate listing of 10 important publications

austria's 1st independent 3D cinema, founder
"Kabelbrand - sounds from the max brand synthsizer", international cd-release
"roomboom", electromechanical concert & installation, kunstraum nö
"ghostwriter" installation, artist international summer meeting, czech rep.
"doppelclicker", computer game with bernd klinger, utrecht, netherlands
"synapsen service" with robert zimmermann, ars electronica festival
"analogsat - perfomance", ars electronica
"tonspur", founder and artist, permanent multichannel installation, museumsquartier

Title: „Nosferatu – A Symphony fading"
Video, digital Video
3min10sec, 2012-2014
720x576 pixels
Author: Uli Kühn

Description

3min10sec, 2012-2014
Video, 4:3 dv-video, stereo sound
Taglist: increasing light, vampire, software, color values, mathematical analysis

The base for this experimental movie is the 1922nd black and white classic „Nosferatu – Eine Symphonie des Grauens“ - one of the first vampire and horror films ever. A mathematical analysis of this found footage material enables the reconstruction of the content. The newly computed content contains – permanently increasing within its timeline - the narrative story of the vampire.

Each frame of „Nosferatu“ (each second consits of 25 frames!) is analyzed by self-written software and rearranged by increasing color values. So each frame of the original movie gets a new number/value by the software – lower numbers for mainly dark/black/night frames, higher numbers for daybright frames. The higher (= brighter) the number, the later this frame is played in the newly arranged experimetal movie.

This leads to the situation that the new movie starts with black/night (like the vampire) and ends with bright daylight – the meaning of the original movie is the mathematical base for the new one. The brighter the frame, the later it is displayed - it is represented more close to the end of the movie, in other words the death of the vampire.

The awakening of the vampire at night and the dying caused by bright daylight in the morning is represented by the time-based position within the new movie.

Originally Nosferatu is a tinted movie (with 5 monochrom colors) without sound.

The sound of „Nosferatu – A Symphony fading“ consists of the modulated sound of a old cinema movie projector – the original „Nosferatu“ is (so called) „silent“.

To enhance the fading-experience of the consumer the experimantal movie „Nosferatu – A Symphony fading“ is diplayed best in dark surroundings (e.g. cinema, dark areas, . . .).

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www.analogs.at
www.praxistest.cc

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Stefanie Leinhos

Artist, Illustrator

Stefanie Leinhos studied illustration in the class of ATAK / Georg Barber at the Burg Giebichenstein University of Art and Design Halle and finished with a Master of Arts And since 2011 she works as a freelance artist and illustrator

2012 scholarship of the Saxony Anhalt Arts Foundation

Group Exhibitions

2014 In Print, Work Detroit Gallery, Detroit
Frédérique Rusch et Stefanie Leinhos, Galerie Le HUIT, Paris

2013 Fanzines! Festival de la Micro-Édition Graphique, Paris VOLK, Westwerk, Leipzig
Hurly Burly, Saxony-Anhalt Arts Foundation, Halle
Now Here – M.A. Editorial Design, RAUMinbetrieb, Halle
International Poster and Graphic Design Festival, Chaumont

2012 Ereignis Druckgrafik 4: Ansichten – Aussichten, International Printmaking Exhibition, BBK Leipzig

2011 7. Nordhäuser Grafikpreis der Ilsetraut Glock-Grabe Stiftung, Kunsthaus Meyenburg, Nordhausen
Die Ungarische Methode, REH Kunst, Berlin and Grauer Hof, Aschersleben
Kalter Hund, Kunsthalle am Hamburger Platz, Berlin
Satellite-Show, Fumetto – International Comix-Festival Lucerne

2010 Die Weihnachtsgeschichte – eine Ausstellung, KMA 36, Berlin
To be Announced, Academy of Fine Arts, Warsaw
Angst Forum, Werkleitz Festival, Galerie dieschönestadt, Halle

2009 Mieterwechsel, Knopffabrik Zwickau

2008 AMERIKA, Werkleitz Festival, Werkleitz Gesellschaft e.V., Halle

Publications

2013 How we met, self published artzine
Throwing Stones, self published artzine
Is there something I should know?, self published comic

2012 Taschenlexikon der Angst, Verlag Hermann Schmidt Mainz, Berlin University of the Arts, 2nd print run
About #3, Gloria Glitzer (ed.), Berlin

2011 LUBOK 10, Christoph Ruckhäberle (ed.), Lubok-Verlag, Leipzig

2010 EIS, 213 Magazin No. 3, Halle
Mixtape, Burg Giebichenstein University of Art and Design Halle
weit von hier, Salon Magazin No. 3, Offenbach

2009 The Wooden House, 213 Magazin No. 2, Halle
TEENAGE KICKS ! ATAK feat. Team 213, Strapazin No. 95, Zürich 213 Magazin No. 1, Halle

Title: 1/6.2044840173323394e+23
Laserprint, DIN A4
2,80 x 1,50 m
Author: Stefanie Leinhos

Description

The wall work is part of the series »It will all be worth it in the end«.This series consists of black and white drawings, in which I am exploring all possible variations within different image motifs. It is based on the idea that a variation is not only a different manifestation of the original but an original itself. By showing those possibilities simultaneously and next to each other, I state them as equal forms of existence. The drawings are made with fineliner and black ink on a DIN A4 paper in a certain grid. The final and presented work consists of real size copies of the drawings.The copying is a logical consequence, that questions the term of »the original«. The amount of variations is depending on the images complexity.

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Milan Ličina

Graphic and Product Designer, Digital Artist, New Media Designer

Milan Ličina, born 5.7.1990 in Belgrade, Serbia. After graduation from High School in 2009 he started studying Graphic and Product design at Belgrade Polytechnic College. While finishing studies at Belgrade Polytechnic, Milan also started education at Faculty of Digital Art (part of Metropolitan University) in Belgrade, in field of Interactive Media Design. Having Bachelor degrees in Graphic, Product and Interactive Media Design, continuation of his studies through New Media Design (Master Program-Metropolitan University Belgrade) seemed as a logical step in 2012. Throughout his New Media Design studies, Milan experimented with various techniques and expressions, varying from print design, art installations, interactive and digital design, video mapping, creative coding, (live) audiovisual performance and sound art. Graduating this program in 2014, with highest grades, he became Docent at Faculty of Digital Art (Metropolitan University), at the Department of Interactive Media Design. Milan is now starting his PhD studies in New Media Design at Metropolitan University. His current interests cover live audiovisual performances, sound art and new media installations.

Notable references:

- "mutual" – interactive sound installation, Gallery UK Parobrod, Belgrade (2014.)
- "Key to Transformation" – interactive video mapping on sculpture, Mikser festival Belgrade ("Best interactive installation" award, part of the team with Marija Ličina, Madlena Domazet and Stefan Ćirić – interaction and visual content designer, 2014.)
- "sinilluminate"- Visual Math educational toolkit developed with Jovana Jezdimirović at Faculty of Applied Arts in Vienna, Austria (2014.)
- „U korak sa vremenom ("In Step with Time")"- Production and development for Faculty of Architecture Annual Exhibition (interactive content development, screenings and video mapping), Applied Arts Museum, Belgrade (2013.)
- "Rebirth of Truth"-fashion performance, Gallery UK Parobrod, Belgrade (VJ, part of the team, live 3-screen visual performance, 2013.)

Title: Polyscape 1
Printed digital artwork
420 x 297 mm
Author: Milan Ličina

Description

Polyscape 1 and Polyscape2 represent artistic exploration of polygons. In this particular case, triangles were used in order to create geometrical shapes that are intended to imitate organic forms which can be found in nature, forming landscapes around us, hence the name "Polyscapes". Speaking of imitation, this work is not about actual recreation of any form in particular, in fact, figures are only nature-inspired in order to signify presence of geometrical order in our habitat. Having in mind that these shapes can not be constructed or mathematically explained (except in theory) by applied mathematical rules, these works can also, in some way, be considered as some artistic expansion of "impossible geometry" concepts and a free cultivation of the motive. Staying true to the original concept of mixing hybrid forms, coloring is also green-inspired, which in comparison with highly futuristic approach gives an artistic statement about importance of nature preservation in our (near) future. Mentioning the word "hybrid" above, artist wants to point out how the borders which define terms "organic", "geometrical", "futuristic" etc. are fading away, letting the contrasts blend together and let us create freeform design, architecture and art pieces, which, in personal belief, present future of (applied) art in general. Pieces are created and post-produced digitally, but are intended to be printed.

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`www.milanlicina.tk`

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11000 Beograd
Srbija

Julie Monaco

Artist

Julie Monaco studied Transmedia Arts at the University of Applied Arts Vienna with Prpf. Brigitte Kowanz and Sculpture with Prof. Michelangeo Pistoletto and New Media with Prof. Peter Kogler and graphics with Prof. Ernst Caramelle at the Academy of Fine Arts in Vienna.

Solo exhibitions (selection in 10 years)

2014 Traklhaus, Salzburg
Chemical Plates, Gallery Ernst Hilger, Vienna
Julie Monaco, Factory der Kunsthalle Krems, Krems
2011 Julie Monaco 19972011, Hilger BROTKunsthalle, Wien
2009 _957, Galerie hilger contemporary, Wien
_957, La Citta, Verona
2006 quot21quot, Galerie Bonni Benrubi, New York
quot21quot, Galerie hilger contemporary, Wien
2003 Focused Daily. Hyperrealistic Landscapes, DAM Gallery, Berlin
Focused Daily. Hyperrealistic Landscapes, KlausEngelhorn20, Wien

Group Shows

2013 Fragile, BAWAG Contemporary, Wien
10 Jahre Lentos, Lentos Kunstmuseum Linz, Linz
Young Austrian Photography, Lockheed Martin Gallery, Manison at Strathmore, Maryland
2012 Young Austrian Photography, L2 Lounge, Washington DC
Spaces amp Faces, DIEAUSSTELLUNGSSTRASSE, Wien
Grafik, Galerie Wolfrum, Wien
2011 Entartainer, Parkfair, Parkdeck Stadion Center, Wien
2010 Story behold, story be told, Kunsthistorisches Museum, Wien
Summer, Galerie Ernst Hilger, Wien
Wunderwelt, Fotogalerie, Wien
You never know what happen next, Lentos Kunstmuseum Linz, Linz
im vorbeigehen, Katholische Theologische Universität, Linz

2009 Now you see it, Bartlemas Chapel, Oxford
Kreuzungspunkt Linz, Junge Kunst und Meisterwerke, Lentos Kunstmuseum Linz,
Stark bewölkt, MUSA Museum Startgalerie Artothek (ehem. Museum auf Abruf),
Wien
Landscape as a Dream, Studio La Citta, Verona
2008 Kowanz und Next Generation, Investkredit Bank AG, Wien
Central Europe Revisited, Esterházy Contemporary, Schloss Esterházy, Eisenstadt
L#x2019 Art en Europe-Pommery Experience 5, France Domaine Pommery, Reims
Lichtspuren, Lentos Kunstmuseum Linz, Linz
2007 Editionen, Momentum Photographie, Wien
Romanticism - a female approach?, Galerie Nusser amp Baumgart Contemporary, München
2006 Human touch, Sala Terrena, Salzburg
Visum et Repertum, Stella Art Foundation, Moskau
Modernes Abenteuer, Russisches Kulturinstitut, Wien
Ankäufe, Sammlung der Universität für angewandte Kunst Wien, Heiligenkreuzerhof, Wien
2005 State of the art, SoS State Department amp Embassy, Wien
Wasserspiegelbild, Galerie Wolfrum, Wien
2004 Wolkenbilder, Alte Nationalgalerie, Berlin
2003 State of the art, State of Sabotage, Wien
Open 2003, Art and the Cinematic Vision, Venedig
Naturbeobachtungen, Galerie Engler + Piper, Berlin
Zugluft - (Draught), Moskau
2002 digital room # II, Fotografisk Center, Kopenhagen
Vitra Austria: Julie Monaco, Isamu Noguchi, Jean Prouve, Vitra Austria GmbH,
Wien
memorybank, KlausEngelhorn20, Wien
2001 Bahnhof in Transition, Nordbahnhof/Praterstern, Wien
2000 Chrysler + Greisler, KlausEngelhorn20, Wien

Title: VV_2178_#E, 2013
Chromogenic print on Aluminium, framed
38,9x42“ , ed 3+2AP
Author: Julie Monaco

Description

World-Laboratory: Recently Discovered Pictures by JULIE MONACO Andrea Gnam

At first sight, whenever we examine pictures or concern ourselves with numbers we are dealing with two completely separate fields: when we look at a picture—assuming we believe that we can recognize a familiar object such as a landscape or a human figure—we enter a world that still seems to be, to some extent, connected with our own. Furthermore, through our memory of things already seen, we are drawn into imaginary worlds of experience. This occurs by the mediation of the language of forms: composition, shades of coloring or grey tones. Alternatively, however, we move into the kingdom of numbers, where a mathematically determined space opens up, a space in which axioms and calculations are defined, but in which also, as in art, boldly innovative modes of perception become possible. In her series of works made in 2013/14, CHEMICAL PLATES, Julie

Monaco is concerned with questions and inquiries that are no less fundamental than her early graphic conceptions involving number sequences. Visitors to her exhibitions are confronted with an installation that is, on the one hand, a precise introduction to her work but, on the other hand, operates on the far side of what can be grasped by the eyes: the data of all the images shown are projected as animations onto a semi-transparent foil or onto a wall. Numbers and letters appear, only to be replaced shortly afterward by series of numbers and letters closely succeeding each other. Here, in a soberly enhanced script, one sees the internal world of the computer brought into the light of the outside world: but this is not achieved by means of an optically undemanding binary code but rather by the hexadecimal system, which uses not only numbers but also certain letters—a work process that takes hours to present. Here we see everything that is before us in the exhibition rooms precisely presented. The secret of the workshop is perfectly exposed, but we are still not concerned with the pictures themselves but rather with processes that make them possible.

Monaco's pictures, which, without exception, were created at the computer but then moved out into the material world, have undergone a transformation during this journey. The artist operates with fractal algorithms but intervenes here and there into the code in order to achieve convincing aesthetic results, while elements of drawing by hand also enter the process. The computer-generated picture is printed out, and then a repro and finally a photograph on silver-gelatin paper or chromogenic colored prints made in the laboratory. At this point, at the very end of the process on the material carrier, we are dealing with photographic procedures, but without a camera creating the umbilical cord to an event in the world. A picture is photographed, not an event outside the picture. We see these images as landscapes of elementary beauty, thinking that we can distinguish between ocean and horizon, a rising sun, the agitated sea, plants and so on. Some things seem cool and unapproachable as if we had reached a limit we cannot pass. That is a purely aesthetic experience, even though parallels to natural experience may exist, when one has the impression that the intensity of the moment could be increased no further. However, these pictures also have no preordained order outside themselves. The artist works by intervening in chemical processes during the materialization of the images and generates—in the interior of the laboratory, so to speak—found images from the building blocks of our digitally permeated world: numbers, letters, dashes, fractals, which form, on the micro-level, the key to the structure of plants and minerals, the organic and the inorganic. Computer-generated, in the way the appearances of the real or photographed world are retold or newly created. But then there is a huge leap, like a journey from one world into another, or even like a birth: the images find their way back into our old life—the pre-press shop and photo laboratory, the walls of the exhibition rooms or the pages of a book.

The fact that Monaco's pictures make landscapes appear underlines this process. From their position in the landscape or in relation to the landscape the viewers can internalize their awareness of self. Are we part of the landscape or do we dominate over what lies before us? The coppery, sometimes earthy, sometimes wonderfully colorful landscapes—in each case the chemical processes during completion of the image production were altered by an admixture of copper sulfates—the landscapes we imagine when we have Monaco's latest computer-based images in front of us pose anew these old questions about dissolution or dominance, beauty and intervention: now against the background of image generation by means of fractals, numeric processes and chemical reactions.

Contact: `monaco@monaco.at`

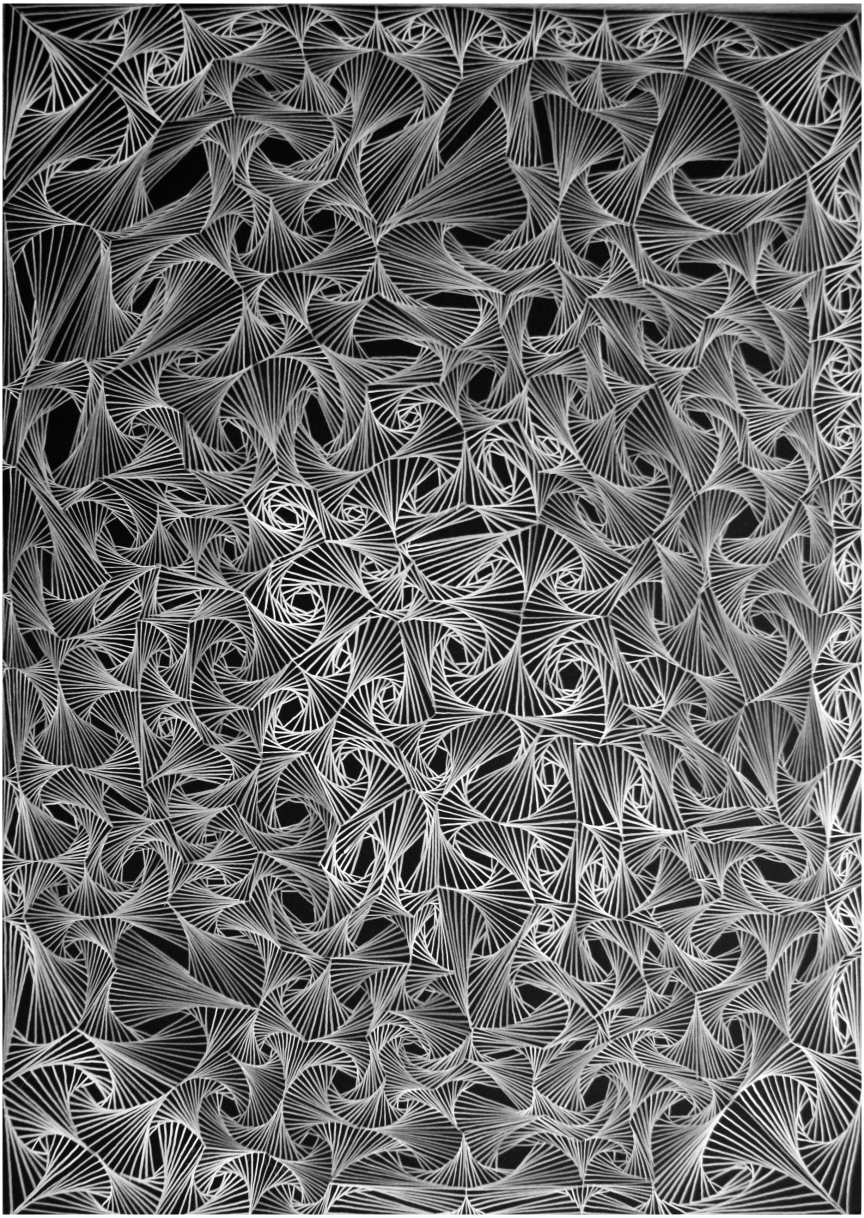
Luís Filipe Salgado Pereira Rodrigues

Artist, Architect

Luís Filipe Salgado Pereira Rodrigues was born in 1971, in Portugal, I took a course in Fine Arts from the University of Porto (Portugal) (finished in 1996), He did an MA in Art Education in Fine Arts from the University of Lisbon (Portugal) (finished in 2007), He is a student of Architecture Doctorate (PhD) in the Faculty of Architecture of the University of Lisbon (Portugal) since 2008. He lives in Santo Tirso (22 Km from Oporto) and currently he is working in a school in Santo Tirso.

Since 1992, he has been participating in nearly 70 exhibitions of paintings (three of them in Amsterdam, two in Spain, one in Seoul (Korea), and the others in Portugal). Since 2002, he has had eight solo exhibitions of painting, one with photographs, and another with drawings. He held 13 lectures about art and drawing and published 23 articles on art and drawings. He also published a book called "Drawing, Creating and Consciousness" with a text made by himself (280 pages) and, in the same book, 11 interviews (250 pages) to 2 (architects Álvaro Siza Vieira and Alcino Soutinho) and 9 artists.

Recent solo exhibitions:
2013:
Ars GEometrica Galéria, Eger.(Hungary)
`http://www.elmenymuhely.hu/?page_id=127`
Monnumental Gallery, Lisbon. Portugal
`http://www.galeriamonumental.com/`



Title: Mathematical Net
100 x 73cm
Author: Luís Filipe Salgado Pereira Rodrigues

Description

I start my research article by asking a question: are my artworks drawings or a mathematical play with representation of the shapes? Then, is drawing always a mathematical application on the registration of proportionality through perception? Let's try to know more about it. My artworks are, really, conceptual representations from which I get an aesthetically sensible set of shapes, as it happens with any type of drawings, as images that they are.

It is my belief that the drawing process is not dissociable either from mathematics or aesthetics or even from perception. When we draw, we record amounts of shapes or of lightness or, in a broad sense, amounts of visual information. When we work literally with our perception, we do that more cognitively; when we make imaginative drawings, we do that more intuitively; when we make geometric drawings, we do that in a way in which there are no borders between the literal imagination and cognition, using, however, the visual perception only as an intuitive mathematics perception, i.e., not using perception as a way to understand the reality that one observes and that one would transpose into a support.

In my assertion of drawing, I put aside the so called (in Portugal) project's drawing, the design, because I consider that the project is not a drawing; it is merely a project (here only mathematics applies, without any intuitive or perceptive nature).

It's here where some controversy of concepts is born. Even though, I consider that drawing is a representation where there is an exercise more or less dependent on the proportionality which perception allows us to understand and translate, and that allows us – directly or not (when we use much more our memory) – to build or rebuild shapes with an aesthetic feature. So, all the representations are drawings if they meet these requirements (the perception and the intuitive translation of proportions that we capture with it) and when it is feasible either graphically or in a sensitive way.

The process of learning to draw involves the exercise of intuitive calculation where some shapes are compared to others and where the dimension of the parts of a shape are compared to the remaining parts; in one word, we compare measures as a mathematical way of thinking. What is the meaning of proportion implicit in any kind of drawing or object/observed shape? It is a measure of dimensions that correlate with one another comparatively. The implicit proportion is an inherent visual thought but it's also a relationship that nature created. For example, "a part of this is the third part of that", "this part is half of that part", et cetera. This is a mathematical thought. What are numbers but measures of quantity? We quantify all kinds of information, and, in this case, even visual information.

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Anđelka Simić

Mathematician, Grammar School Teacher

Anđelka Simić was born in 1986 and is mathematician and was educated in mathematics and computer studies, informatics at the Faculty of Mathematics, University of Belgrade. She worked as a a mathematician teacher at elementary school and in a technical school. She participated at the project: Training farmers to work with computers (within project "Creating conditions for rural development in the municipality of Ub", program "EXCHANGE 3")

Title: Jewellery inspired by optical ornaments
Author: Anđelka Simić

Description

An example of modular structures, which is the borderline between art and mathematics, is asymmetric ornaments derived from several basic elements – "Optical mosaic".

Adequate black-and-white optical ornaments are obtained from basic black-and-white elements.

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János Szász

Painter, Sculptor

János Szász was born in 1964 and is a freelance geometric artist. He graduated from the Óbudai University, and post-graduated from the University of Art and Design in Budapest. His constructivist masters were Csiky and Fajó in Budapest, then MADI founder Arden Quin in Paris. He has participated in hundreds group exhibitions in Europe and many other countries. He has 50 personal exhibitions up to today and his works belong to private and public collections in many countries. He had two times the Pollock-Krasner Grant in New York.

He is a creator and inventor of the Poly-Universe system in the art. His artistic endeavors are characterized by the Poly-Dimensional Plan Painting on yellow-white colors, with many spaces-holes.

He is co-founder of Mobile MADI Museum in Hungary, he edited two art periodicals and many art book in his life. In this time he is member and participant of ESMA – European Society for Mathematics and Art (Paris), Bridges Organization (USA), Experience Workshop – Experience Centered Education of Mathematic (Hungary), Salon des réalités nouvelles (Paris), International Symmetry Association (Budapest)...

Individual exhibitions (selection in 10 years)

2013 Budapest, ELTE Matematikai Múzeum, MAMA Galéria Vác (H), Madách Imre Művelődési Központ

2012 San Francisco (USA), Don Soker Contemporary Art: Haraszthy200 Festival, wine paintings Helsinki, Galleria U, Unkarin kulttuuri- ja tiedekeskus: MOBIILIT ja MODUULIT

2011 Eger (H), Vitkovics-ház: KEPES/OROSZ/SAXON/KAPITÁNY - Az interdiszciplinaritás esztétikája Vác (H), Piarista Gimnázium: ŰRŐK Széchenyi István emlékére Budapest, Óbuda Universiti: POLY-UNIVERSE of SAXON Dallas (USA), Museum of Geometric and MADI art: Konstructivists from East-Central Europe

2010 Budapest, Gallery B55: POLY-UNIVERSE of SAXON Veresegyház (H), Váci Mihály Művelődési Ház: Kris-tájak és Poli-mezők New York (USA), Jardin Galerie a l'ouest Pécs (H), House of Civil Communities: POLY-UNIVERSE of SAXON

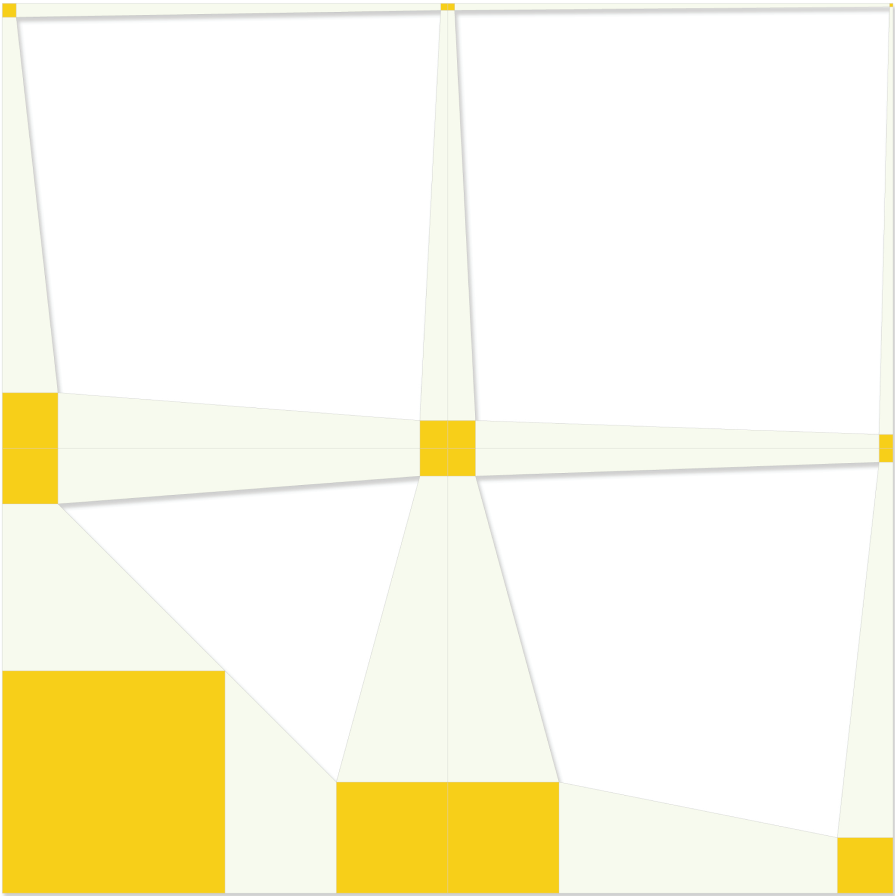
2009 Kaposvár (H), Kaposvári Egyetem Campus: Poliuniverzum Pécs (H) Cella Septichora: Szakrális Geometria

2008 Fortaleza (BR), Museu de Arte contenporanea Győr (H), MTA-MADI Galéria: szupremadik

2007 Ettlingen (D), Galerie Emilia Suciu: polydimensionale Arbeiten Montigni (F), Le Conservatoire des Arts Plasctiques Budapest, KAS Galéria: „supre-MADIsm"

2006 Košice (SK), Východoslovenská galéria: Polydimenzionálne polia Nyíregyháza (H), ZIG Galéria

2005 Győr (H), MTA-MADI Galéria Paris, ORION centre d'art, géometrique MADI



Title: SAXON: The Poly-Dimensional Fields and Spaces
Author: János Szász

Description

As is generally known, Constructivist geometric artists, including me, work with geometric forms. While working, it often happens that if we place geometrical elements of varying size or proportion, but of similar form, on a sheet of paper, our eyes will perceive the connections between large, small and even smaller elements in perspective. We perceive the starry sky, the plane projection of the Cosmos perceptible for us, in a similar way, where we see the nearer celestial bodies bigger, the further ones smaller. In reality the bodies that look bigger may not be bigger than the others. In our present experiment, however, the plane forms, i.e. those trapped in two dimensions, possess the parameters in correspondence with their actual scale. What looks the biggest is the biggest and what looks the smallest is the smallest.

The question arises, what happens if we connect and combine the same forms? Take the square - the most abstract geometric form – as a starting point. Let us choose outward building as direction of progress (exterior = adding to the area), marking the corners as connecting points. We attach smaller squares obtained from the previous form in 1 : 3 proportion to each corner. Let us repeat the process several times. We can see that it is possible to attach four smaller squares to the first one, and three squares to the free poles of the four squares, and so on to infinity...

In the meantime the area of the original square ($T_0 = 1$) has been expanded $T_3 = 1+[4/9]+[4/9\cdot 3/9]+[4/9\cdot 3/9\cdot 3/9] = 1,64197\ldots$ times in three steps, while the number of squares has increased to $D_3 = 76$. We can get the further number of pieces by the simple formula $D_{n+2} = 5 + 4[3 + 3^2 + 3^3 + \cdots + 3^{n-1} + 3^n]$. If a means the segmentation of sides, that is 2, 3, 4, 5 etc., and n means the number of connection rings, then we can use the formula $T_n = T_0 + [4/a^2]\cdot[1 + 1/a + 1/a^2 + 1/a^3 + \cdots + 1/a^{n-1} + 1/a^n]$. We can point out that if ($n = \infty$), $T_n < 2$, that is, much as our new form tends to multiply itself up to infinity, it cannot double itself.

However, we can also see that it is a system creating itself on the basis of its own laws - perspective ceases to be effective, and we arrive at new structures constituted by the different forms attached to one another. During the past thirty years, studying these basic geometrical shapes (the square, the circle, the triangle) I have named these image structures 'poly-dimensional fields'. Now I had the analogy of my childhood observations in nature, since the 'poly-dimensional fields' thus emerging are able to model the abundance of nature (trees, blood and water systems, crystals, cell division, etc.) and the infrastructural growth of human civilization (networks of roads, pipe systems, networks of communication, etc.). On the other hand, they can represent the dimension structures of atomic and stellar systems, which have a similar structure, but are realized on extreme scales.

The systems based on their own laws queried individual creative principle, therefore I gave up the didactics of mathematics since as an artist, I had not only logical but aesthetic construction requirements as well. After this my works of art became so-called condensed pictures, universal event-figures, since it is physically impossible to represent all the stations in the infinite process. With proper respect, I can emphasize or rearrange certain parts without causing harm to the essence; thought will then glide out anyway, skipping on the biggest or smallest element of the open system.

There is only one step towards the creation of the poly-dimensional space from the poly-dimensional field taking shape from the squares as seen in the previous chapter. There is nothing else to do but replace the plane figure with a corresponding cube, then attach $1, 1/27, 1/729, 1/19683 \ldots [1/(a^3)^n]$ size cubes to each possible corner point, the sizes deriving from the $1 : 3$ proportion of the previous scale, and continue the process to infinity.

This poly-dimensional space-construction will fire your imagination. At first sight, it looks like an unimaginable crystal structure, in which microscopic systems are connected step by step, in an indirect way to macroscopic worlds. In order to have a clearer insight into the interconnectedness of spaces or dimension structures on various levels, I placed identically formed chess pieces on a poly-dimensional field consisting of squares. The proportion of the pieces should follow the different sizes of the planes. Then I constructed two wooden stools of sixteen legs each, continuing the splitting of the legs mentally $(4, 16, 64, 256, 1024, 4096, 16384 \ldots 4^{n-1}, 4^n)$ up to infinity (Figure 3). The infinite-legged chair is a concomitant of the poly-dimensional chess. It would not be possible to break out of the parameters of our present world unless our thoughts rested on such an object. In a physical sense, the infinite number of legs is not a very reassuring idea, since the segmentation of the plane involves a smaller and smaller surface for the legs to support themselves on, and reaching infinity ($n = \infty$) the plane crumbles ($T_n = [4/9]^n$) and the legs rest on an infinite number of points with no dimension. Thus, getting at the infinite-legged chair we can speak about the singularity of the chair, that is, about a legless chair, or as I named it, footless chair.

After the completion of the Dimension Chess (Figure 4), I sank onto an infinite-legged dimension chair, and, after having taken a short rest, it occurred to me that this game does not follow the usual stereotypes. The chess table lying in front of me is a poly-dimensional field, practically the horizontal projection of the micro- and macro-world's vertical texture. One of the pieces lined up is me, and I can move about in the unfolding Poly-universe freely, by disposing of the parameters of the actual dimensions at every single move.

Contact: `saxon-szasz@invitel.hu`
`www.poly-universe.com`
`www.saxon-szasz.hu`

Ron Wild

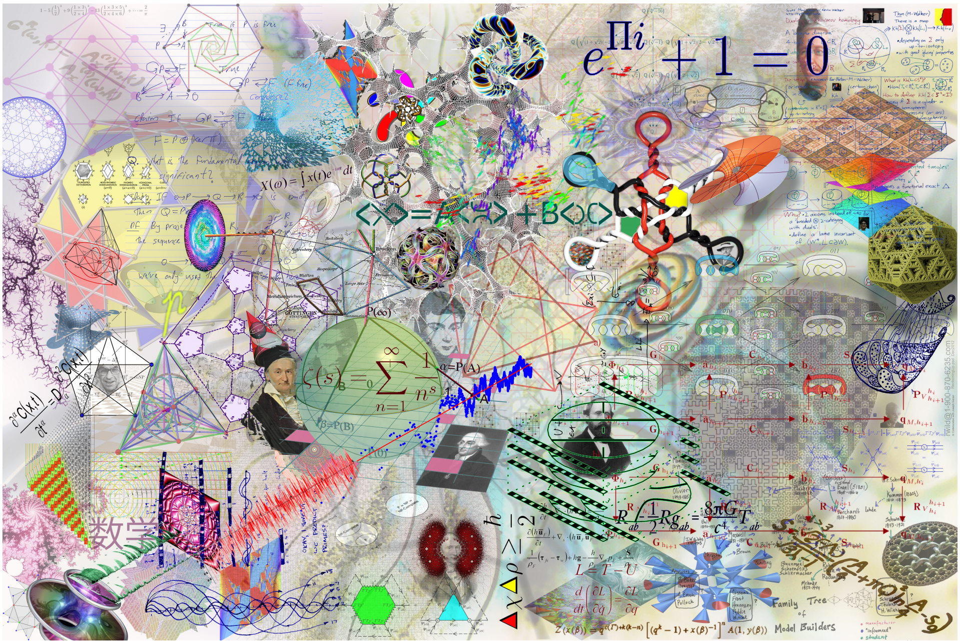
Digital Artist

Dr. Joseph Geraci

Mathematical Physicist

Ron Wild Aka Dr. Joseph Geraci is a mathematical physicist working in medicine and drug discovery. He uses mathematical structures to strip away the cacophony of noise that is inherent in medical data sets where the goal is the ability to predict the most effective treatment for individual patients. Currently he works on several cancer projects and mood disorder treatments. He utilizes graph theory, topology, geometry, statistics, dynamical systems theory and beyond.

Ron Wild has quickly established a readily recognizable and memorable art style, unlike anything done by anyone else in the world. At a glance people know it's a Wild map when they see one. Even members of the general public typically spend an inordinate amount of time studying his art in great detail. They are fascinated both by the resulting imagery and the time and attention to detail required to generate it. In large juried show competitions his work has received notable awards and honorable mentions. In an art history context, he likes to think that his imagery is what innovators like Kandinsky would have produced if he had access to digital technology and Photoshop software.



Title: Reckoning
Chromogenic original on Canvas
60 x 90 cm
Author: Ron Wild and Dr. Joseph Geraci

Description

Reckoning is the result of an effort, which attempted to create an aesthetically pleasing piece involving an explosive brew of mathematical imagery. Many of the images are unlikely to be understood by the general public, however the hieroglyphic characteristic of modern mathematics is none the less, beautiful. The piece features characteristic elements from geometry, topology, combinatorics, physics, analysis, and algebra. Mathematics is an endeavor of pure creativity confined to a purely adamant cage of diamond hard rules. Here we utilize Ron Wild's digital style to extract the beauty of the forms used in mathematics accompanied by the rigor of the subject matter. We wanted to share a portion of this wondrous universal landscape that affects our lives through our technologies and beyond. The rules that bind the universe together are embedded within - the same rules that unify all of us.

Contact: `mrowade@gmail.com`

Ron Wild
44A Castle Frank Road
Toronto, ON M4W 2Z6

Art Collectives

Project – Password

[keys – codes – signs - sounds]

is an interdisciplinary and international artist collective which presented its works in USA, China, Mexico, Italy etc.

Writers, artists and musicians are cooperating worldwide to create a joint art project. Internet is the primary medium. Writers choose fragments from their work which are then put online on our website. The artists draw from this pool of texts. Whatever inspires them passes to the next stage.

password project = quantum x (text fragments) turns to x+1+2+3+4+5+6+... The basis for the project are texts (x). Images inspired by texts are computer-generated and modified ("painted over") by any number of artists $(+1+2+3+4+5+6....)$

Different from real paint-overs, originals and all phases of the pictures are preserved and remain visible. Instead of conventional tools the artists use IT tools for visualizing the text fragments. Every picture they create is uploaded on the password project homepage and all artists get informed about new works. All password project artist may then alter the pictures, react with texts or music compositions.

The result is a complex construct based on the participation of many writers and artists. All the created works of art can be uploaded by password artists themselves.

This means that the complete process is shown online. Series relating to texts can be from individual artists or can be international interpretations of text or even a MATRIX in a reduced, web-appropriate scale, so that everybody has always access to information about the current state of the project work. All created data files are centrally collected and organized in series on the password project homepage, in order to keep the process always transparent.The whole process is online and in a matrix you can observe the interdisciplinary work. After a working and collecting phase of one year (2005), password project shows its work on international exhibitions. *Each exhibition is based on intercultural dialogue*, i.e. artists from each country we exhibit are invited to join password project.

Sponsors/Partners

Styria /Culture Forum / Firma Wilhelm Schmidt Stahlbau

Copyrights-Urheberrecht

Authors

Marco Albertazzi, Rodolfo Anaya Larios, Susan Castillo, Ingrid Coss, Ramon Farrés, Kurt Flecker, Gabriele Foissner Weinländer, Gerald Ganglbauer, Petra Ganglbauer, Walter Grond, Walter W. Hölbling, Santos Lopez, Daniel Thomas Moran, Theres Moser, Ruth O'Callaghan, Gabriele Pötscher, Dorine Ratulangie, Wolfgang Ratz, Stefan Schmitzer, Sabine Scholl, Charlotte Strobele, Aura Maria Vidales Ibarra, Peter Wolf, Dietmar Wächter

Artists

Robert Bodnar, Christine Brunner, Malgorzata Bujnicka, Lily Chan, Beata Ciunowicz, Jörg Dobrovich, Felipe dos Santos, Lei Feng, Lukas Friedrich, Teresa Frodyma, Renée Gadsden, Maria Luisa Grimani, Dirk Huylebrouck, Yuko Ichikawa, Heidi Inffeld, Emanuel Jesse, Bahman Kalantari, Beate Landen, Slawomir Lipnicki, Ruth Mateus Berr, Consuelo Mendez, Moje Menhardt, Waltraud Mohoric, Stephan Rauch, Radmila Sazdanovic, Matthias Silveri, Tatia Skhirtladze, Petra Suko, Jing Sun, Herta Tinchon, Franco Trasatti, Christoph Ueberhuber, Shobha Untersteiner, Gerald Wenzl, Heliane Wiesauer-Reiterer

Musicians

Igor Lintz-Maues , Zbigniew Bargielski, Rupert Huber, Friedrich Moser, Michael Moser, Xinshun Xu

Description

The project LEONARDO was designed for a conference and exhibition organized by The Faculty of Architecture of the KU Leuven (Dirk Huylebrouck). The Symposium "Leonardo da Vinci" was held at the MUNTPUNT in Brussels on 4/5th of April 2014 (`http://www.wiskunst.be/`). Members of Password Project worked on the topic of Leonardo/Leonarda:
Dirk Huylebrouck, Ruth Mateus Berr, Lei Feng, Moje Menhardt, Waltraud Mohoric, Daniel Thomas Moran, Radmila Sazdanovic, Heliane Wiesauer-Reiterer, Gerald Wenzl

Leonard

Leonardo Flies Home

Leonardo Da Vinci, took
his place across the tiny isle
from me, and buckled
the belt of his seat.

I was certain it was him.
Too timid to speak,
I listened as he told
the attendant in the
blue-black skirt-suit,
he was returning
from a long overdue visit
with his kid sister in Phoenix.

The New Mexico sun
stayed red and settled upon
the bareness of his cheeks.
The layover at Kennedy had
granted him just enough time
to grab a Coke and a slice.

He studied the movements of
her calves carefully as she
trolled ahead in the cabin,
gauging all of the tender
architectures of her sway.
Sketching upon his napkin,
sipped his orange juice, and
leaned back into a reverie.

Beneath his hair, wiry and limp,
Against the raggedy blanket of his beard,
I could almost hear the notes
of machines assembling in his head.

The gears and wings, the
rockets and parachutes, the
motions that made the
birds lift skyward, and those
contraptions of battle
that would live to eclipse
horse and sword and shield.

Traipsing the heights of
the atmospheric perspective,

describe Earth's lights reflected
against the cold black of the Moon.

Devoted as I have been to the
celebration of life's soft edges.
I thought to touch his shoulder,
hoping we might speak.

I longed to understand the
miracles of chisel at stone, how the
oily applications of ground pigments,
became lambency and shadow,
The virgin he settled among the rocks.
The tenderness in the eyes of his angels.
The satin cheeks of the Italianate ladies.

The despair of Jerome in his wilderness.
Judas clutching his purse on the wall in Milan.
The mounts of his imagination in pitched battle.

Soon, my time had past, our dinners arrived,
and I was left to the contentments of flight
and a wandering, wondering mind.
Below us the ascending cloud mountains,
below them, the dappled cerulean sea.

2014 Daniel Thomas Moran

Daniel Thomas Moran

Poet, Dentist

Poet Laureate Suffolk County, New York 2005-2007. Daniel Thomas Moran, born in New York City in 1957, is the author of six volumes of poetry, the most recent of which, Looking for the Uncertain Past, was published by Poetry Salzburg at The University of Salzburg in 2006. He earned a Bachelor's Degree in Biology from Stony Brook University (1979) and a Doctorate in Dental Surgery from Howard University (1983). He has read widely throughout New York City and Long Island and has done readings in Ireland, Italy, Austria, Great Britain, The Library of Congress, and at The United Nations.

His work has appeared in such prestigious journals as Confrontation, The Recorder, Nassau Review, Oxford, National Forum, Opium, Commonweal, Parnassus, Opium, Istanbul Literature Review, Sulfur River, Mobius Pedestal, Rattapallax, LUNGFULL, Poetry Salzburg Review, Prairie Poetry, The New York Times, The Journal of The American Medical Association, and The Norton Critical Anthology on Darwin.

He was the subject of a profile on New York's Public Television station WNET in 2006. From 1997-2005 he served as Vice-President of The Walt Whitman Birthplace Association in West Hills, New York where he instituted The Long Island School of Poetry Reading Series and has been Literary Correspondent to Long Island Public Radio where he hosted The Long Island Radio Magazine.

His work has been nominated for a Pushcart Prize on six occasions. He was profiled on New York Public Television's Setting the Stage and on The Poet and on The Poem from The Library of Congress hosted by Grace Cavalieri. He was profiled in the 2009 edition of Poet's Market and is a member of the prestigious New England Poetry Club. A selection of his poems were read in translation on Romanian Public Radio in 2008. His collection, "From HiLo to Willow Pond" was translated into Romanian by Iulia Gabriela Anchidin at The University of Bucharest. He is a participating writer to The Password Project, an international collaboration between visual artists and writers based in Austria. In 2005 he was appointed Poet Laureate by The Legislature of Suffolk County, New York the birthplace of Walt Whitman.

His work has been translated into German, Spanish, Romanian, Chinese and Italian. He has been listed in Who's Who in America, The International Who's Who, and The International Who's Who in Poetry. He was profiled in the 2009 Edition of Poet's Market. He is a member of PEN American and has been ordained a Celebrant by The American Humanist Association. He Edited of The Light of City and Sea, An Anthology of Suffolk County Poetry 2006 (Street Press). His collected papers are being archived by Stony Brook University where he also serves on The Dean's Council at The Frank Melville Library. His newest collection of poems, "A Shed for Wood" was just released by Salmon Poetry in Ireland, and a bilingual edition in Spanish, translated by Mariela Dreyfus of New York University will be released by Diaz Grey Ediciones in New York City in the Spring of 2014.

He has recently retired as Clinical Assistant Professor at Boston University's School of Dental Medicine. He and his wife Karen live along the Warner River in Webster, New Hampshire.

Homepage: `http://www.danielthomasmoran.net/`

Address:
141 Dustin Road
Webster, NH 03303
USA

Feng Lei

Architect, Artist, Designer

FENG LEI was born in 1976 in Xuzhou, Jiang Su, P. R. China

Occupation

Professional Affiliations
03. 2008 Coop Himmelb(l)au Wolf D. Prix & Partner
ZT GmbH / Vienna, Austria
Architect

2001–2003 China University of Mining & Technology Architecture Institute / X. Z. J. S. P. R. China Interior Designer

2001–2002 „Da Yang Architecture Design Institute" Shanghai, P. R. China Interior Designer

Teaching Affiliations

1999–2003 China University of Mining & Technology, Architecture Institute / X. Z. J. S., P. R. China Assistant Professor

Education & Training

03. 2008– Study at University of Applied Art Vienna, Austria, Art Education and Communication

1994–1999 Study at Chinese University for Mining & Technology, Architecture Institute / X. Z. J. S., P. R. China

Selected Exhibitions

09. 2008 Jugendstiltheater am Steinhof, Wien
11. 2007 Password Project / Washington D.C. U.S.A 812 7th Street, NW Washington, D.C. 20001-3718 U.S.A Goethe-Institut Washington & Austrian Cultural Forum DC
09. 2007 Password Project / Beijing P. R. China Beijing 4. Dangdai International Art Festival 2007 (DIAF)
08. 2007 Shadow Report – It depends where you stand Short-film Evening from Works of Art-University. (Wien, Budapest, Belgrade, Zurich, Maidstone) Taban Art Cinema, Budapest, Ungarn.
12. 2006 Shadow Report The Next Vienna / New Crowned Hope, Mozart Jahr 2006 Festival Center Künstlerhaus, Vienna, Austria.
06. 2005 Light Space / Space Light, Heiligenkreuzer Hof, Vienna, Austria

Selected Publications

02. 2013 Tangeled Up in Blue — Martin Luther Church, Hainburg, Austria / Interior Design-China
03. 2012 Coop Himmelb(l)au — World Top Architectural Studio / Text Chinese Version
09. 2007 易 / 移北京当代国际艺术节 2007 (DIAF) / Game Process / Beijing 4. Dangdai International Art Festival 2007 (DIAF)
12. 2006 The Next Vienna / New Crowned Hope, Mozart Jahr 2006
12. 2002 Lonely Courtyard, Quiet Altars — Search Disappearing Old Hu Bu Shan Residence< Interior Design and Decoration>
10. 2000 Strategy and Education of Design

Address:
Badgasse 24/24, 1090 Wien

Ruth Mateus-Berr

Artist, Scientist, Designresearcher

Ruth Mateus-Berr is Professor at the University of Applied Arts Vienna at Institute of Art Sciences and Art Education, Dept. Design, Architecture and Environment and at Institute of Art and Society Dept. Master Program Social Design - Arts as Urban Innovation. She is artist, scientist and design researcher on the interface of art/design & science. She is author of several articles and publications on her research interests which includes: Inter/transdisciplinary art & design, art & design education, Design Research, social design, interdisciplinary approaches, multisensual art & design transfer, maths & design & fashion, intercultural and social projects, member of Design Research Society, and Research directory of Sensory Studies. Numerous exhibitions, workshops and publications in Austria and abroad. Lives & works in Vienna.

Exhibitions

2014 PROJECT PASSWORD MUNT-PUNT, Brusseles, Belgium PROJECT PASSWORD MIKSER HOUSE, Belgrade, Serbia Snapshots of Design Patterns Macy Gallery 525 WEST N.Y., USA

2013 COOL CITY VIENNA. Empathy for climate-Hangovers in a city, Biennale Internationale Design Saint-Etienne, France Textile Design as Social Fabric, with Inspiration from '4-layers of Sari' and the Material Culture of SilkKasetsart University Bangkok, Thailand Don't Worry: Staged Photography: bit:NC/Vienna, Austria Social Design Project: 4 layers of sari, -ELIA Biennal Conference Vienna, MQ Vienna; -Conference of the European League of the Institutes of the arts, Vienna, Austria -Heiligenkreuzerhof Sala Terrena; Symposium „Mythos Praxis" Vienna, Austria

2012 Social Design Project: 4 layers of sari, University of Porto/Portugal Social Design Project: INTERACCT, Performance: Royal Institute of Technology (KTH), Stockholm and The Centre for School Technology Education (CETIS), 2012, Linköpings Universitet /Sweden

2011 www.earthwatercatalogue.net (Uwe Laysiepen (Ulay)) 2011- today Scent of Ruth: Flipbook of smells, different fragrances used by the artist, Olfactory Flipbook, Retrospective of the Austrian Flipbookfestival, Atelierhaus Salzamt, Linz/Austria Gleichzeitig hat die Zeit keinen Reflexionsraum (Simultaneously time has no space for reflection): Energie-SPUREN III aus der Perspektive von Kunst : Politik : Wissenschaft : Wirtschaft (Traces of Energy – from the perspective of art, politics and economy.) K3, Alte Schuhfabrik, Gewerbepark 8212 Pischelsdorf/Styria/Austria

2010 Cool City Vienna: Cool Design: implementable Konzeptkunst, staged photography, installation, mixed media Project Vienna: A Design Strategy. How to React to a City? MAK, Departure Call,; Design Award MAK, 2010, Museum of Applied Arts Vienna, AUSTRIA 100 coolest cities. Global Cooling 1/100 Vienna, 2/100 New York, Galerie 18/Franz Morgenbesser/Vienna

2009 SANTA BARBARA: smell/sound installation, pendulum, steel, St. Stephans Cathedral (Stephansdom,Vienna, Austria) Numbers and Mismeasurement. Socialdarwinism and rascism of today. Staged Photography, Performance, Künstlerhaus Vienna, Austria Why lilac and concer will be expelled from Vienna? smell installation, space installation, VIENNART: Wiener Gerücht, MUSA (Museum auf Abruf) Vienna New Beautox: staged photography, Paint4life, 2009, Cultural Forum Berlin, Germany Ein Stein liegt mir am Herzen und Steine geben statt Brot. (There is a load on my heart; Stones instead of bread – rotation of proverbs and methaphors)Die andere Ausstellung: Die andere Hälfte. (The other exhibition: The other half) Kunstwerk Krastal. Galerie Freihaus Villach, Kärnten, Austria Password Project, Casa

Strobele Borgo Italy

2008 ReMember your Heart, VIENNART, MUSA (Museum auf Abruf) Vienna Password Project: Multimedia Installation, Cultural Forum Milano, Italy

2007 Password Project: Multimedia Installation, DIAF. Game Process. Beijing Dangdai International Art Festival, 798 district (curated by Huang Rui)

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Homepage: `www.ruth-mateus.at`

Address: Silbergasse 42/4, 1190 Vienna, Austria

Moje Menhardt

Artist

Moje Menhardt is Austrian and is living and working in Weitenegg and Vienna, Austria. Before she had been living in Buenos Aires, Rio de Janeiro, Munich, Hamburg, Cologne and in Eindhoven, Holland. After finishing College, she studied Law in Vienna and later Painting at the Koninklijke Akademie voor Kunst en Vormgeving s'Hertogenbosch (Holland), finishing studies at the Akademie der Bildenden Künste, Vienna (Austria), Meisterschule für Malerei Professor Eckert, 1980 with a Diploma.

Wolfgang Hilger: ... For the process of her work, Moje Menhardt needs to be free for associative thinking, for the admission of memories, and for perpetual variation. She adapts the compass of her artistic elements to the subject at hand. She works in series, and her chosen concept is only valid within a particular theme. Her style is variable on purpose, nevertheless, her hand is unmistakeable ...

Exhibitions (choice)

2009 BILDER OHNE WORTE, Künstlerhaus Vienna, São Paulo (Brazil), Centropalia Bad Radkersburg and Straden, alpha Vienna, MUSA Wien (participation „clouds up high - stark bewölkt"), October 2009: Ausstellungsbrücke St.Pölten (Austria)

2008 Foyer Kubinsaal Schärding (Austria), Gotischer Kasten Gern, Eggenfelden (Germany)

2005 International Symposium Valtice (Czequia)

2004 Art Museum Los Gatos (Kalifornien)

2001-2003 Museums in Quito (Ecuador), Bogotá, Medellin, Ibague (Columbia) and Caracas (Venezuela): IMAGINARY BEINGS SERES - IMAGINARIOS INSPIRED BY Jorge Luis Borges, WEINSTADTmuseum Krems (Austria): DANUBE PAINTINGS

1999 Museum Moderner Kunst Passau (Germany): DANUBE PAINTINGS
1996 Museu Nacional de Belas Artes Rio de Janeiro (Brazil) NÖ Landesmuseum Vienna: SEVEN WONDERS OF THE ANCIENT WORLD VIENEN DE VIENA HOLLEGHA MENHARDT MIKL: Valladolid, Toro, Zamorra, Valencia, Madrid

Corporate collectors: Austrian Government, Vienna, Austria, Austrian National Bank, Museum Moderner Kunst Passau, Germany, Stadt Wien, Stadt Passau, Germany, Stadt Krems, Passauer Neue Presse, Germany, Austrian Industries Vienna, Kapsch AG Wien, Barclay's Bank, Miami, USA, Banco de Bilbao, Miami, USA, Klinik Hirslanden, Zürich, Switserland, Nicolaus SA, São Paulo, Brazil, NÖ Landesregierung, St.Pölten, NÖ Dokumentationszentrum für Moderne Kunst, St.Pölten, Bundeskanzleramt, Wien, Bundesministerium für Kunst, Wien, Sdružení Výtvarných Umečlců Jihovýchodní Moravy, Czequia, Donauversicherung Wien

Homepage: `http://www.menhardt.com/Moje`

Radmila Sazdanovic

Mathematician

Radmila Sazdanovic is Assistant Professor at North Carolina State University, Department of Mathematics.

Accademic Appointments

North Carolina State University
Assistant Professor 2013-
University of Pennsylvania, Philadelphia
Postdoctoral Fellowship 2010-2013
Supervisor Robert Ghrist
KITP Knotted Fields Program, Santa Barbara
Visitor June 2012
Simons Center for Geometry and Physics, Stony Brook
Visiting scholar June 2010
MSRI Berkeley
Postdoctoral Fellowship Spring 2010
Columbia University, New York
Visiting Scholar
Fall 2008
Mathematical Institute, Belgrade, Serbia
Research Assistant 2004-2010

Education

George Washington University, Washington DC
2010
PhD in Mathematics, Advisor J.H. Przytycki
Categorication of Knot and Graph Polynomials and the Polynomial Ring
George Washington University, Washington DC
2007
Master of Arts, Mathematics
Faculty of Mathematics, University of Belgrade
2005
Dipl.Mat. (equiv.)

Fellowships and Awards

AMS Simons Travel Grant
2012-2014
Simons Center Invitee
June 2011
James H. Taylor Graduate Mathematics Prize
George Washington University 2009
Marvin Green Prize
George Washington University 2006
Presidential Merit Fellowship
George Washington University 2005-2010
Scholarships for a promising generation
2000
Scholarship of the Royal Norwegian Embassy in Belgrade

Homepage: `http://www4.ncsu.edu/~rsazdan/`

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Herta Tinchon

Artist, Teacher

Living and working at Gleisdorf,Austria;
born 1931 at Gleisdorf.
Until 1984 working as a teacher.

First education in arts by Kurt Weber. Further training at the summer academy Salzburg(Rivka Rinn),academy Graz (Paul Rotterdam) and other seminars.

Exibitions and partcipations
2007 Steirischer Herbst:"next code love" Gleisdorf
2001 - 2008 donna mobile - project SUPERNOVA
Austria,Italy,Slovenia,Poland,Slovakia
2004 - 2008 project PASSWORD
2002 - 2006 Werkbund Graz - Künstlerhaus and others

Homepage: `www.kultur.at/see/tinchonhtm`

Address:
Johann Josef Fux Gasse 32
8200 Gleisdorf
Austria

Heliane Wiesauer-Reiterer

Artist, Sculptor

Heliane Wiesauer-Reiterer was born in 1948 in Salzburg

CV

1948–57 Argentinien Buenos Aires
1957–68 Marne/Holstein BRD
1968–1972 Academy of Fine Arts Vienna (Prof. Gustav Hessing),
1968-2007 Mitarbeit im [kunstwerk] krastal
1982–1992 Aufarbeitung des Nachlasses von Otto Eder in Zusammenarbeit mit Ernst Reiterer
2004 Foundation of FOCUS [kunst frei raum] Painiting, Sculpture, interdisciplinary Projects
Since 1969 Participation and Organization of divers symposia at [kunstwer]krastal

Awards

1973 Förderungspreis der Österreichischen Nationalbank, Wien
1978 Preis der Bundeshauptstadt Wien beim 16. Österreichischen Graphikwettbewerb Innsbruck
Arbeitsstipendium der Stadt Wien
Arbeitsstipendium des Bundesministeriums für Unterricht und Kunst, Wien
1980 Förderungspreis des Bundesministeriums für Unterricht und Kunst, Wien

1982 1. Preis beim Franz-von-Assisi-heute-Wettbewerb, Krems/A
1984 Arbeitsstipendium der Stadt Wien, Anerkennungspreis des Landes Niederösterreich
1985 Preisträgerin der Länderbank – Galerie Würthle Kooperation, Wien
Förderungspreis des Landes Niederösterreich
1987 Arbeitsstipendium der Stadt Wien
1988 Preis des Landes Salzburg beim 21. Österreichischen Graphikwettbewerb Innsbruck
1989 Förderungspreis der Stadt Wien
2010 Gewinnerin des Kunst-am-Bau-Wettbewerbs Oberstufen-Realgymnasium Neulengbach/A

Art in Public Space

2006 Teilung III (Dorfergrün), Haus an der Traisen – Niederösterreichisches Landespensionistenheim, St. Pölten/A
2009 Die andere Hälfte – Denken in Stein. 42. Internationales Bildhauersymposion [kunstwerk] krastal, Skulpturen in Landschaftsräumen am Dobratsch/A
2010 Kunst am Bau, Skulptur und Bild, Oberstufen-Realgymnasium, Neulengbach/A
2012 Teilung eines Würfels (01) 2005–2012, NÖ Kunst im öffentlichen Raum, Breitenfurt/A

Solo-Shows (Selection)

2013 Malerei, Grafik, Skulptur. Ein Querschnitt seit 1970, Galerie in der Freihausgasse, Villach/A
Elementare Fotografie, Stadtgalerie Klagenfurt, living studio, Klagenfurt/A
Körper-SekundenBilder, Galerie am Lieglweg, Neulengbach/A
2012 Chromatisches Schwarz, Österreichisches Papiermachermuseum, Steyrermühl/A
Notationen, eine Installation, Galerie im Schloss Porcia, Spittal an der Drau/A
Tonalität im Raum, Malerei–Skulptur-Fotografie, Artothek, Krems/A
Konzentrationen, NÖ Dokumentationszentrum für moderne Kunst, St. Pölten/A
2009 primär schwarz, Blau-Gelbe Viertelsgalerie, Zwettl/A
Zwischenräume (mit Rosa Maria Plattner), Kunstverein Mistelbach/A
2006 Imaginäre Räume. Malerei–Skulptur–Objekte–Text, Galerie der Stadt Salzburg, Holzpavillon (Katalog), Salzburg/A
Imaginäre Räume 2006, Blau-gelbe-Viertelsgalerie im Schloss Fischau, Bad Fischau-Brunn/A
Imaginäre Räume. Malerei–Skulptur, Galerie ARS, Brünn-Brno/CZ
2005 Reduktion in Malerei und Skulptur 1980–2005, Landhausgalerie, St. Pölten/A

2003 Teilung. Papierarbeiten–Malerei–Skulptur, Galerie Göttlicher, Krems/A
2001 Innere Ordnung – äußere Ordnung. Papierarbeiten–Skulpturen, Kunstwerkstatt Tulln/A
2000 Polarities/Polaritäten, Arbeiten auf Papier, Haus der Kunst der Stadt Brünn (Broschüre), Brno/CZ
1999 Polarities/Polaritäten 1990–1998, Austrian Cultural Institute London (Katalog), London/UK
Polaritäten 1990–1998, Stadtgalerie im Elbforum, Brunsbüttel/D
Polaritäten 1990–1998, Stadtmuseum St. Pölten und Niederösterreichisches Dokumentationszentrum für moderne Kunst, St. Pölten/A

Exhibitions (Selection)

2014 Anklopfender Widerstand – Hommage an Jörg Schwarzenberger, Kunst in der Natur am Wachtberg, Gars a. Kamp/A eyes on – Monat der Fotografie Wien, Focus kunst frei raum Wien am Getreidemarkt, Wien/A
2013 making of secession | edition /150. Die Kunstedition der Mitglieder der Secession (Broschüre), Wiener Secession, Wien/A
2012 Girls Girls Girls, Galerie Lang Wien, Wien/A Am selben Tag, 10 Jahre Blaugelbezwettl, Kunstverein Zwettl, Galerie Blaugelbezwettl, Zwettl/A Ausgesucht, Galerie Lang Wien, Wien/A 10 Jahre Artothek, Artothek, Krems/A 25 Jahre Galerie artmark, Galerie artmark, Wien/A
2011 quer, Galerie artmark, Wien/A
2010 Die Blöße, Galerie Lang Wien, Wien/A Farbe Licht Maß, Städtische Galerie in der Haderbastei, Ingolstadt/D MASS – eine Annäherung, Kunstforum Salzkammergut, Gmunden/A natur.PUR.2010, Kunstverein Kärnten (Katalog), Klagenfurt/A 42. Internationales Bildhauersymposion 2009 [kunstwerk] krastal, Buchpräsentation, Wiener Secession, Wien/A
2009 Wiener Gerüchte – das Private und das Öffentliche, MUSA, Wien/A Die andere Hälfte. Künstlerinnen im [kunstwerk] krastal 1968–2009, Galerie der Stadt Villach, Galerie Freihausgasse, Villach/A Die andere Hälfte – Fokus Bildhauerinnen denken in Stein. 42. Internationales Bildhauersymposion 2009 [kunstwerk] krastal (Katalog), Krastal/A Wie im Traum. Aus der Sammlung des Museums der Moderne Salzburg, MdM – Rupertinum (Katalog), Salzburg/A Spotlight – Neuzugänge seit 2006, Museum der Moderne (DVD), Salzburg/A FrauenBilder, Galerie Lang, Wien/A Project Password, Casa Strobele, Borgo Valsugana/I
2008 Eroticon, Galerie Lang, Wien/A über zeichnen, Kunstraum Arcarde, Mödling/A zeitraumzeit, Künstlerhaus Wien (Katalog), Wien/A Erfahrung[en], Galerie artmark, Wien/A The Faces of Eve. International Women's Exhibition 2008, The Bible Museum (Katalog), Tel Aviv/Israel K08. Emanzipation – Konfrontation. Kunst aus Kärnten 1945 bis heute (Katalog), [kunstwerk] krastal, Krastal/A Project Password, Forum Austriaco di Cultura, Palazzo del Liberty, Milano/I
2007 99mal21mal21. Querschnitt zur Kunst der Gegenwart, Kunstforum Salzkam-

mergut (Katalog), Gmunden/A Aus & auf Papier, Österreichisches Papiermachermuseum, Steyermühl/A Zu Gast: Kunstforum Salzkammergut, Deutschvilla, Strobl/A 1.X–tended, Medienkunst aus Österreich, Schiele-Museum (Internetkatalog), Neulengbach/A Meisterwerke auf Papier, Galerie für zeitgenössische Kunst im Kunsthaus Rapp (Internetkatalog), Wil/CH

2006 Bunte Steine, Nachlese. Kunstpositionen zu Adalbert Stifter aus Deutschland, Tschechien, Österreich, Kammerhofgalerie der Stadt Gmunden (Katalog), Gmunden/A 5. SCHIELEwerkstattFESTIVAL, 2006, Verbotene Blicke, Schiele-Museum (Internetkatalog), Neulengbach/A Spotlight – Neuzugänge seit 2006, Museum der Moderne (DVD), Salzburg/A

2005 Expedition Skulptur 2 – Gefährten. Kärntner Künstler in Südtirol, Galerie Prisma (Katalog), Bozen-Bolzano/I 4. SCHIELEwerkstattFESTIVAL, 2005, Schiele-Museum, Neulengbach/A Reflexion. 30 Jahre Galerie Göttlicher, Galerie Göttlicher (Katalog), Krems-Stein/A Skulpturenstraße durchs Krastal – vom Fluss zum See (Broschüre), Krastal/A 99mal21mal21. Querschnitt zur Kunst der Gegenwart, Kammerhofgalerie der Stadt Gmunden (Katalog), Gmunden/A Bunte Steine. Kunstpositionen zu Adalbert Stifter aus Deutschland, Tschechien, Österreich (Katalog), Regionalmuseum Krumau, Czesky Krumlov/CZ und Museum Kunstforum Ostdeutscher Galerien, Regensburg/D Geschlossene Gesellschaft. 38. Bildhauersymposion Krastal, [kunstwerk] krastal (Katalog), Krastal/A

2004 Phänomen Landschaft, Niederösterreichisches Landesmuseum, St. Pölten/A 3. SCHIELEwerkstattFESTIVAL, 2004, Lengenbachersaal, Stadtkeller u. Schiele-Museum, Neulengbach/A

Homepage: `http://www.heliane.wiesauer-reiterer.com/`

Gerald Wenzl

Artist, Multimedia Artist

Gerald Wenzl was born in 1975 in Vienna, Austria; lives and works in Vienna. 2004 founding member of artists' group MIR (Hannah Swoboda, Gerald Wenzl, Sylvia Winkelmayer)

Studies

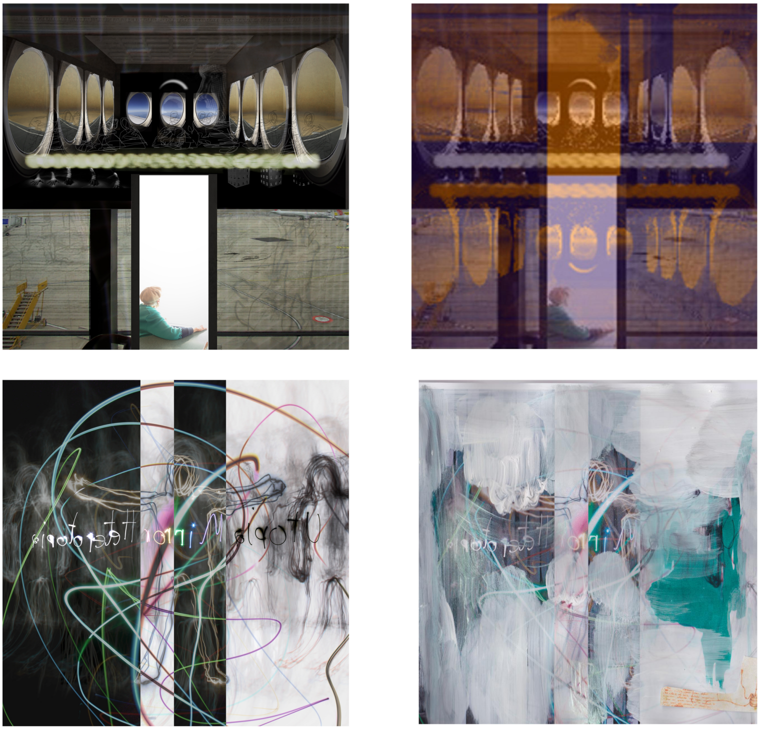
2003–2008 Intermedia Art / Sculpture and Multimedia at the University Of Applied Arts Vienna Studies in Political Science and Mass Media / Communication Science at the University of Vienna

Selected Projects & Exhibitions

2014 Project Password Munt-Punt, Brusseles, Belgium
Project Password Mikser House, Belgrade, Serbia
2010–2014 Establishment of his ‘Experimental Projection Lab’ in Vienna, Austria (until July 2014)
Extensive research and studies on the Cultural History of ‘Projection and the Arts’
Several projects of projection in private and public spaces (to be shown in 2015)
2010 MIR „System“, Galerie Zeitvertrieb, Vienna, Austria
2009 MIR (Group Exhibition) „Invitation“, Rondo, Graz, Austria
Project Password, Casa Strobele, Borgo Valsugana, Italy
2008 „Run 2008“, Rustenschacher Allee, Vienna, Austria
„Closed Societies“, Diplomausstellung, Universität für Angewandte Kunst, Vienna, Austria
Project Password, Forum Austriaco Di Cultura, Milano, Italy
2007 MIR (Group Exhibition): „The Faceless Gaze“, Brukenthal Museum, Sibiu, Romania „Mediale Aggression“, Schikaneder, Vienna, Austria „misused media“, Sterngasse 13, Vienna, Austria MIR (Solo Exhibition): „Beyond The Borderline“, project space, Kunsthalle Karlsplatz, Vienna, Austria MIR (Group Exhibition): „Der Gesichtslose Blick“, Kunstraum Niederösterreich, Vienna, Austria 2006 „Video des Monats #19“, Kunsthalle Vienna, Ursula Blickle Videolounge, Vienna, Austria „The Essence 06“, Museum für angewandte Kunst Vienna, Austria MIR (Solo Exhibition): „kopf.geld.jagd.“, project space, Kunsthalle Karlsplatz, Vienna, Austria „Heinestraße“, Heinestraße 25/ 4-6, Vienna, Austria

Homepage: `http://www.geraldwenzl.com`

Address:
Luckenschwemmgasse 3
A-1210 Wien
Austria



DANIEL THOMAS MORAN
RUTH MATEUS-BERR
HERTA TINCHON
GERALD WENZL
www.password.or.at

Title: Leonardo/Leonarda
Digital Print
Author: Daniel Thomas Moran, Ruth Mateus-Berr, Herta Tinchon, Gerald Wenzl

Title: Leonardo/Leonarda
Digital Print
Author: Daniel Thomas Moran, Moje Menhardt, Heliane Wiesauer-Reiterer

Title: Leonardo/Leonarda
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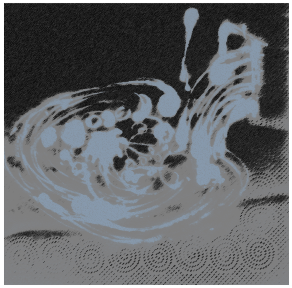


DANIEL THOMAS MORAN
RUTH MATEUS-BERR
FENG LEI
www.password.or.at

Title: Leonardo/Leonarda
Digital Print
Author: Daniel Thomas Moran, Ruth Mateus-Berr, Feng Lei

Contact: `rumabe@chello.at`, `www.password.or.at`

Sama Mara

Artist, Geometer

Sama Mara is an artist and geometer based in London. He was awarded the Barakat Trust Prize at the Prince's School of Traditional Arts where he graduated with an MA in Traditional Arts. He completed his BA in Music and Visual Performance in the University of Brighton. Sama has a wide range of skills and interests including traditional geometry, programming, painting, music theory, studio photography, video editing, fractal geometry and quasi-crystals, all of which inform, inspire, and enable his practice as an artist.

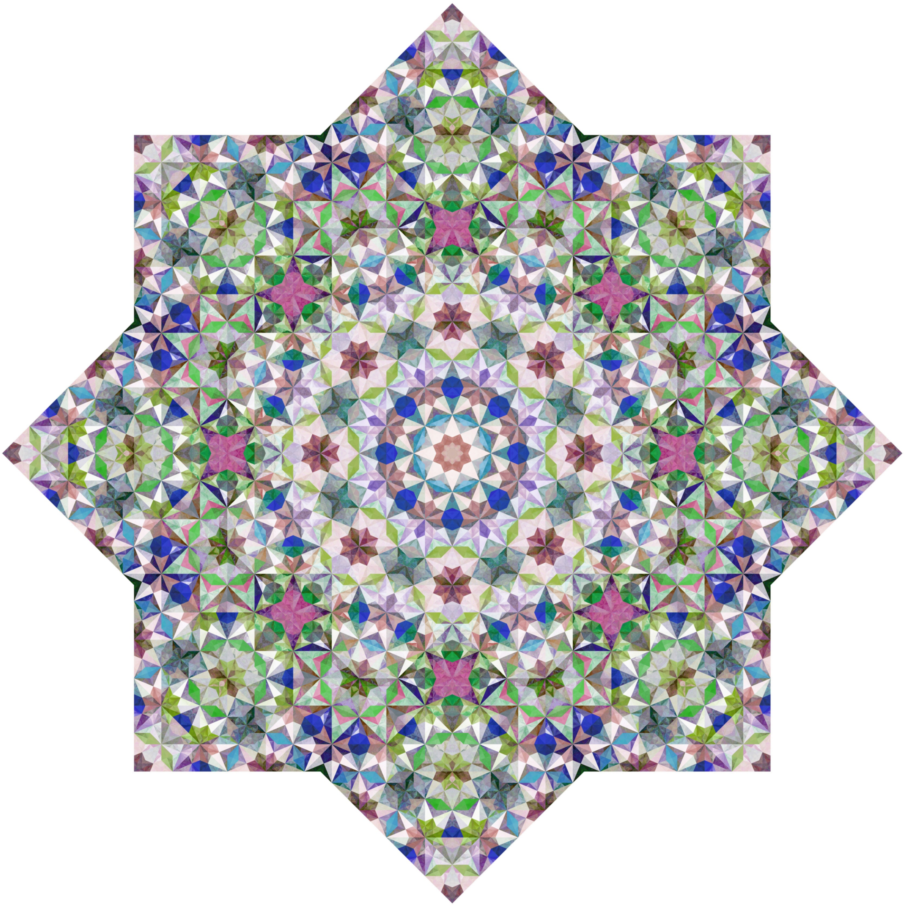
Contact: sama@musicalforms.com
www.samamara.com

Lee Westwood

Composer, Guitarist

Lee Westwood is both a composer and guitarist. His music draws influences from a wide field of cultures and traditions, including Modern Jazz, Folk, and the past 150 years of Classical Music. Lee has studied with a number of composers including Martin Suckling, Martin Butler and Alison Kay. He is currently one of Sound & Music's 'New Voices' Composers 2014-15, and has toured his music extensively throughout the UK and Europe, releasing both a large back-catalogue of solo albums and a wide selection of sheet music. The Musician's Union described Lee as "one of the UK's most exciting and versatile musicians". Lee performed for 5 years with Songlines Award nominees Dizraeli & The Small Gods, and currently tours and records with experimental trio Le Juki.

Contact: www.lee-westwood.com



Title: Octagon I – Flute & Marimba
Screenshot
A Hidden Order Suite – Short Films
Video, 1080p - 28minutes
Author: Sama Mara, Lee Westwood

Link download:
`www.musicalforms.com/mf/downloads/AHiddenOrderSuite-2.mp4`

Andreas Karaoulanis

Computer Scientist, Animation Artist

Andreas Karaoulanis (b. 1980) comes from a Computer Science and Animation background (Newport Wales UWCL and Bristol UWE). His current work deals with interactive media design and animation. He has present his work in Museum Of London, Moscow, Paris and in various Galleries and spaces around Europe. One of his latest projects is bestbefore, an online interactive showcase blog with hundreds of daily visitors. He is member, along with Antonis Anissegos of the duo "Best before Unu", focusing in improvised audio-visual relationship.

His work concentrates on the deconstruction and abstraction of visual narration. He creates animations as anti-narrations that analyze the relation between object and recipient. With various techniques he discovers the interplay between sound and abstraction as well as movement and animation.

Lives and works in Berlin.

Contact: `www.bestbefore.gr`

Patrick K.-H.

Sound Artist, Video Artist, Composer

Patrick K.-H. (Anton Iakhontov, b. 1980) sound artist, video artist, composer. Currently works as electroacoustic music composer, live-acousmatic performer, video artist and animation maker. His wide-range art experiences turns him more into interactive forms and reflects his belief that most of the laws as well as paradoxes of each single media can be mapped to other medias for producing a certain (un)expectable result. Composes music and video for theater. Member of Theremin Center for Electroacoustic Music at Moscow Conservatory since 1999. Master degree, Cum laude diploma, sound producer (GITR'09). Since 2013, student of University of Applied Art, Vienna. Art-director of Media Studio in Alexandrinsky Theater, St. Petersburg, co-founder of Floating Sound Gallery.

Contact: `http://drawnsound.org/patrickkh.html`
`http://soundartgallery.ru/`

Title: SpechtRaum
Video
1280 x 720. 08:36 Min.
Author: Patrick K.-H. / Andreas Karaoulanis