Visual Mathematics in Practice

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Theme of the lesson:	Trigonometry
Place in curriculum: (type of school, grade)	High school, 2^{nd} grade
Age of the students/pupils:	16
Title of the lesson:	Function $y = a\sin(bx+c), a \neq 0, b > 0$

Description of the lesson

Time	Exercises, matters, parts of the lesson	Methods and forms of student activities	Developable competencies
	The amplitude, period, and phase shifts will be explored interactively. This investigation will be carried out by changing the parameters a,b, and c. To understand the effects of each parameter on the graph of the function, we will begin by changing one parameter at a time, then later change all the parameters at once.		
10 min.	We first start with the graph of the basic sine function $f(x) = sin(x)$ The domain of function f is the set of all real numbers. The range of f is the interval $[-1,1]$. Also function f is periodic with period equal to 2π. The graph of f over one period can be sketched by first finding points that give important information such as x intercepts, y intercept, maxima and minima. We need to understand how do the	*Frontal instruction,* *Individual work ,* *Group discussion*	*Image creating skills,* *Looking for connections,* *Problem representation,* *Problem solving,* *Generalization,* *Recognizing relations*

15 min.	parameters a, b and c affect the graph of $f\ (x) = asin(bx + c)$ when compared to the graph of $sin(x)$? The domain of f is the set of all real numbers. The range of expression $bx\ +\ c$ is the set of all real numbers. **Period of *f*** Examples $$y = sin2x$$ $$y = sin\frac{x}{2}$$ For f to complete one cycle (period), expression bx needs to vary from *0* to 2π. $$0\ \leq\ bx\ \leq\ 2\pi$$ Period $T = \frac{2\pi}{b}$. **Phase shift** Examples $$y = sin\left(x - \frac{\pi}{4}\right)$$ $$y = sin\left(x + \frac{\pi}{4}\right)$$ We now consider the whole argument $bx\ +\ c$. For f to complete one cycle (period), expression $bx + c$ needs to vary from *0* to 2π. $$0\ \leq\ bx + c\ \leq\ 2\pi$$ Assume $b\ >\ 0$ and solve for x $$-c\ \leq b\pi\ \leq\ 2\pi - c.$$ $$\frac{-c}{b} \leq\ x\ \leq \frac{2\pi}{b} - \ \frac{c}{b}$$ Period of f is $T = \frac{2\pi}{b}$ c does not affect the period. Let us now compare the cycle $$\left[0, \frac{2\pi}{b}\right]$$ when $c\ =\ 0$ with the cycle $\left[-\frac{c}{b}, \frac{2\pi}{b} - \ \frac{c}{b}\right]$. This indicates that there is a shift of $-\frac{c}{b}$. $-\frac{c}{b}$ is called the phase shift. If $-\frac{c}{b} <\ 0$, the shift will be to the		

	left. If $-\frac{c}{b} > 0$, the shift will be to the right. Examples $$y = sin\left(2x - \frac{\pi}{2}\right)$$ $$y = sin\left(\frac{x}{2} + \frac{\pi}{2}\right)$$		
20 min.	**Amplitude** $$y = -2sinx$$ $$y = -\frac{1}{2}sin$$ Range of $sin(bx + c)$ is $[-1,1]$. Hence $$-1 \leq sin(bx + c) \leq 1$$ Multiply both sides by a. If $a > 0$ $$-a <= asin(bx + c) <= a$$ If $a < 0$ (change symbols of inequality) $$-a \geq asin(bx + c) \geq a$$ or $a \leq asin(bx + c) \leq -a$ We can say that parameter a affect the range of f which can be written as $[-\lvert a\rvert , \lvert a\rvert]$. $\lvert a \rvert$ is called the amplitude. Examples $$y = -2sin\left(2x - \frac{\pi}{2}\right)$$ $$y = -\frac{2}{3}sin\left(2x + \frac{\pi}{3}\right)$$		

Summary
Inspiration came from workshop Mathematical Modeling with Geogebra by Đurđica Takači. Using computer applets, students visualize and explore graphs to explain the effects of transformations (amplitude, period, and phase shift). Students were enthusiastic and motivated to work on this type of lesson.

Supplements	
Used materials:	*Projector, computer, Geogebra* *asin(bx+c).ggb*
Photos:	