Software package LaTeX for preparation teaching materials for mathematics

Lecturer: PhD Rale Nikolić, Belgrade Metropolitan University
e-mail: rale.nikolic@metropolitan.ac.rs

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The birthrate in Serbia rapidly decreasing and secondary school will be in a position to fight for every student – better teaching materials, more students

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- files are very small - LaTeX file is just a text file

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For instance, book in serbian language can be downloaded for free at
`http://poincare.matf.bg.ac.rs/~janicic//books/latex2e.pdf`

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To use LaTeX, you first must create a file using a plain text editor. There are many text editors for Windows: WinEdt, TexMaker, BaKoMa TeX, ...

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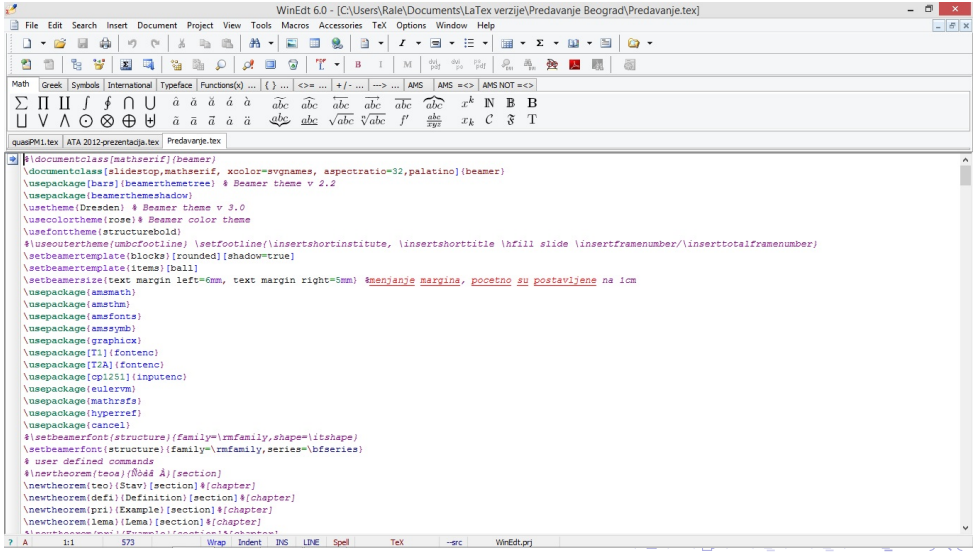

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You have to know LaTeX in order to use *Beamer*! Then you can use your knowledge of LaTeX also when creating a presentation, not only when writing publications.

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- your presentation is in a light text file (+ media if present)

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4. *transitive if it satisfied* $(\forall a_1, a_2, a_3 \in A)\ (a_1, a_2) \in \rho \wedge (a_2, a_3) \in \rho \Rightarrow (a_1, a_3) \in \rho$.

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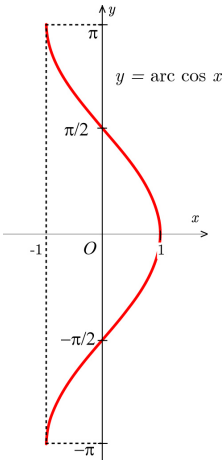

Picture. The graph of function y = arccosx

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Solution. *By using the truth-table*

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge q$	$(p \Rightarrow q) \wedge q \Rightarrow p$
$\top$	$\top$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\top$	$\top$	$\bot$
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we obtain that above formula is not a tautology.

In easy way you can present the n-th order determinant

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$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1i} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2i} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ii} & \dots & a_{in} \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{ni} & \dots & a_{nn} \end{vmatrix},$$

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or Sarrus rule

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - \\ - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}.$$

Also, you can present a system of linear equations involves m equations and n variables:

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1j}x_j + \ldots + a_{1n}x_n &= b_1,\\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2j}x_j + \ldots + a_{2n}x_n &= b_2,\\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots + a_{3j}x_j + \ldots + a_{3n}x_n &= b_3,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + \ldots + a_{ij}x_j + \ldots + a_{in}x_n &= b_i,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mj}x_j + \ldots + a_{mn}x_n &= b_m,
\end{aligned}$$

Example

Determine the first derivation of next function

$$f(x) = -\frac{\cos x}{2\sin^2 x} + \ln\sqrt{\frac{1+\cos x}{\sin x}}.$$

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$$f'(x) = \left(-\frac{\cos x}{2\sin^2 x}\right)' + \left(\ln\sqrt{\frac{1+\cos x}{\sin x}}\right)' =$$

$$= -\frac{-2\sin^3 x - 4\sin x\cos^2 x}{4\sin^4 x} + \frac{1}{\sqrt{\frac{1+\cos x}{\sin x}}}\cdot\left(\sqrt{\frac{1+\cos x}{\sin x}}\right)' =$$

$$= \frac{2\sin x(\sin^2 x + 2\cos^2 x)}{4\sin^4 x} + \sqrt{\frac{\sin x}{1+\cos x}}\cdot\frac{1}{2\sqrt{\frac{1+\cos x}{\sin x}}}\cdot\left(\frac{1+\cos x}{\sin x}\right)' =$$

$$= \frac{\sin^2 x + 2\cos^2 x}{2\sin^3 x} + \frac{\sin x}{2(1+\cos x)}\cdot\frac{-\sin^2 x - \cos^2 x - \cos x}{\sin^2 x} =$$

$$= \frac{\sin^2 x + 2\cos^2 x}{2\sin^3 x} + \frac{\cancel{\sin x}}{2\cancel{(1+\cos x)}}\cdot\frac{-\cancel{(1+\cos x)}}{\cancel{\sin^2 x}} =$$

$$= \frac{\sin^2 x + 2\cos^2 x}{2\sin^3 x} - \frac{1}{2\sin x} = \frac{\cancel{\sin^2 x} + 2\cos^2 x - \cancel{\sin^2 x}}{2\sin^3 x} = \frac{\cancel{2}\cos^2 x}{\cancel{2}\sin^3 x} = \frac{\cos^2 x}{\sin^3 x}.$$

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$$= \frac{1}{\ln 3}\arcsin\left(\frac{3^x}{5}\right) + c.$$

Example

We should solve the system of linear equations

$$\begin{aligned} x + \quad y - \quad z &= 1, \\ 2x + \quad 3y + \quad az &= 3, \\ x + \quad ay + \quad 3z &= 2, \end{aligned} \tag{1}$$

where a is a real parameter.

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Solution. We will apply Cramer's rule. The determinant of the system D is

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & a \\ 1 & a & 3 \end{vmatrix} = -a^2 - a + 6 = -(a-2)(a+3),$$

and the determinants of variables x, y and z denote as D_x, D_y i D_z respectively, are:

$$D_x = \begin{vmatrix} 1 & 1 & -1 \\ 3 & 3 & a \\ 2 & a & 3 \end{vmatrix} = -a^2 - a + 6 = -(a-2)(a+3),$$

$$D_y = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & a \\ 1 & 2 & 3 \end{vmatrix} = -a + 2, \quad D_z = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & a & 2 \end{vmatrix} = -a + 2.$$

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Discustion: We have next three cases. Firstly, if $a \neq -3$ and $a \neq 2$, we have $D \neq 0$, and system has unique solution:

$$x = \frac{D_x}{D} = 1, \quad y = \frac{D_y}{D} = \frac{1}{a+3}, \quad z = \frac{D_z}{D} = \frac{1}{a+3}.$$

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In last case, for $a = 2$, we have $D_x = D_y = D_z = 0$, and we can't apply the Cramer's rule. Now, we will directly put the value $a = 2$, in the system (1). Then we obtain:

$$\begin{aligned} x + \quad y - \quad z &= 1, \\ 2x + \quad 3y + \quad 2z &= 3, \\ x + \quad 2y + \quad 3z &= 2, \end{aligned} \tag{2}$$

For solving system (2) we will apply Kronecker-Capelli theorem.

We have

$$A|B = \left[\begin{array}{rrr|c} 1 & 1 & -1 & 1 \\ 2 & 3 & 2 & 2 \\ 1 & 2 & 3 & 2 \end{array}\right] \begin{array}{c} V_{12}(-2) \\ \sim \\ V_{13}(-1) \end{array} \left[\begin{array}{rrr|c} 1 & 1 & -1 & 1 \\ 0 & 1 & 4 & 1 \\ 0 & 1 & 4 & 1 \end{array}\right] \begin{array}{c} V_{23}(-1) \\ \sim \end{array}$$

$$\begin{array}{c} V_{23}(-1) \\ \sim \end{array} \left[\begin{array}{rrr|c} 1 & 1 & -1 & 1 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

From mentioned theorem we obtain $\mathrm{rang}(A) = \mathrm{rang}(B) = 2 < 3$, and initial system has infinitely many solutions, where number 3 denotes the number of variables in the system (2).

This means that system (2) is equivalent to the next system:

$$\begin{aligned} x + \quad y - \quad z &= 1, \\ y + 4z &= 1, \end{aligned}$$

If we take the variable z to be free, than we can express variables x and y in term of z:

$$x = 5z, \quad y = 1 - 4z.$$

In this case solution set for system (1) is $(5z, 1 - 4z, z)$, $z \in \mathbb{R}$.

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- math teacher was not a good lecturer,
- I have not practiced enough math,
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The consequence of this is that students are not motivated to continue to learn math, because they no longer have enough knowledge for it.

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Now, I will give one example.

The simplest kind of linear system involves two equations and two variables:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + a_{22}x_2 &= b_2 \end{aligned} \tag{3}$$

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If we solve system (3) with some of well-known methods, for example, with Addition method (The method of opposite coefficients), we obtain that a unique solution to the system (3) is given by

$$\begin{aligned} x_1 &= \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{21}a_{12}} \\ x_2 &= \frac{a_{11}b_1 - a_{21}b_2}{a_{11}a_{22} - a_{21}a_{12}} \end{aligned} \tag{4}$$

under condition $a_{11}a_{22} - a_{21}a_{12} \neq 0$. ▸ more about Addition method.

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The existence of a unigue solution given by (4) for system (3) is determined by the condition $a_{11}a_{22} - a_{21}a_{12} \neq 0$, which is called the second order determinant.

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- the main text is not overloaded with unnecessary information,
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- students learn at a pace that suit them,
- all the necessary mathematical knowledge are at the same location,
- more broadly, all math teachers from the same school can create repository of mathematical knowledge, from which they can pick up necessary teaching materials for creating some mathematical lecture.

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Beamer can be used either for making teaching materials for traditional students and e-learning students.

THANK YOU FOR ATTENTION!

Metod suprotnih koeficijenata predstavlja jednu od elementarnih metoda za rešavanje sistema linearnih jednačina. U slučaju sistema od dve linearne jednačine, sa dve promenljive metoda se primenjuje jednostavno. Ilustruvaćemo primerom.

Rešiti sledeći sistem metod suprotnih koeficijenata:

$$\begin{aligned} 2x + 5y &= 204 \\ 9x - 3y &= 204 \end{aligned} \tag{5}$$

Iz datog sistema eliminisaćemo promenljivu y tako što ćemo prvu jednačinu pomnožiti sa 3, drugu sa 5:

$$\begin{aligned} 2x + 5y &= 204 \quad / \cdot 3 \\ 9x - 3y &= 204 \quad / \cdot 5 \end{aligned}$$

Dakle, cilj je da se ispred promenljive y naprave suprotni koeficijenti u jednačinama sistema. Tada dobijamo:

$$6x + 15y = 612$$

$$45x - 15y = 1020$$

Sabirajući poslednje dve jednačine imamo $51x = 1632$ tj. da je $x = 32$. Vraćajući ovu vrednost u neku od jednačina početnog sistema (5), na primer prvu imamo da je $64 + 5y = 204$, tj. $5y = 140$, tj. $y = 28$.

Posmatrajmo sada opšti oblik sistema od dve linearne jednačine sa dve nepoznate:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1, \\ a_{21}x_1 + a_{22}x_2 &= b_2. \end{aligned} \tag{6}$$

Odredićemo njegova rešenja metodom suprotnih koeficijenata. S ciljem da eliminišemo promenljivu x_2, uradićemo sledeće:

$$a_{11}x_1 + a_{12}x_2 = b_1 \quad / \cdot a_{22}$$

$$a_{21}x_1 + a_{22}x_2 = b_2 \quad / \cdot (-a_{12})$$

$$a_{11}a_{22}x_1 + a_{12}a_{22}x_2 = a_{22}b_1$$
$$-a_{12}a_{21}x_1 - a_{12}a_{22}x_2 = -a_{12}b_2$$

Sabiranjem poslednje dve jednačine imamo

$$(a_{11}a_{22} - a_{12}a_{21})x_1 = a_{22}b_1 - a_{12}b_2,$$

odnosno

$$x_1 = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{21}a_{12}}.$$

Vraćajući dobijenu vrednost za x_1 u npr. prvu jednačinu početnog sistema (6), možemo da izračunamo promenljivu x_2:

$$a_{11}\frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{21}a_{12}} + a_{12}x_2 = b_1, \quad \text{odnosno} \quad a_{12}x_2 = b_1 - a_{11}\frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{21}a_{12}}$$

Sredjivanjem poslednje jednačine dobijamo:

$$a_{12}x_2 = \frac{a_{11}a_{22}b_1 - a_{21}a_{12}b_1 - a_{11}a_{22}b_1 + a_{11}a_{12}b_2}{a_{11}a_{22} - a_{21}a_{12}},$$

odnosno nakon potiranja prvog i trećeg člana iz brojioca i izvlačenja zajedničkog elementa a_{12} za preostale članove dobijamo

$$a_{12}x_2 = \frac{a_{12}(a_{11}b_2 - a_{21}b_1)}{a_{11}a_{22} - a_{21}a_{12}},$$

Dakle, nakon skraćivanja sa a_{12}, konačno je

$$x_2 = \frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{21}a_{12}}.$$

Svakako, promenljive x_1 i x_2 je moguće odrediti pod pretpostavkom da je $a_{11}a_{22} - a_{21}a_{12} \neq 0$ i tada kažemo da sistem (6) ima jedinstveno rešenje. U suprotnom sistem ima ili beskonačno mnogo rešenja ili ih nema uopšte.

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