Visuality & Mathematics: Experiential Education of Mathematics through Visual Arts, Sciences and Playful Activities

Mathematical modeling in education

Sunčica Zdravković, Đurđica Takači

University of Novi Sad,

Novi Sad

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Content

- Đurđica Takači
  - Mathematical model, modeling,
  - Modeling-Technology in education,
- Sunčica Zdravković
  - Modeling cognitive functions
  - Visual Ilussions

David Tall: Emeritus Professor University of Warwick,

Integrating History, Technology and Education in Mathematics, July 15, 2013

Mathematics begins from our perceptions of and actions on the natural world around us,

first through **practical mathematics** as we build on our perceptions of shape and space and our actions in counting and measuring that lead to the operations…..

We use language to describe objects, and perform operations, such as constructions in geometry, and counting, measuring and more sophisticated operations in arithmetic, algebra, calculus and other areas of mathematics.

A ***mathematical model*** *http://en.wikipedia.org/wiki/Mathematical_model* is a description of a system by using mathematics.

Mathematical modeling is the process of developing a mathematical model.

Mathematical models are used in sciences, engineering, social sciences, economy.

A model may help to explain a system and to study the effects of different components, and to make predictions about its behavior.

Mathematical models:

- differential equations (their solutions)
- dynamic Systems
- statistical models,.....

Mathematical modeling in education

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Mathematical modeling in education

Aslan Doosti , Alireza M. Ashtiani

The idea of using mathematics modeling in mathematics education began in the mid-‘70’s at PUC-RJ, by Aristides C. Barreto.

The mathematical modeling deals with the process of creating a model that should then be applied in solving the mathematical problems.

There is no unique definition of what is mathematical modeling.

The mathematical model is obtained when we translate the problems from the hypotheses language into the formal symbolic language, in other words, when we extract the essence of the problem situation and turn it into systematic mathematical language.

Teaching and Learning Mathematical Modelling with Technology

Keng-Cheng Ang

The approaches to teaching mathematical modelling have been influenced by the development and introduction of technologies such as graphing calculators and computer software.

Many researchers and teachers have reported the successful use of technology in introducing mathematical ideas through exploration and investigation.

Not surprisingly, the use of technology continues to prevail in the mathematics classroom at all levels.

Galbraith, p., Stillman, G., Brown, J., Edwards, I.,

Facilitating middle secondary modelling competencies.

In C. Haines, P. Galbraith, W. Blum, S. Khan (eds.), Mathematical modelling (ICTMA12): edu., Eng. Economics, chichester, UK: horwood (2007), 130-140

Paper-Stilman

....However, a "technology-rich teaching and learning environment" (TRTLE) affords new ways of engaging students in learning mathematics.....

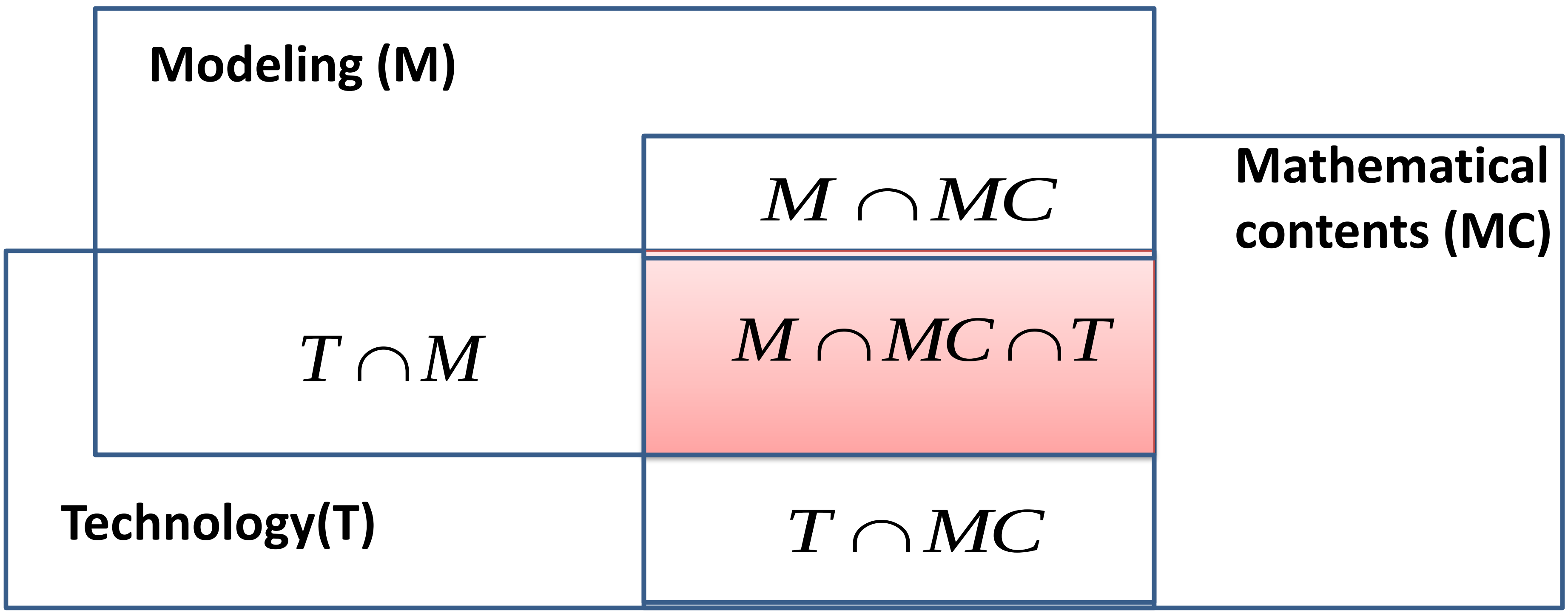
......As we know, the presence of electronic technologies in the classroom can fundamentally change how we think mathematically and what becomes privileged mathematical activity........

European Commission TEMPUS

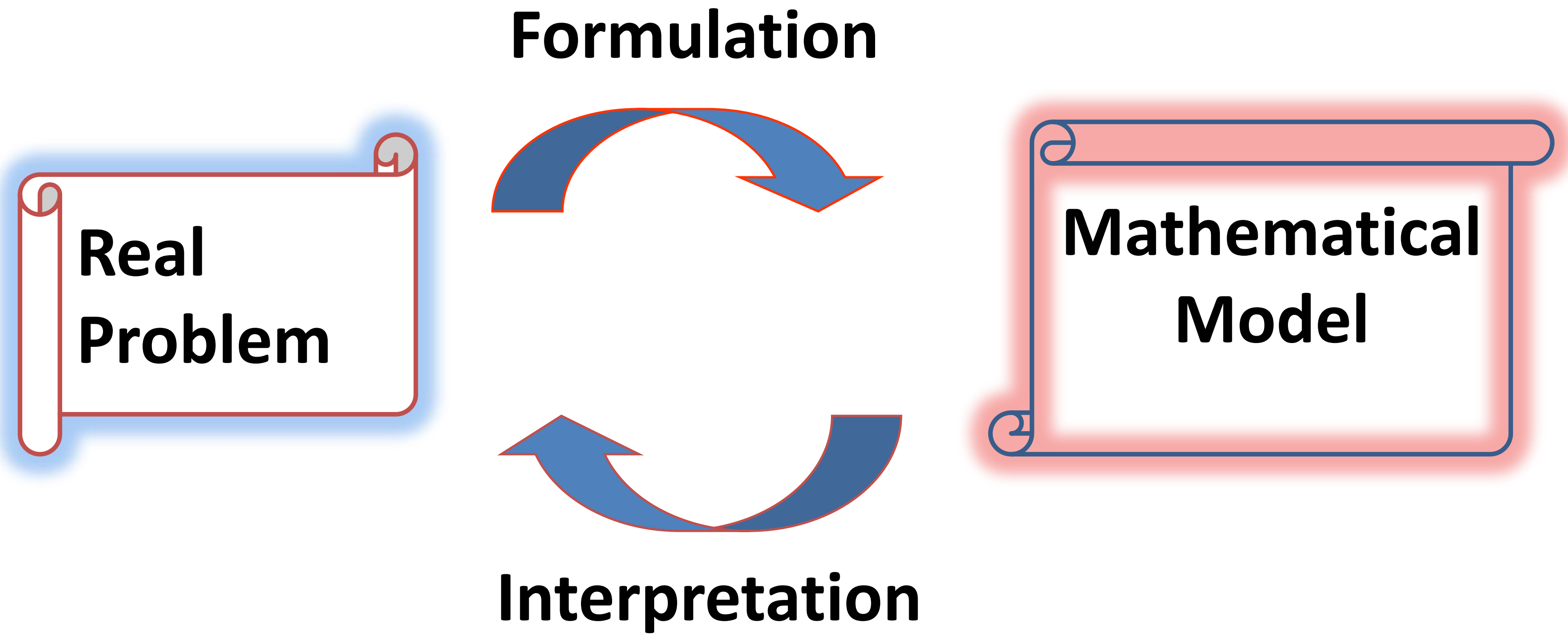
Interactions between

- modelling - M,
- mathematics content - MC,
- technology-T

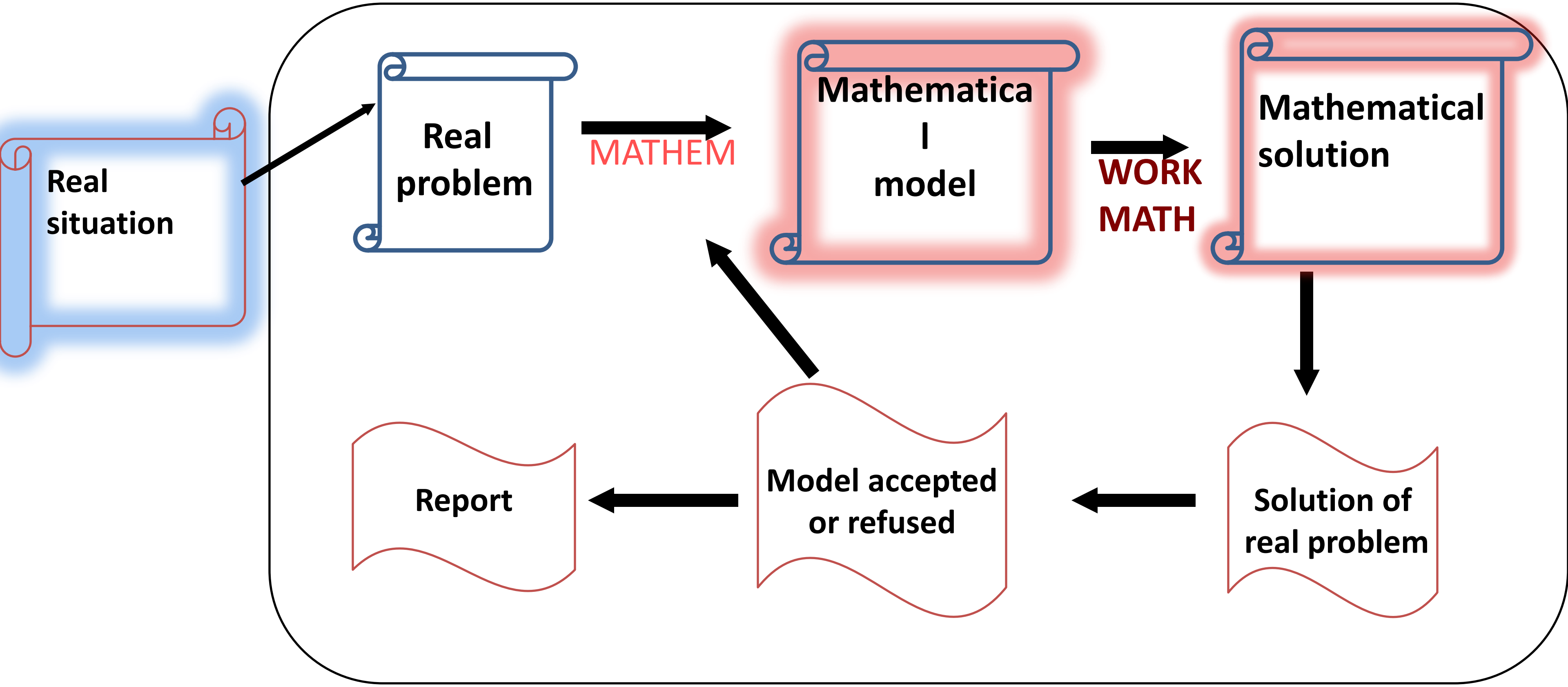
Interactions between modelling, mathematics contents, technology



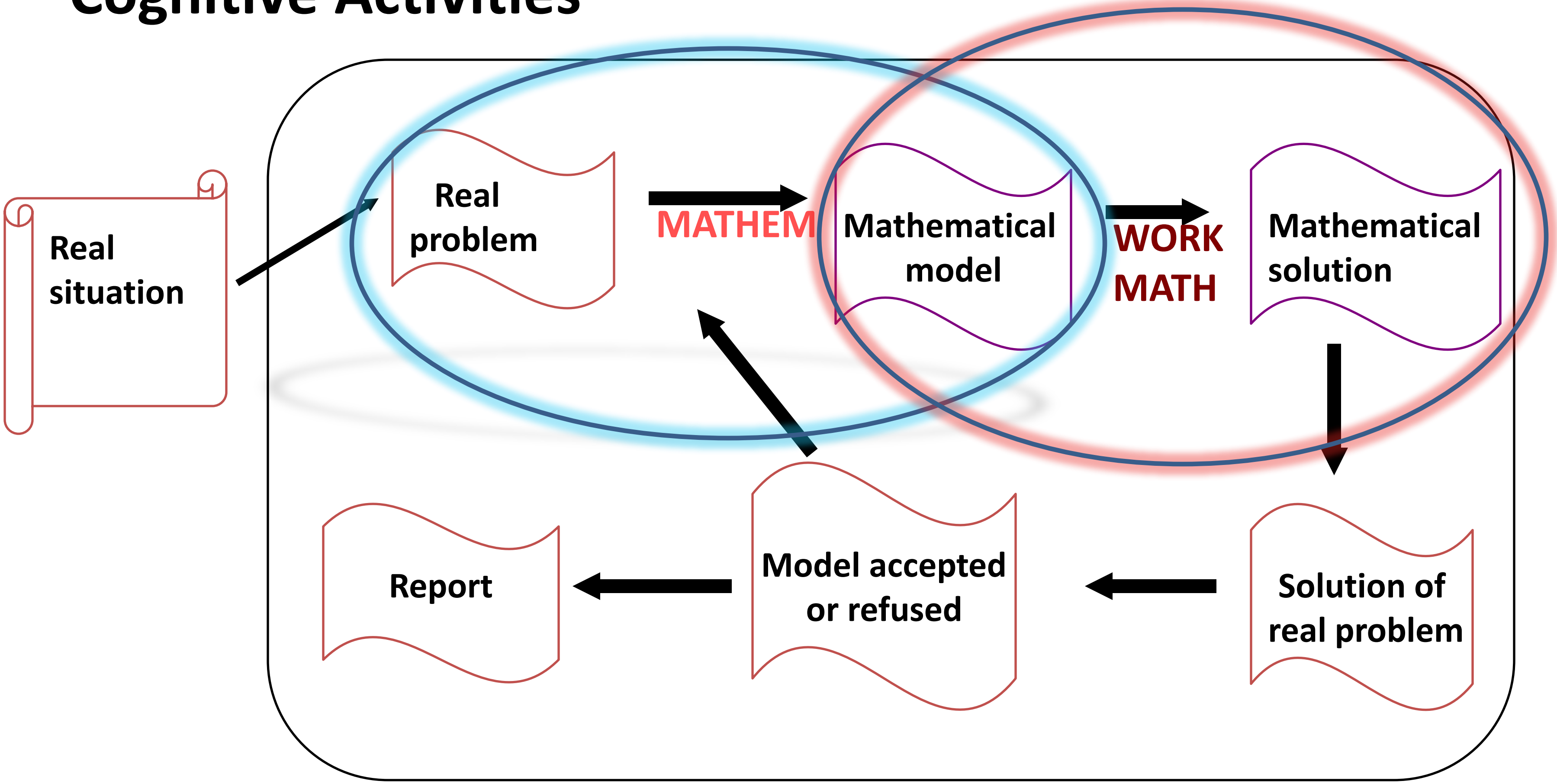
Mathematical modeling process



Mathematical modeling process-Stilman



Cognitive Activities



Cognitive Activities

Real Problem → Mathematical model

Understanding , simplifying , interpreting context

Mathematical model → Mathematical solution

Assuming, formulating, working mathematically

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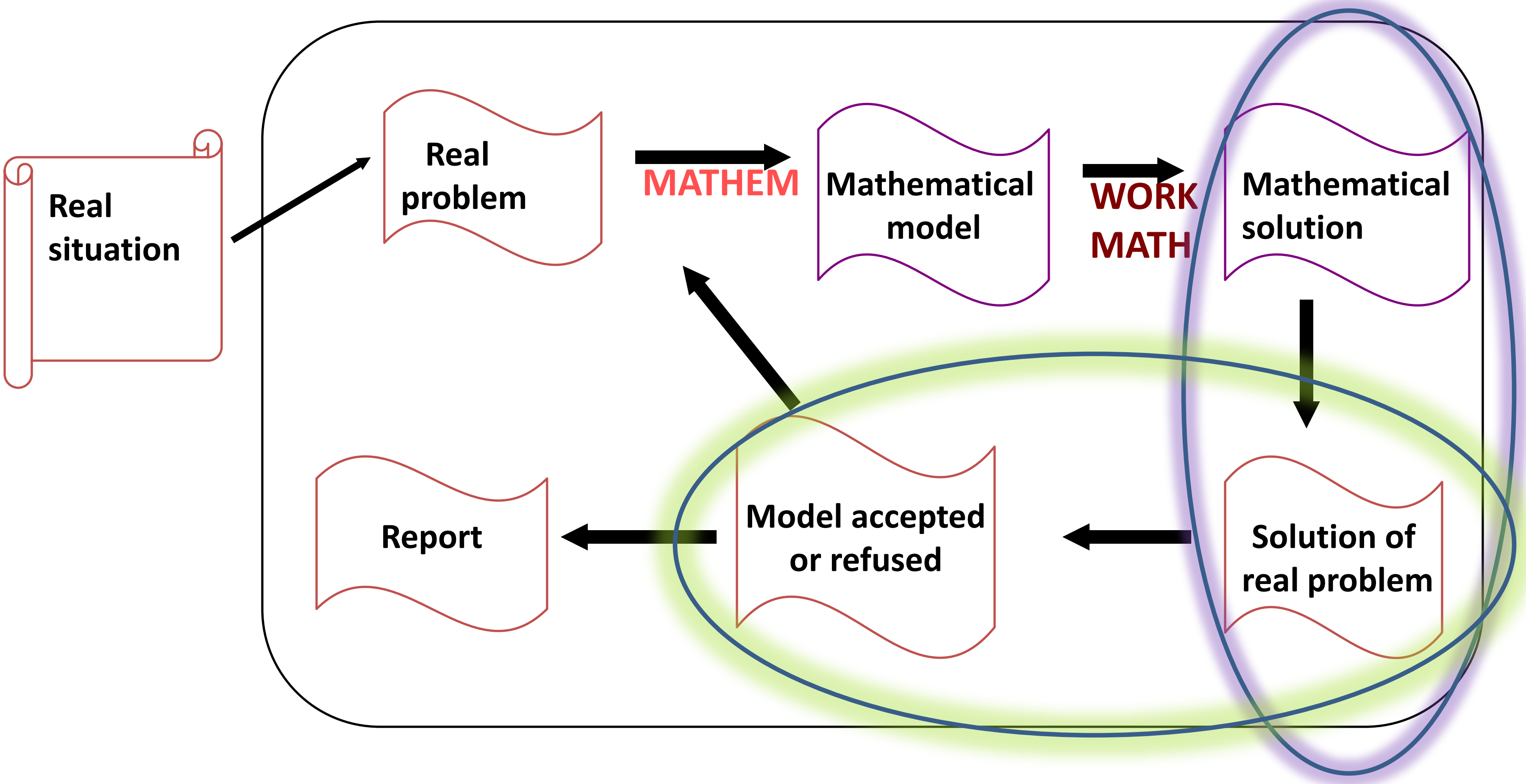
Cognitive Activities



Mathematical solution → Real solution

Interpreting mathematical output

Real solution → Model accepted or refused

Comparing, critiquing, validating

George Pólya http://en.wikipedia.org/wiki/Gyoergy_Polya

Pólya György

He was a professor of mathematics

- 1914 - 1940 at ETH Zurich
- 1940 to 1953 at Stanford University

He made fundamental contributions to combinatorics, number theory, numerical analysis, probability theory

He is also noted for his work in heuristic and mathematics education.

Heuristics

- Trying to characterize the methods that people use to solve problems, and to describe how problem-solving should be taught and learned. He wrote four books on the subject:
- *How to Solve It,*
- *Mathematical Discovery: On Understanding, Learning, and Teaching Problem Solving*;
- *Mathematics and Plausible Reasoning Volume I: Induction and Analogy in Mathematics,*
- *Mathematics and Plausible Reasoning Volume II: Patterns of Plausible Inference*.

How to solve it: http://www-history.mcs.st-and.ac.uk/Biographies/Polya.html

- What is good education?

Systematically giving opportunity to the student to discover things by himself.

- Wise advice:

If you can't solve a problem, then there is an easier problem you can't solve: find it.

George Pólya (http://www-history.mcs.st-and.ac.uk/Biographies/Polya.html)

Mathematics is a good school of thinking.

But what is thinking?

- *The thinking that you can learn in mathematics is, for instance, to handle abstractions.*

Mathematics is about numbers. Numbers are an abstraction. When we solve a practical problem, then from this practical problem we must first make an abstract problem. ...

George Pólya (http://www-history.mcs.st-and.ac.uk/Biographies/Polya.html)

....But I think there is one point which is even more important. Mathematics, you see, is not a spectator sport.

To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems.

George Pólya http://www-history.mcs.st-and.ac.uk/Biographies/Polya.html)

Teaching is not a science; it is an art.

If teaching were a science there would be a best way of teaching and everyone would have to teach like that.

Since teaching is not a science, there is great latitude and much possibility for personal differences.

Perhaps the first point, which is widely accepted, is that teaching must be active, or rather active learning.

... the main point in mathematics teaching is to develop the tactics of problem solving.

George Polya developed four-step solving process:

- *understanding*
- *devising a plan*
- *carrying out the plan*
- *looking back*

http://teacher.scholastic.com/lessonrepro/lessonplans/steppro.htm

1. Understanding the problem

- Can you state the problem in your own words?
- What are you trying to find or do?
- What are the unknowns?
- What information do you obtain from the problem?
- What information, if any, is missing or not needed?

2. Devising a plan

- Examine related problems, and determine if the same technique can be applied.
- Examine a simpler or special case of the problem to gain insight into the solution of the original problem.
- Make a table.
- Make a diagram.
- Write an equation.
- Use guess and check.
- Work backward.
- Identify a subgoal.

3. Carrying out the plan

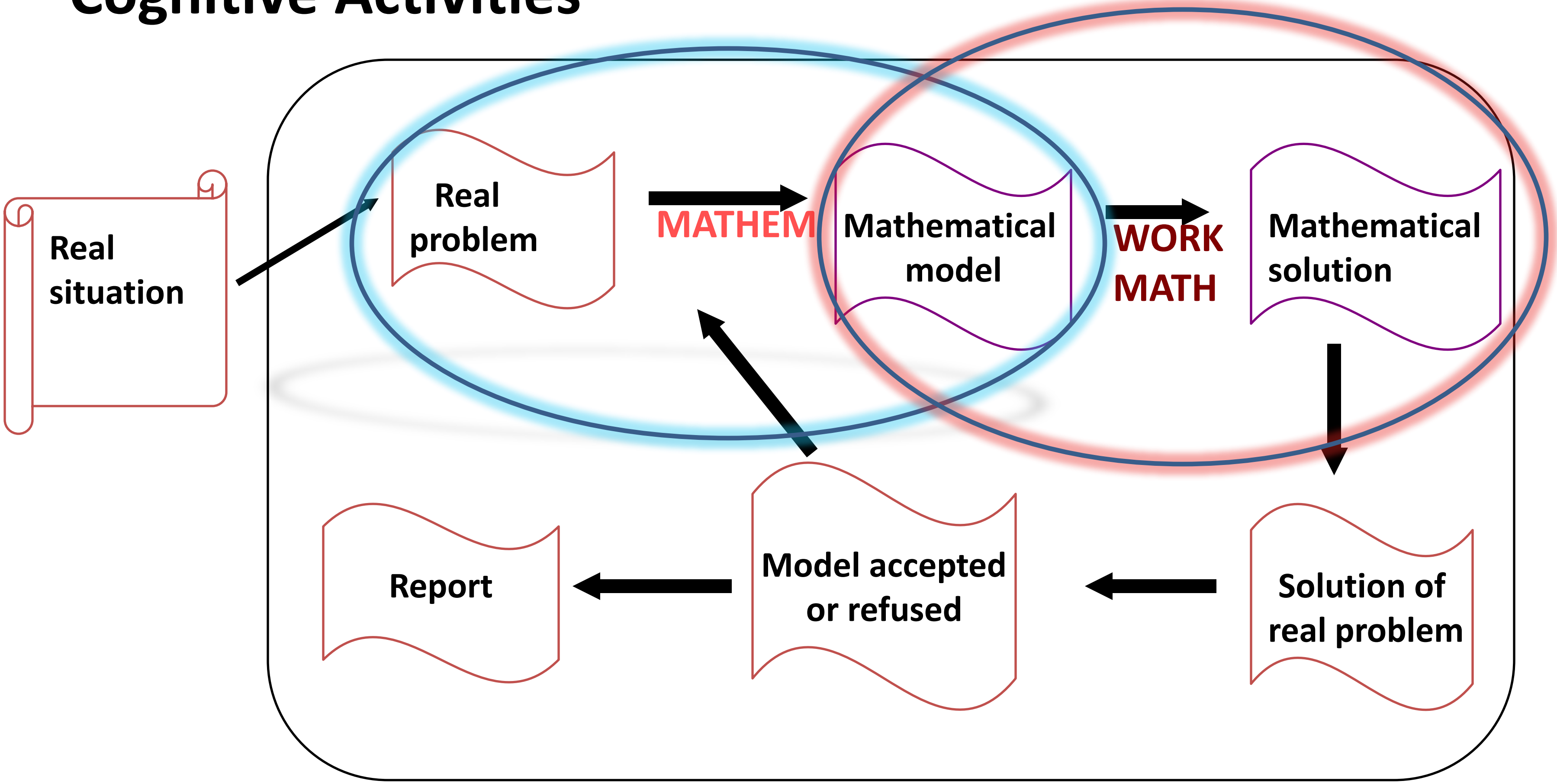
- Implement the strategy or strategies in step 2, and perform any necessary actions or computations.
- Check each step of the plan as you proceed. This may be intuitive checking or a formal proof of each step.
- Keep an accurate record of your work.

4. Looking back

- Check the results in the original problem. (In some cases this will require a proof.)
- Interpret the solution in terms of the original problem. Does your answer make sense? Is it reasonable?
- Determine whether there is another method of finding the solution.
- If possible, determine other related or more general problems for which the techniques will work.

George Polya and Mathematical modeling

Cognitive Activities



Cognitive Activities

Real Problem - Mathematical model--- Mathematical solution --- Real solution---

Understanding , Devising the plan, Carrying out the plan, Looking back

Sunčica Zdravković

Laboratory for Experimental Psychology

Department of Psychology, University of Novi Sad, Serbia

MODELING

- Describing phenomena using mathematical concepts and language
- Usage in psychology and neuroscience
  - *Statistics*
    - Frequency statistics
    - Bayesian statistics
  - *Statistical modes*
    - Filters, neural networks, swarm….

Modeling cognitive functions

Hermann Ebbir

Hermann Ebbinghaus

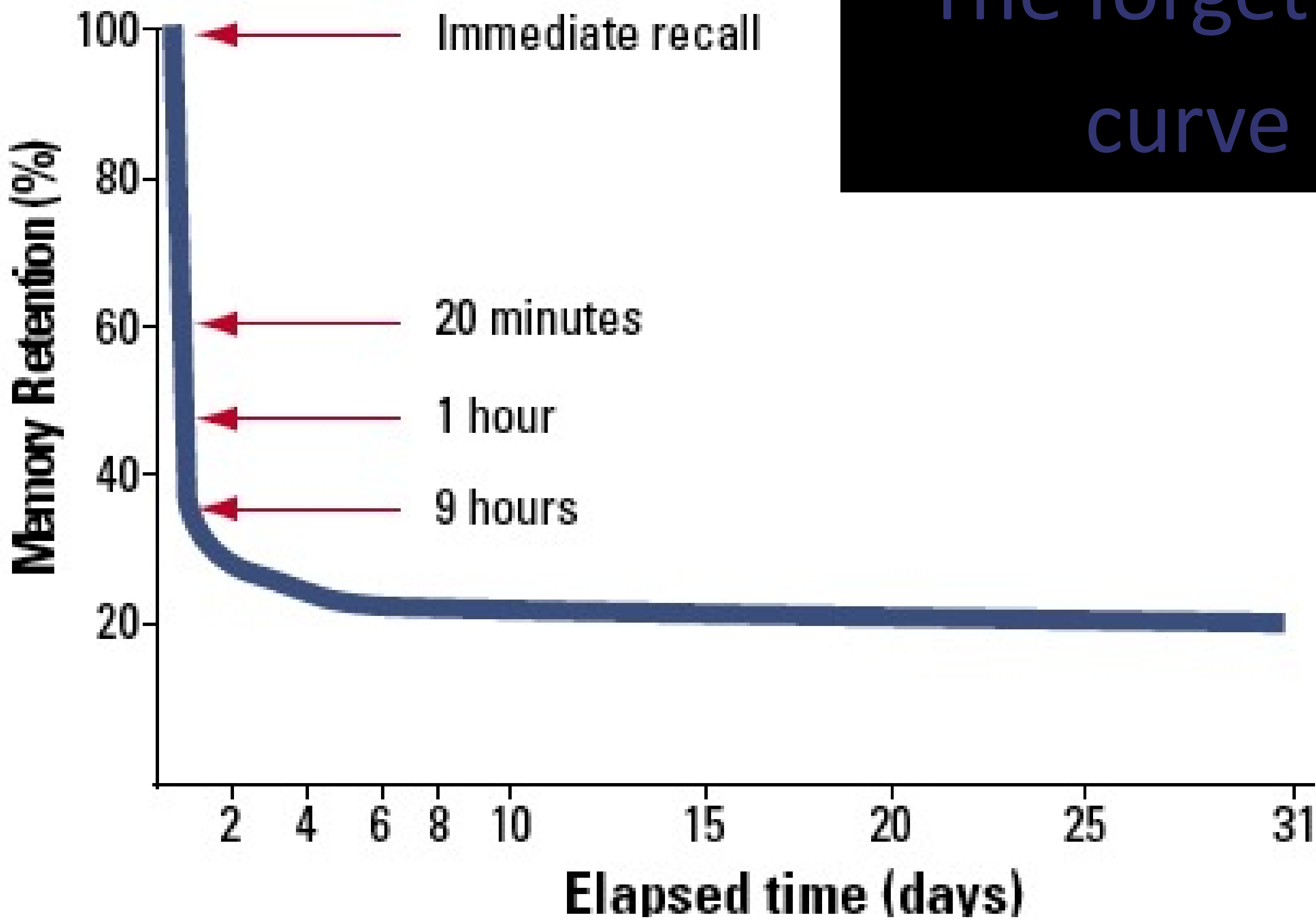
- *Über das Gedächtnis* ("On Memory")
- Experimental study
  - *Learning of "nonsense syllables"*
  - *Collection of 2300*
    - to the regular sound of a metronome, and with the same voice inflection, he would read out the syllables, and attempt to recall them at the end of the procedure
    - one investigation alone required 15,000 recitations

Hermann Ebbinghaus

- The forgetting curve
  - forgetting curve describes the exponential loss of information that one has learned
  - the curve levels off after about one day
- The learning curve
  - refers to how fast one learns information
- Position effects

Hermann Ebbinghaus

The forgetting curve



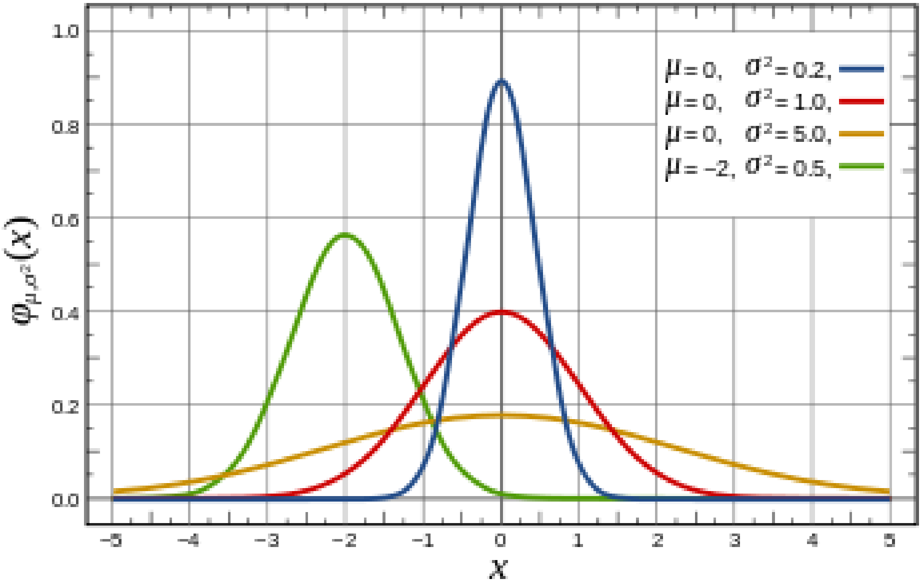
Modeling DATA distributions

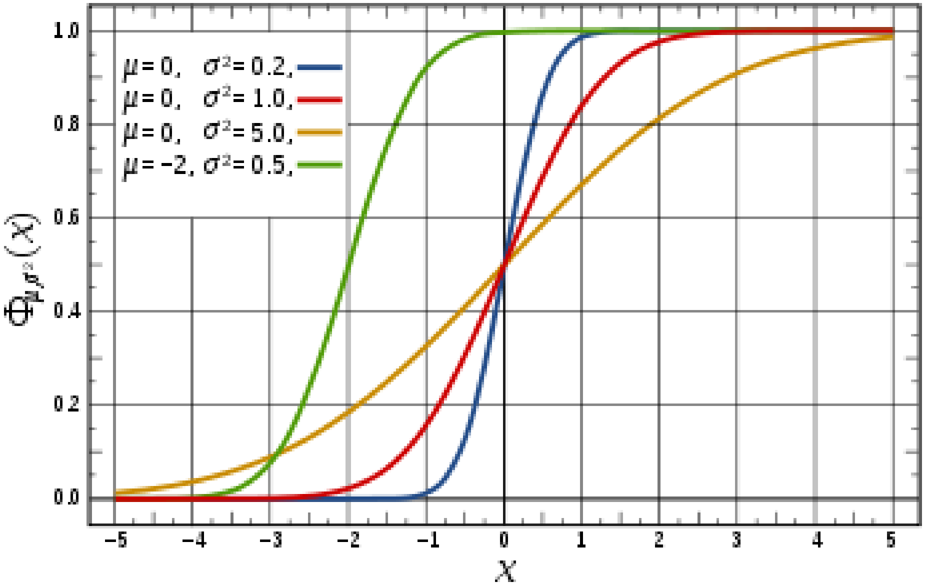
TRAITS

- Traits
  - *habitual patterns of behavior, thought, and emotion*
    - neuroticism, perfectionism, impulsivity, agreeableness…
- Questionnaires
- Test scores approximate a normal distribution given a large enough sample of people

Normal distribution

$$\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$





Reaction time

- Elapsed time between the presentation of a sensory stimulus and the subsequent behavioral response
- An index of speed of processing
- Response time is the sum of reaction time plus movement time.

http://biae.clemson.edu/bpc/bp/Lab/110/reaction.htm#Type%20of%20Stimulus

Reaction time

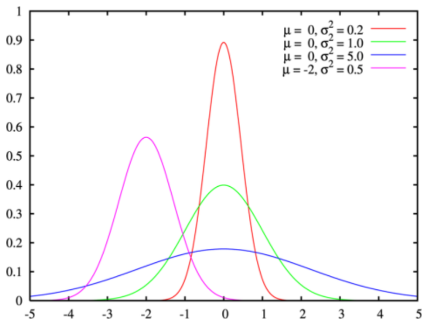
- Four basic types
  - *Simple reaction time*
  - *Go/No-Go reaction time*
  - *Choice reaction time*
  - *Discrimination reaction time*

Reaction time

- Hick's Law
- individual's reaction time increased by a constant amount as a function of available choices
- reaction time is found to be a function of the binary logarithm of the number of available choices (n)

$$RT = a + b \log_2 (n + 1)$$

Distribution



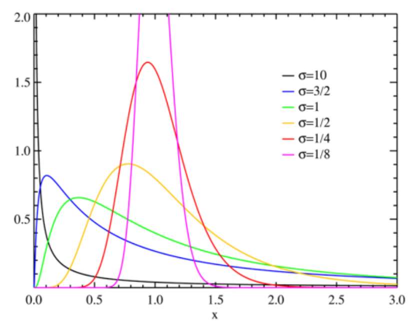
Gaussian, normal distribution

Parameters: mean (central tendency) and standard deviation (variability)

Standard: AS = 0, SD = 1

Fixed number of parameters

$$\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

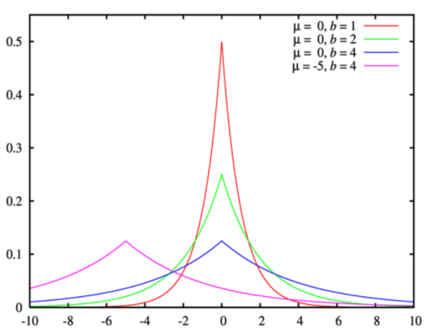


Log-normal distribution

Single-tailed probability distribution of any random variable whose logarithm is normally distributed

Parameters: mean (central tendency) and standard deviation of the variable's natural logarithm (variability)

$$\exp\left(-\left[\frac{\ln(x)-\mu}{\sigma}\right]^2/2\right)/(x\sigma\sqrt{2\pi})$$



Laplace distribution

Continuous, double exponential

Parameters: median (central tendency) and absolute deviation (variability)

Models the symmetrical data with long tales

Narrower confidence intervals

Non-fixed number of parameters

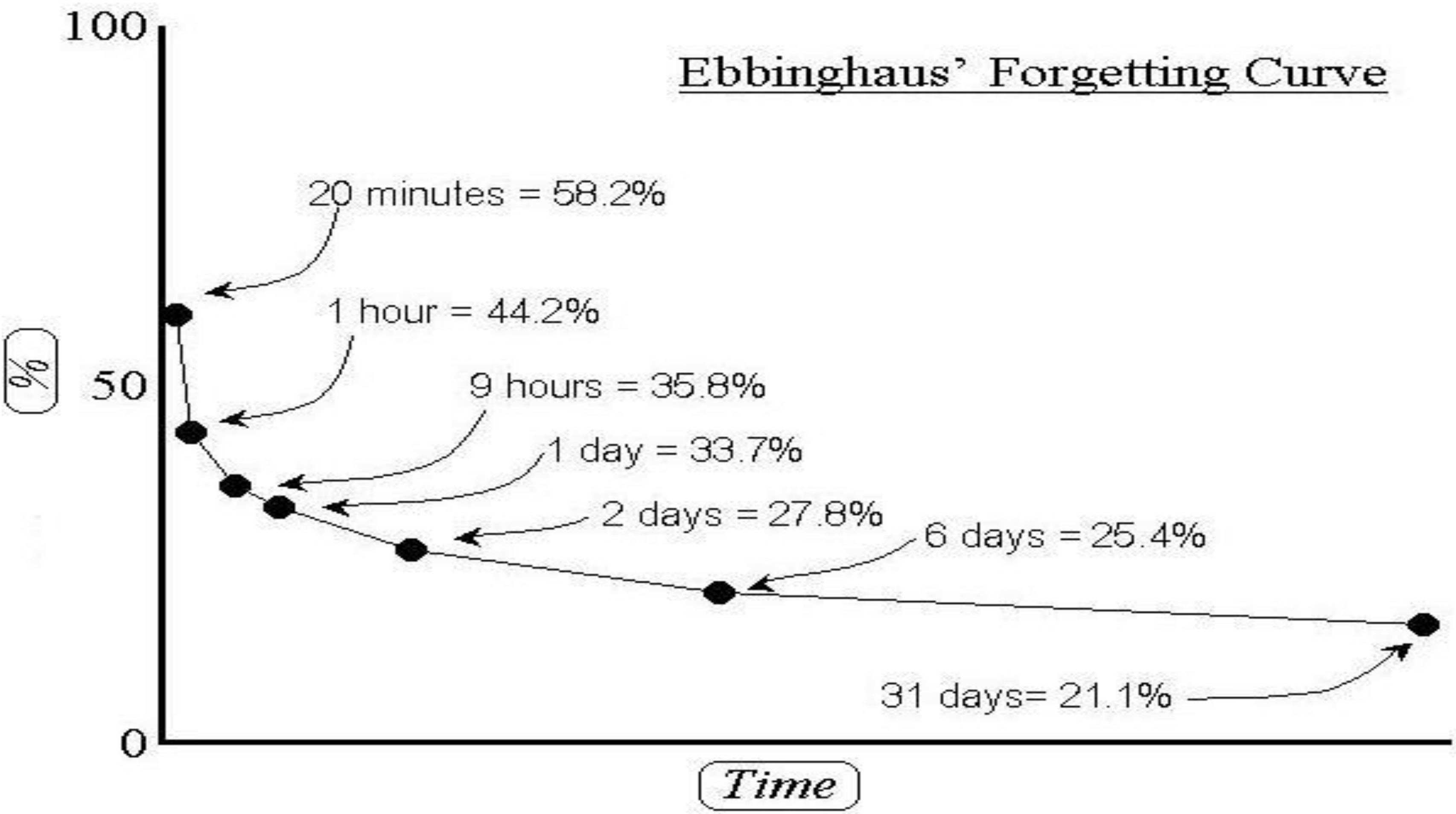
$$f(x|\mu,b) = \frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right)$$

$$\frac{1}{2b}\begin{cases}\exp\left(-\frac{\mu-x}{b}\right), & if\ x<\mu\\ \exp\left(-\frac{x-\mu}{b}\right), & if\ x\geq\mu\end{cases}$$

DISTRIBUTION

- Why is it important to know the distribution of the data?

- Parametric vs. non-parametric statistical methods



Ebbinghaus' Forgetting Curve
100
50
0
%
Time
20 minutes = 58.2%
1 hour = 44.2%
9 hours = 35.8%
1 day = 33.7%
2 days = 27.8%
6 days = 25.4%
31 days= 21.1%

Real Problem:

After 20 minutes (or 1/72 day) one forgets 40% materials
After 1 hour (1/24 day) one forgets 50% materials,
After 9 hours (9/24 day) one forgets 60% materials,
After 1 day one forgets 65% material,
After 2 days one forgets 70% material,
After 6 days one forgets 75% material,
After 30 days one forgets 80% material.

Cognitive activities --- Understanding the problem

Real Problem-Mathematical model

What are we trying to find or do?

We are trying to

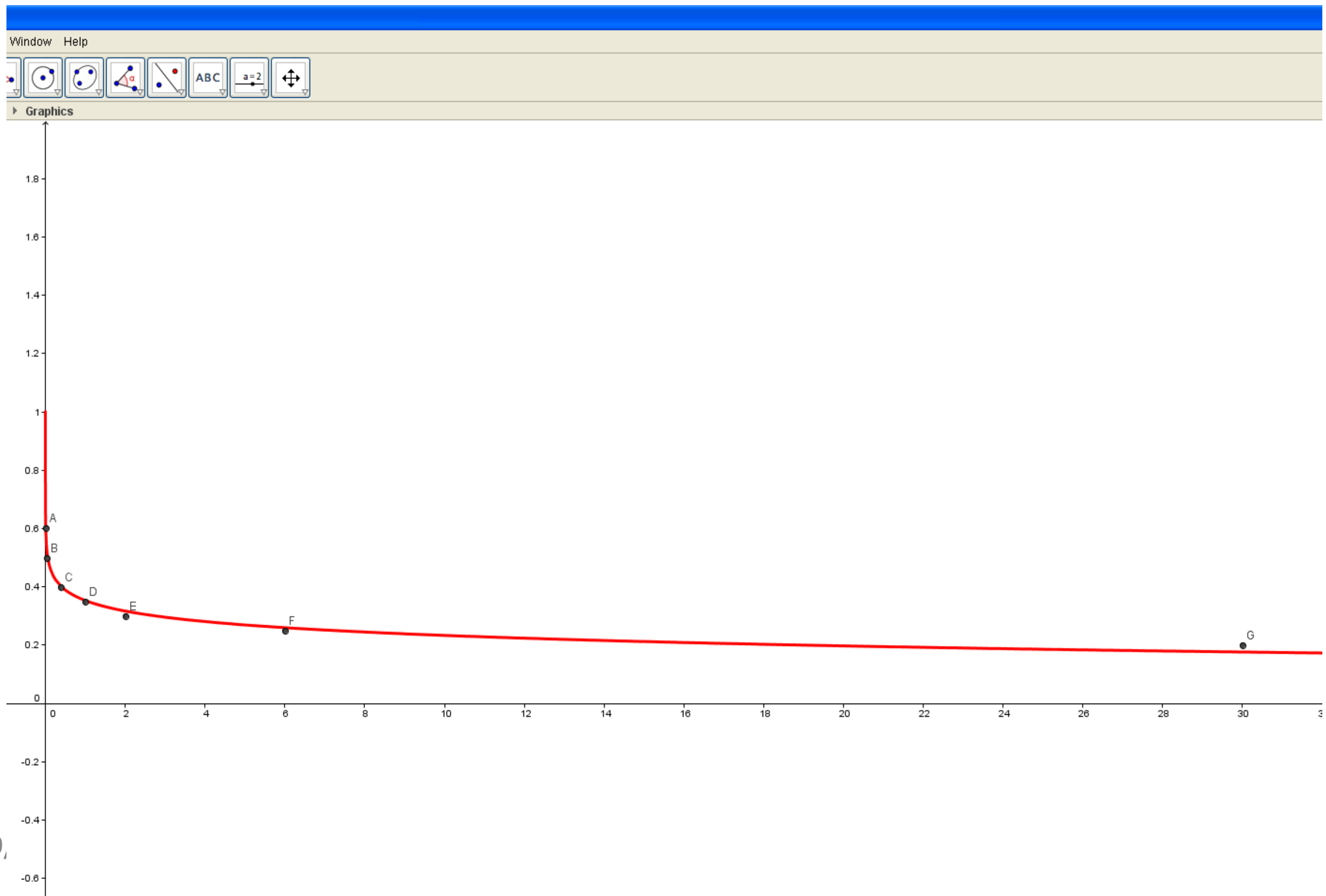
- **fit the curve corresponding to given data,**
- **to compare with the given graph**
- **to extend the problem mathematically**

Cognitive activities --- Devising a plan

Real Problem-Mathematical model

- Make a graph– Use GeoGebra
- Write an equation.
- Examining the graph
- Compare with the given graph

$$f(x) = 0.35 - 0.05\ln(x)$$



Carrying out the plan

Mathematical model - Mathematical solution

EGER-Material\Forget1.ggb

$$f(x) = 0.35 - 0.05\ln(x)$$

We implemented the strategy from previous and present the graph but we need to analyze the graph from mathematical point of view and then to go to

Real solution

Looking back

- Check the results in the original problem

Forgotten and left material

- Interpret the solution in terms of the original problem.
- It looks like Ebinhause curve
- But mathemaically ???
- What is reasonable?

Looking back

Forgotten material

How about the speed of lost material???

First derivative or tangent of the corresponding angle???

It turns out to be very difficult for students to analyze mathematically.

For us, may be not???

New: Understanding and Devising new plan

Let us consider: forgotten material

This another method.

New: Carrying out the plan

Real problem (the same)

Mathematical model - Mathematical solution (different)

Fitting log curve

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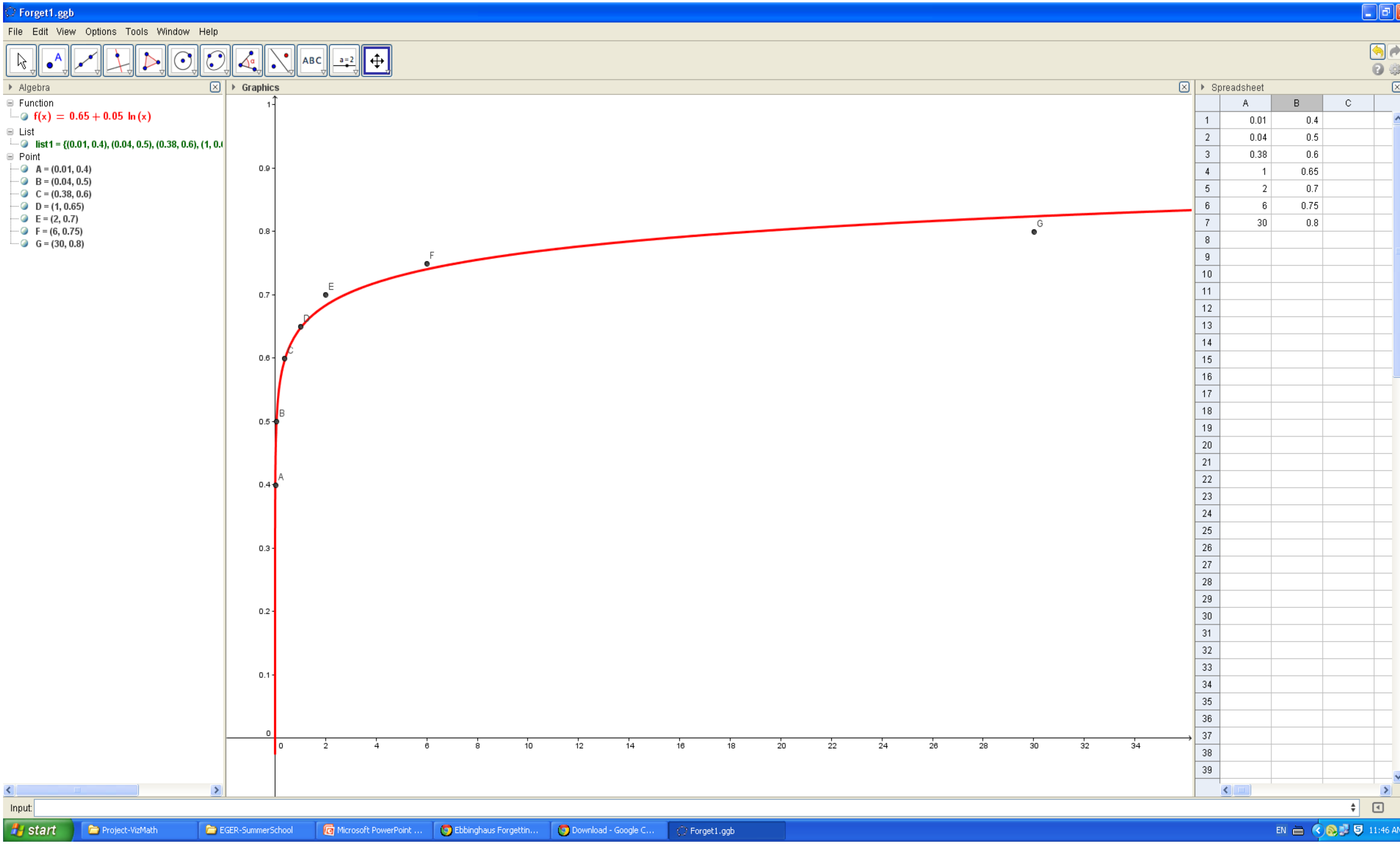

Real solution

Different interpretation but the same meaning

Explanations,......

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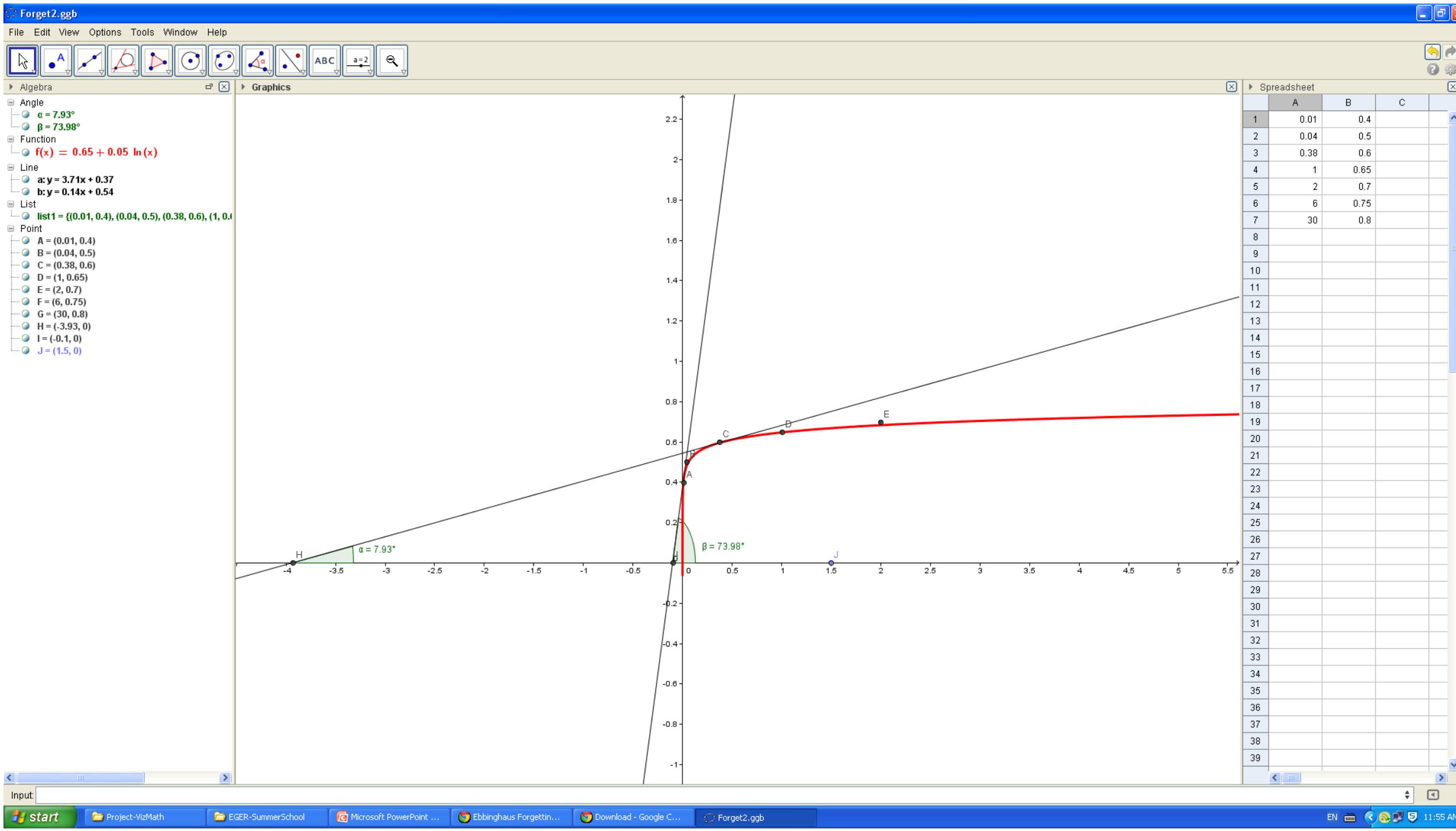

More mathematical works *in the stage of* *mathematical solutions*

Considering the speed of forgetting-

Differential equations $y' = \frac{0.05}{x}$ ***the condition*** $y(1) = 0.65$

Different analysis

Comparing the graphs

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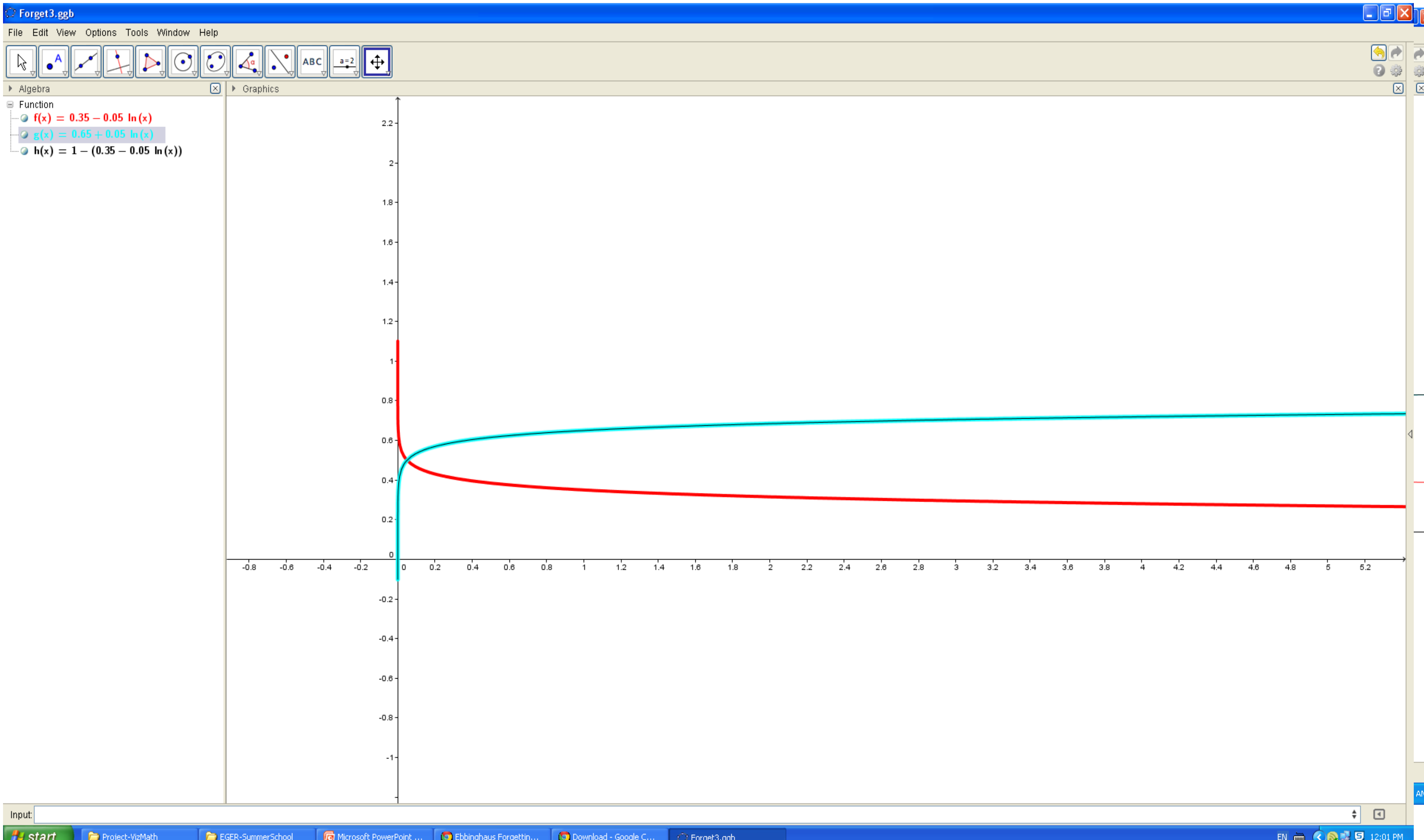


- The first derivative can be compared in order to say about the speed of forgetting.

Conclusion can be seen better from the considerations of rest materials.

The speed of forgetting

is greater at the beging than later....

Modeling cognitive functions

Sunčica Zdravković

Laboratory for Experimental Psychology

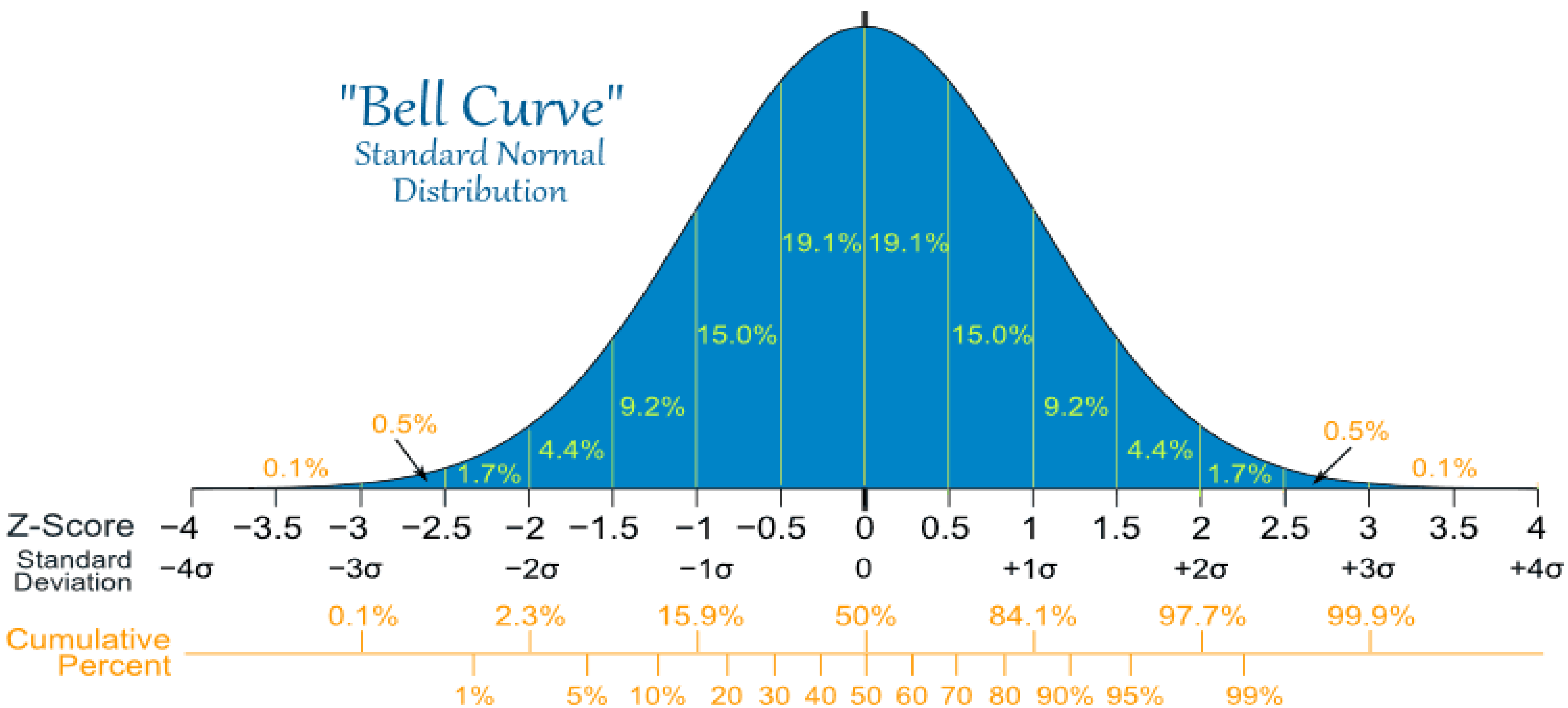
Department of Psychology

University of Novi Sad

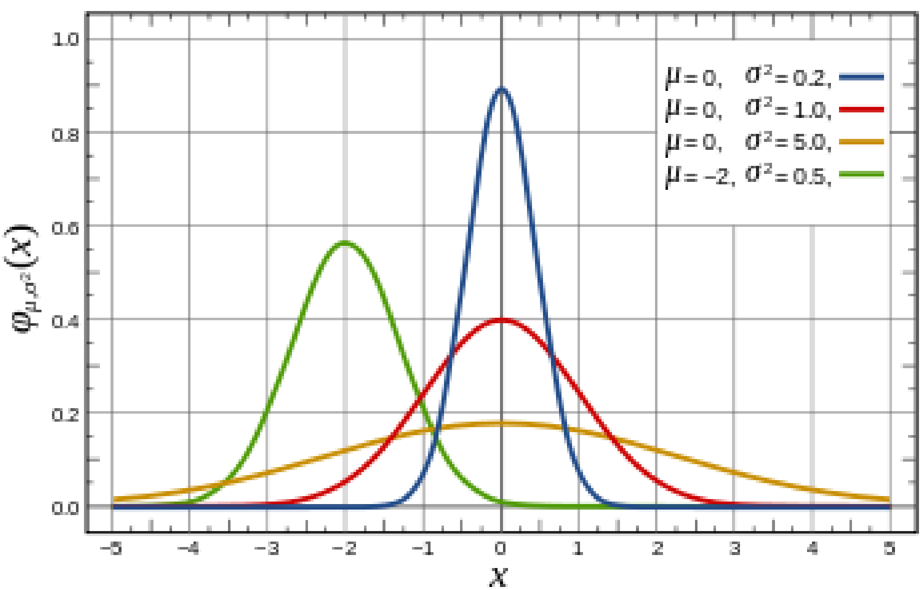
http://www.ff.uns.ac.rs/fakultet/ljudi/SuncicaZdravkovicEng.pdf

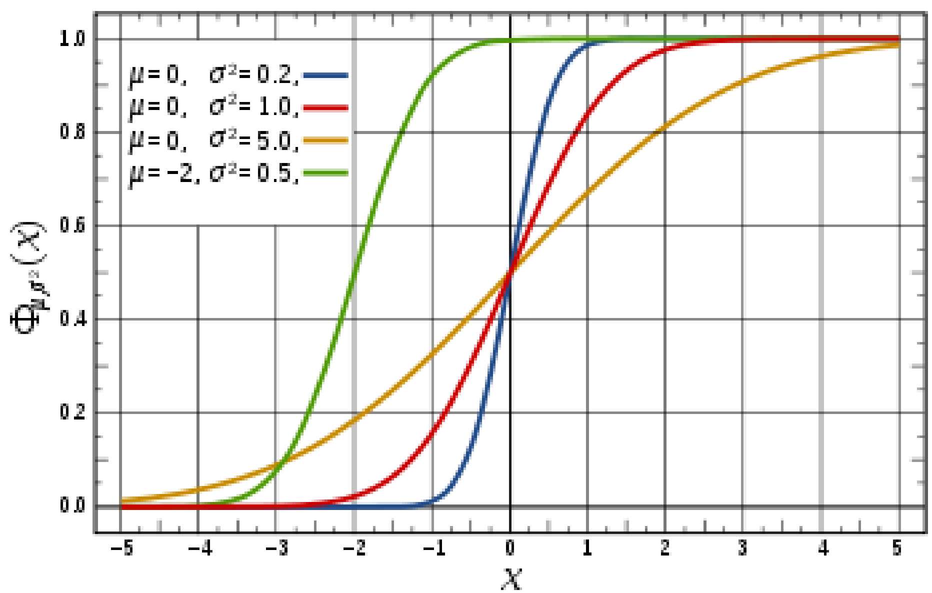
http://lepns.psihologija.edu.rs/?lang=en

MODELING the DATA distributions: Traits and RT



$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$





Different discussions about the **Real solution** and about

Model accepted or refused

Comparing, critiquing, validating

VISUAL ILLUSIONS

Sunčica Zdravković

Laboratory for Experimental Psychology

Department of Psychology

University of Novi Sad

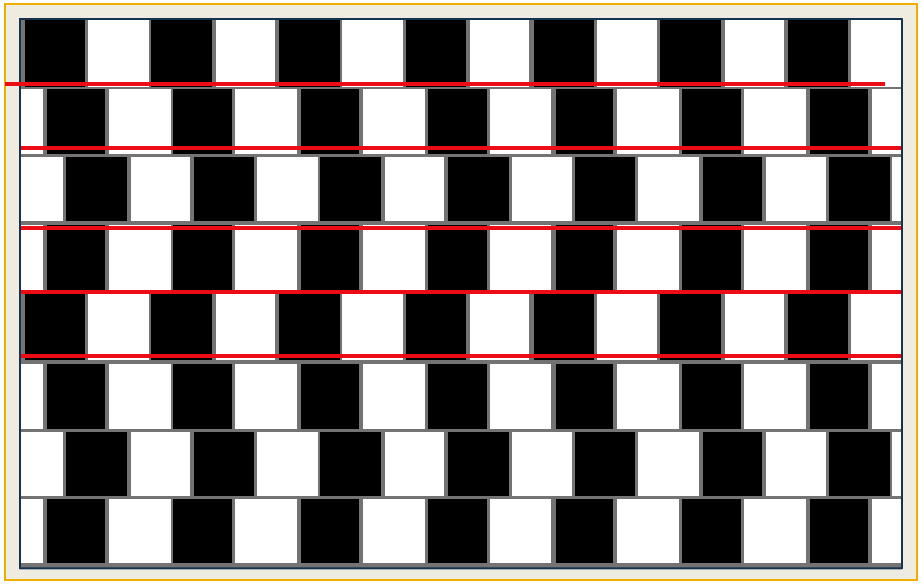
http://www.ff.uns.ac.rs/fakultet/ljudi/SuncicaZdravkovicEng.pdf

http://lepns.psihologija.edu.rs/?lang=en

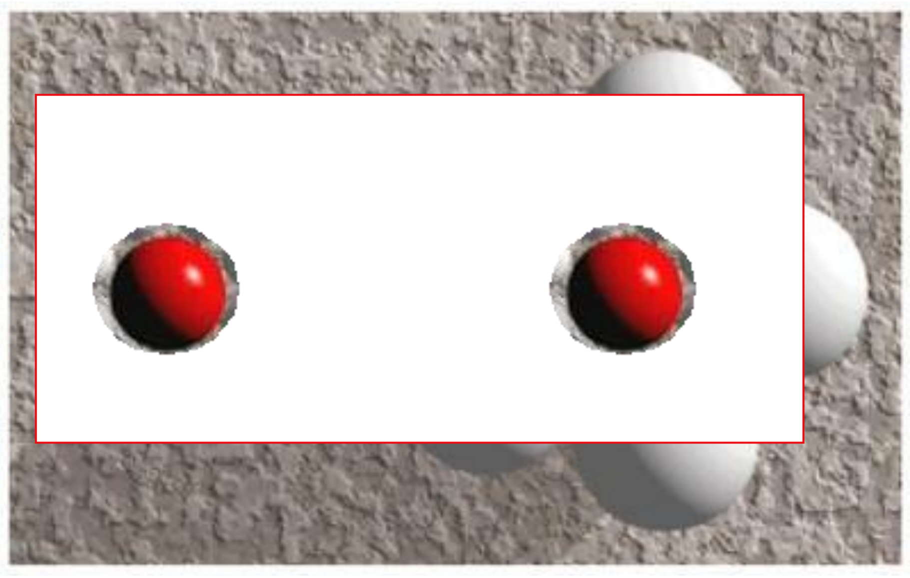
VISUAL ILLUSIONS

VISUAL ILLUSIONS

- Amazing phenomena of normal vision



Gregory (1923-2010)



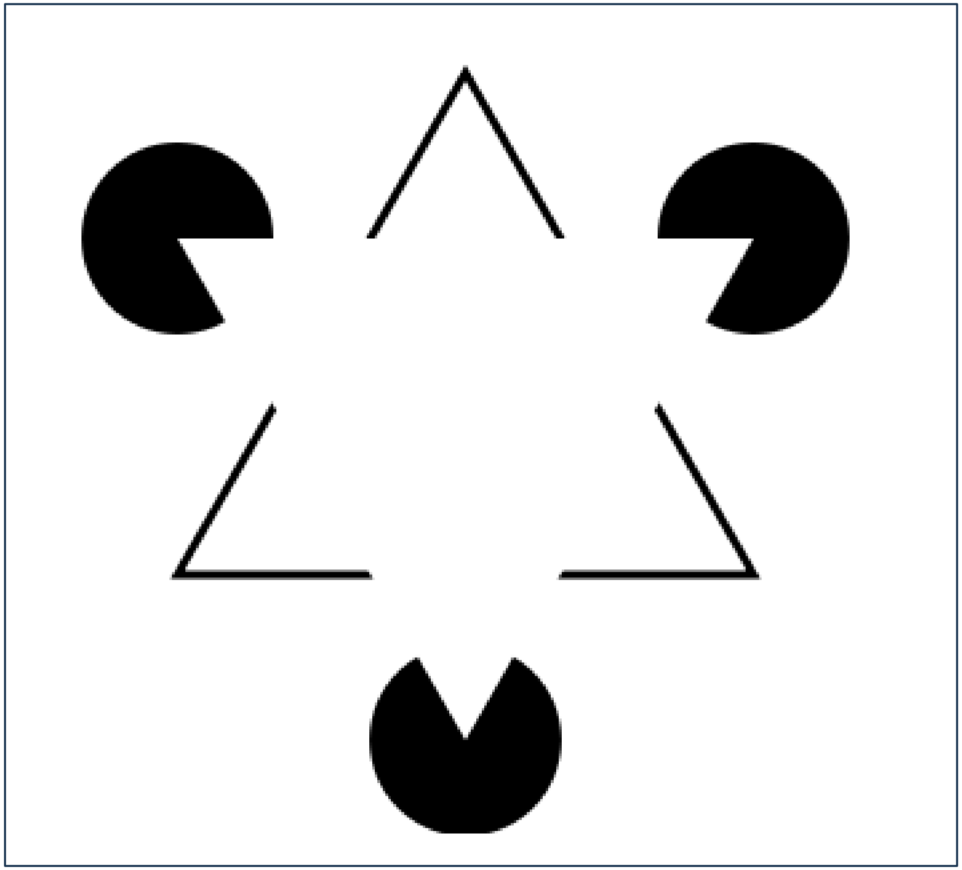
Ebbinghaus (1850-1909)

VISUAL ILLUSIONS

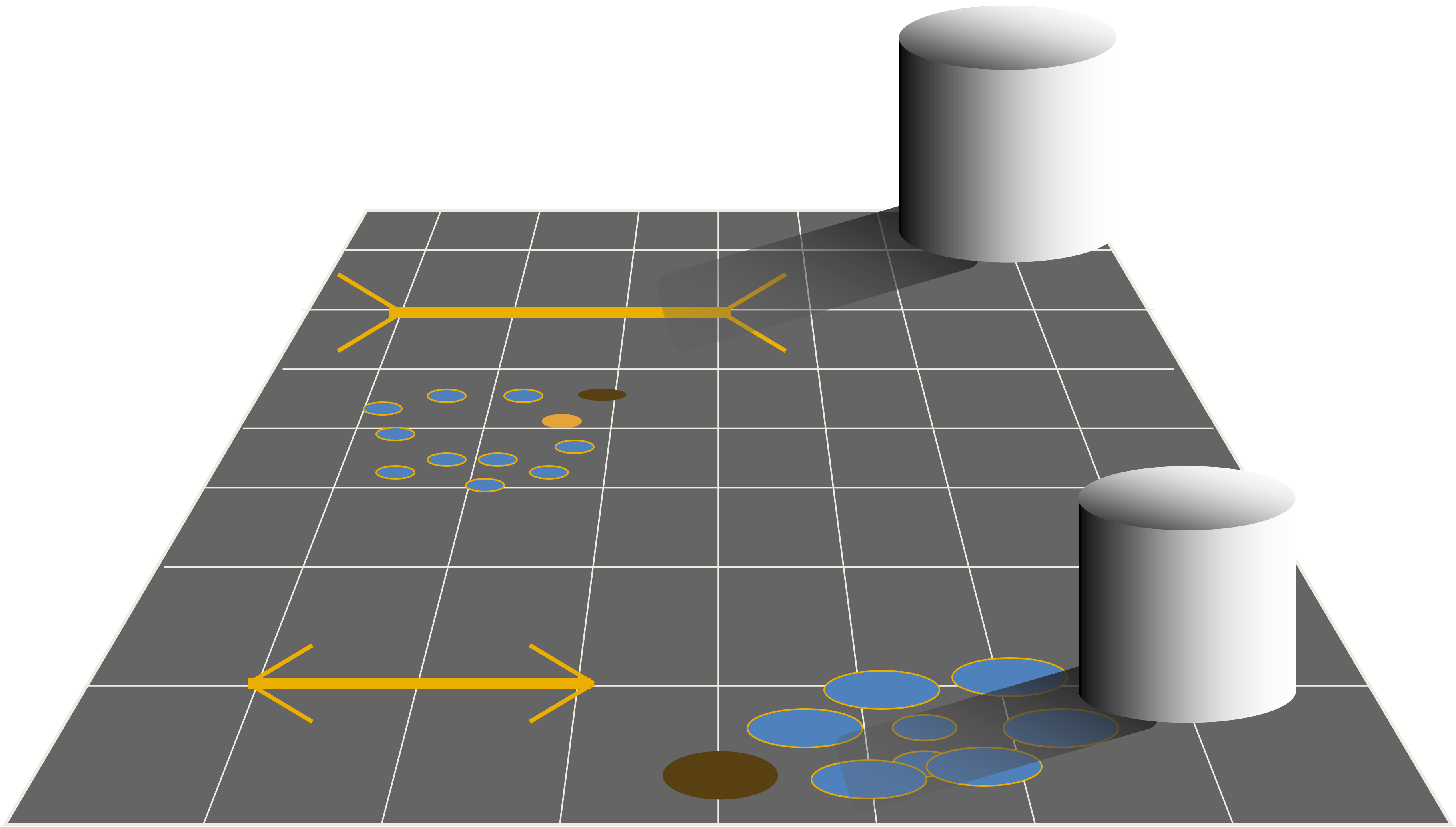
- Perceived images that differ from objective reality
  - *Geometrical-optical illusions are characterized by distortions of size, length, position, curvature…*
- Perceptual organization

VISUAL ILLUSIONS

Kanizsa triangle

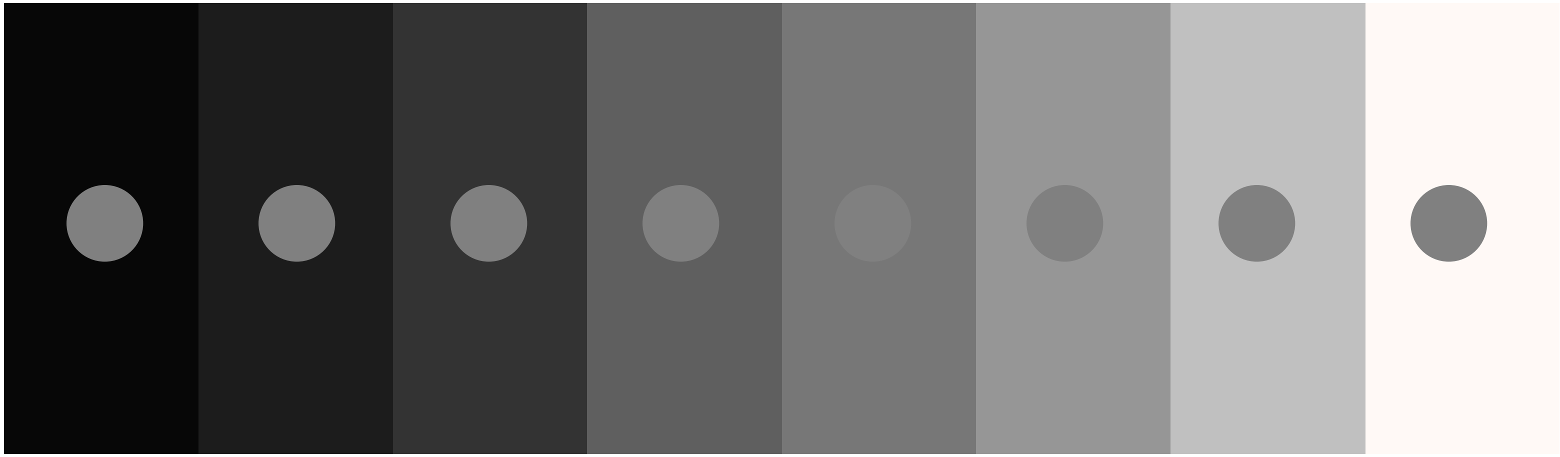


Visual illusions



Visual illusions

Simultaneous contrast



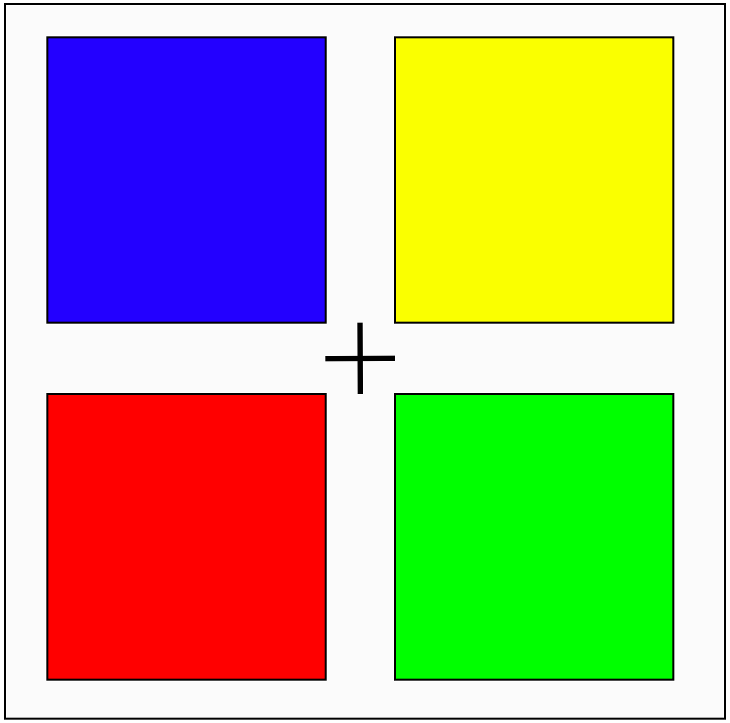
Hering (1874)

Visual illusions

Simultaneous contrast

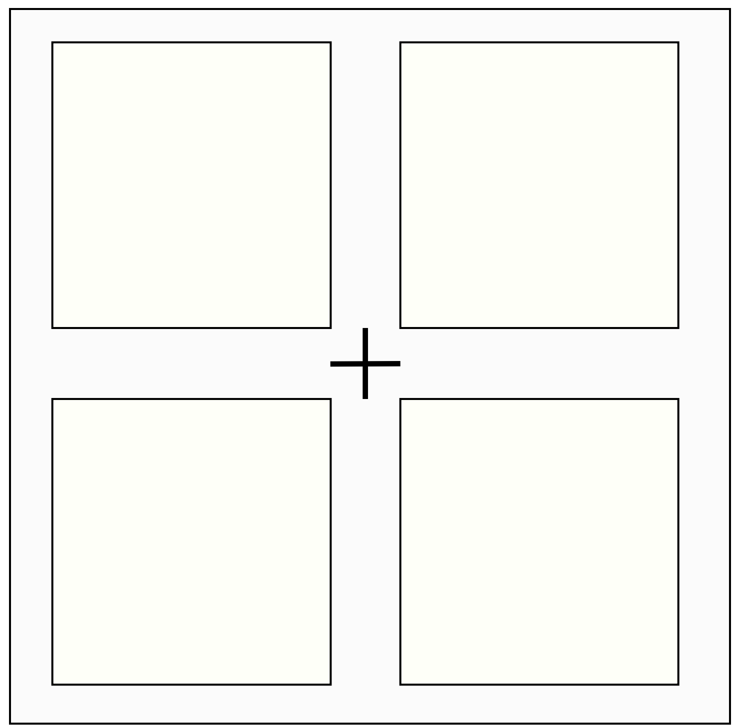
Visual illusions

Aftereffect



VISUAL ILLUSIONS

Aftereffect



ANCHORING THEORY

- Highest Luminance Rule
  - *Highest luminance appears white, and other shades are seen in relation to the the highest luminance*

$$PR = L_t/L_h \times 90\%$$

- Area Rule
  - *Darker surfaces become lighter as they become larger*

$$PR = (100-A_d)/50 \times (L_t/L_h \times 90\%) + (A_d-50)/50$$

- Scale normalization
  - *The perceived range of grays tends toward that between black and white (30 : 1)*

Alan Gilchrist (1999)

THANK YOU

THANKS FOR YOUR ATTENTION